Chapter 5 Interleaving Technique and Concatenation

Interleaving Technique and Concatination

5.1 Block Interleaving

Let C be an (n, k) linear code.

- **Suppose we take λ code words from C and arrange then intoλrows of a λ× n array as shown in the following figure. This structure is called block interleaver.**
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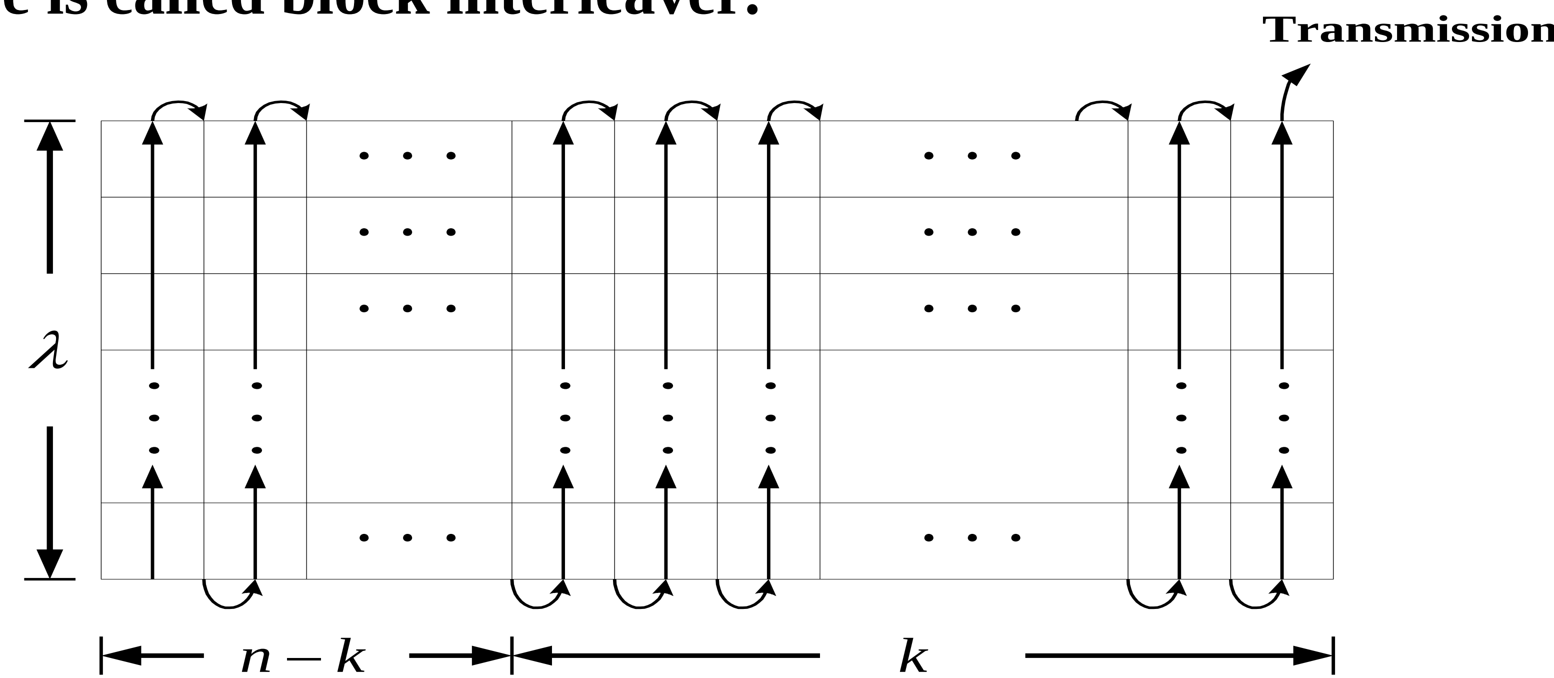


- **Then we transmit this code array column by column in serial manner. By doing this, we obtain a vector of λn digits.**
- **Note that two consecutive bits in the same codeword are now separated by λ-1 positions.**
- **Actually, the above process simply interleaves λ codewords in C. The parameter λ is called interleaving degree (or depth).**
- **There are $(2^k)^\lambda = 2^{k\lambda}$ such interleaved sequences and they form a $(\lambda n, \lambda k)$ linear code, called an interleaved code, denoted C(λ).**
- **If the base code C is a cyclic code with generator polynomial , then the interleaved code C(λ) is also cyclic.**

- **Error Correction Capability of an Interleaved Code :**

A pattern of errors can be corrected for the whole array if and only if the pattern of errors in each row is a correctable pattern for the base code C.

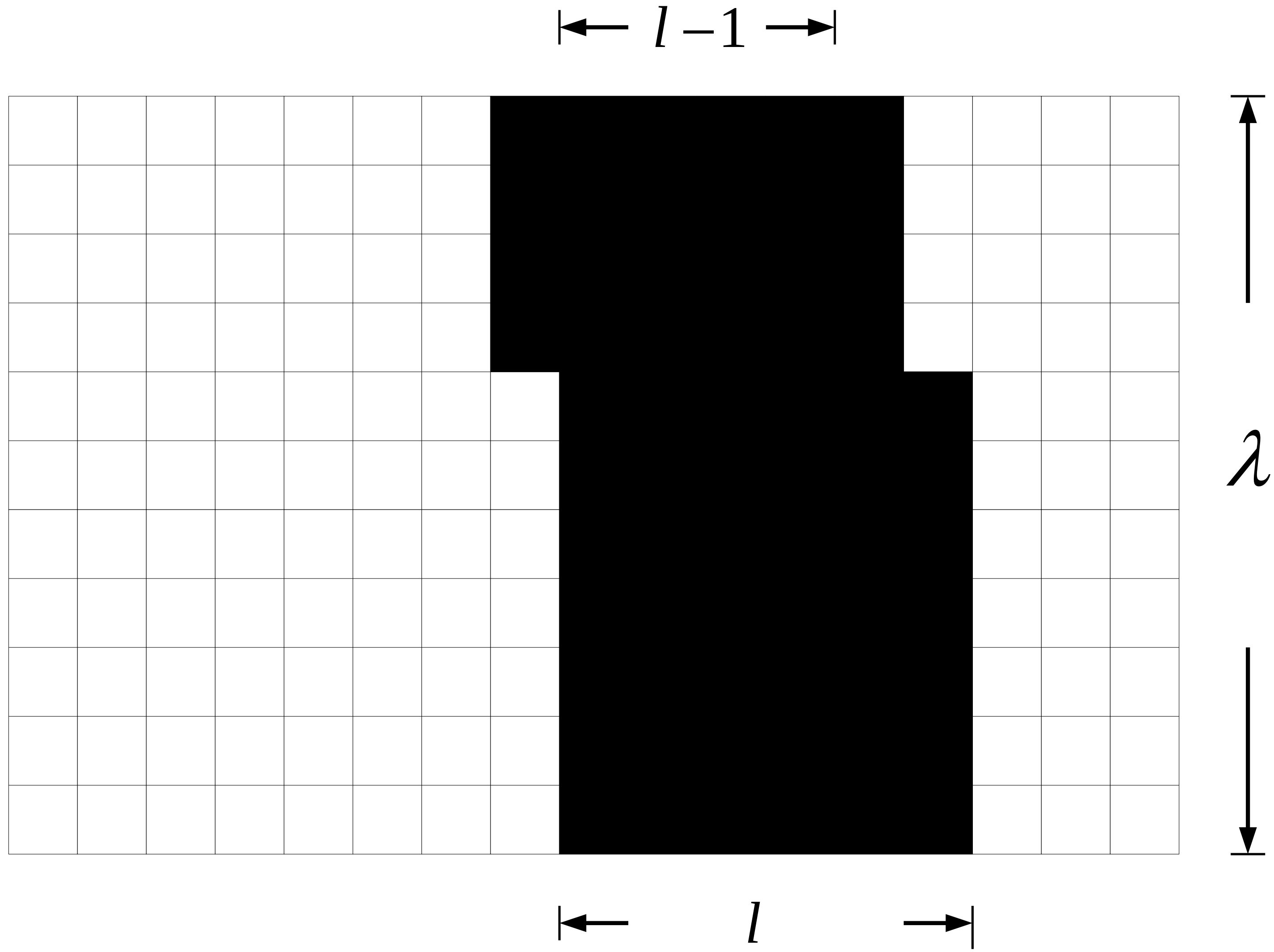
Suppose C is s single-error-correcting code.

Then a burst of length λ or less, no matter where it starts, will affect no more than one digital in each roe. This single bit error in each row will be corrected by the base code C.

Hence the interleaved code C(λ) is capable of correcting any error burst of length λ or less.

5.2 Decoding of Block-Interleaved Code

- **At the receiving end, the received interleaved sequence is de-interleaved and rearranged back to a rectangular array of λ rows.**
- **Then each row is decoded based on the base code C.**
- **Suppose the base code C is capable of correcting any burst of length l or less.**
- **Consider any burst of length λl or less. No matter where this burst starts in the interleaved code sequence, t will result a burst of length l or less in each row of the corresponding code array as shown in the following Figure 4.7.2**



$l-1$
λ
l

- **As a result, the burst in each row will be corrected by the base code C.**

  Hence the interleaved code C(λ) is capable of correcting any single error burst of length λ*l* or less.
- **Interleaving is a very effective technique for constructing long powerful burst-error correcting codes from good short codes.**
- **If the base code is an optimal burst-error-correcting code, the interleaved code is also optimal.**

5.3 Convolutional Interleaver

- **A convolutional interleaver can be used in place of a block interleaver in much the same way.**
- **Convolutional interleavers are better matched for use with the class of convolutional codes.**

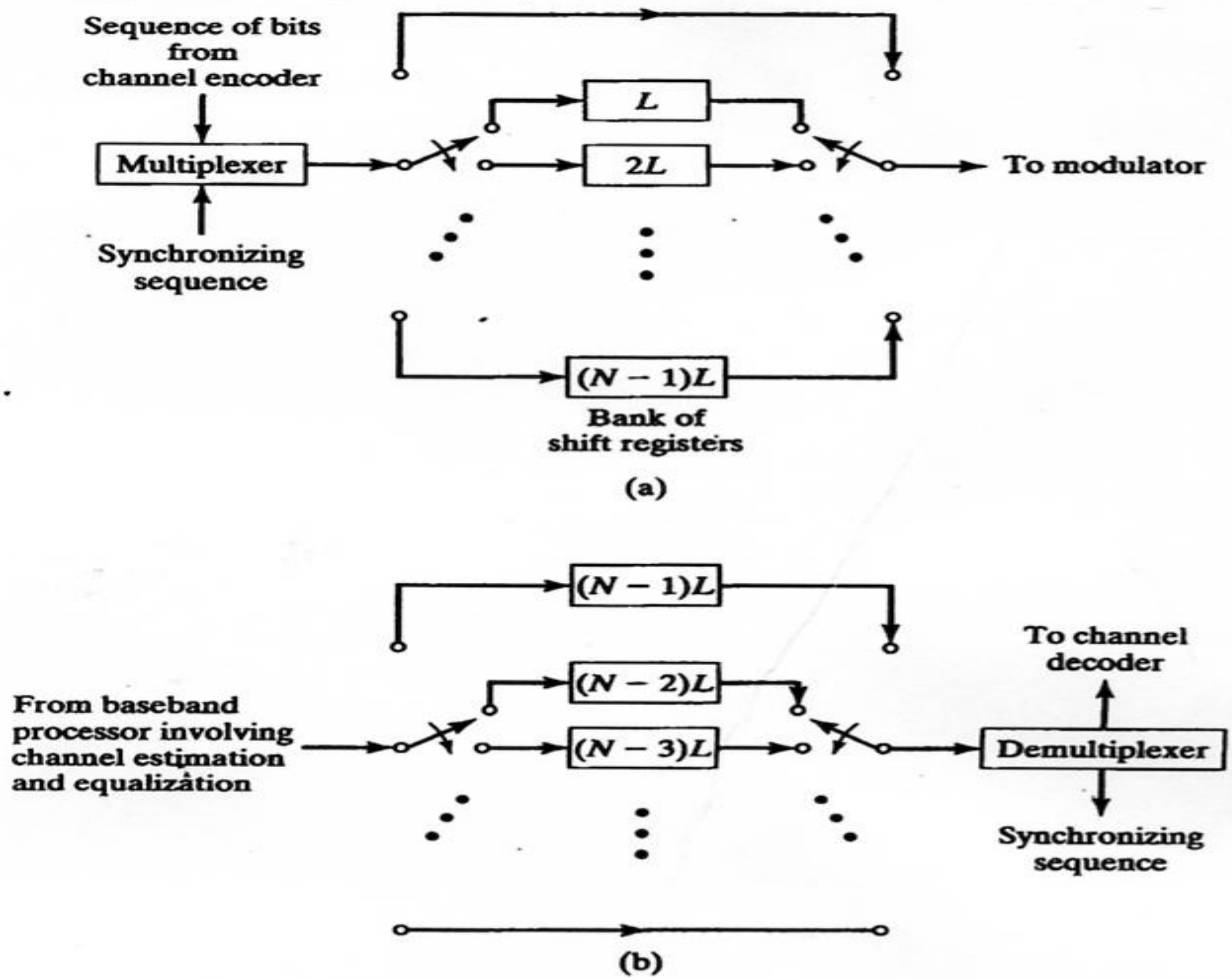


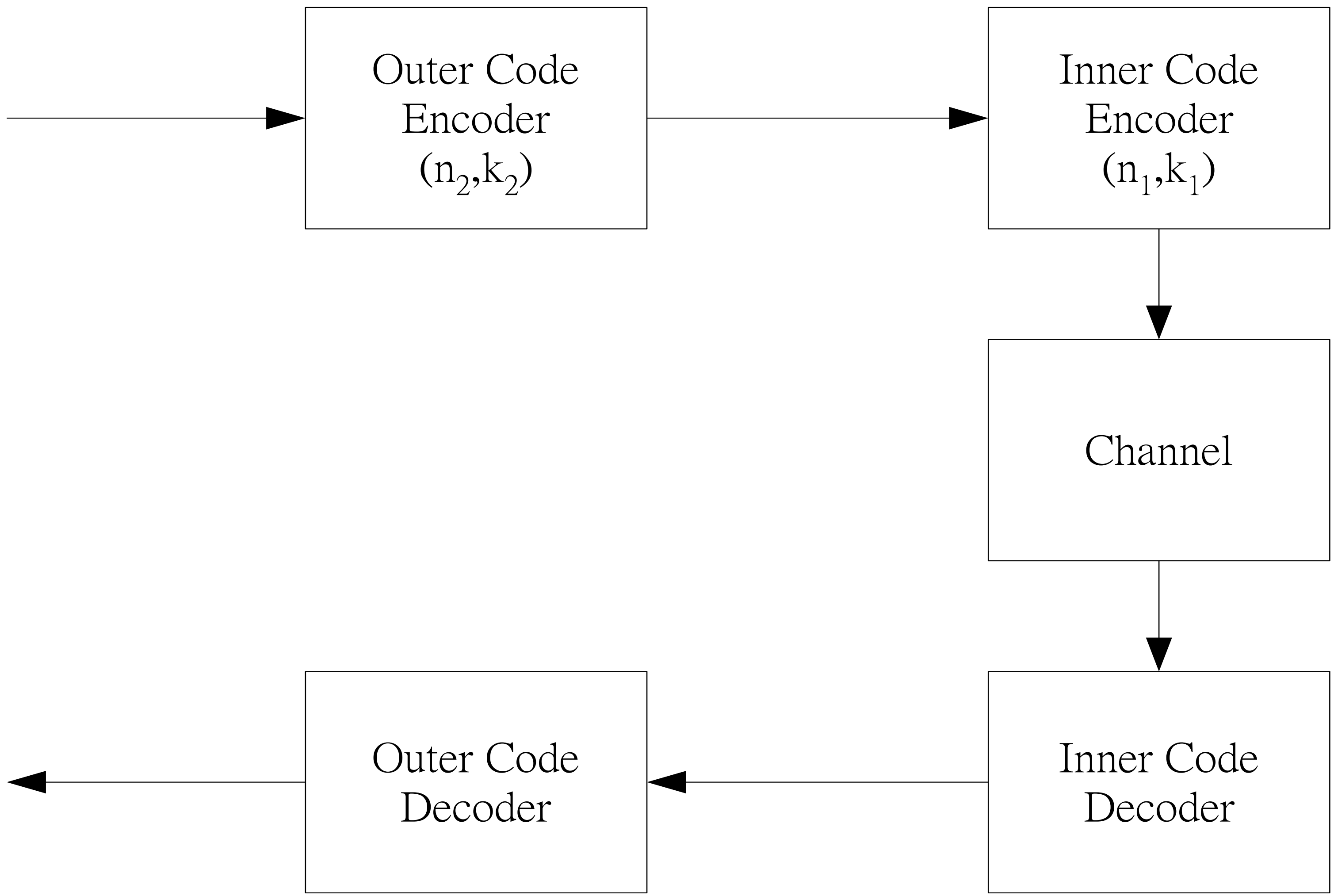
FIGURE 4.12 (a) Convolutional interleaver. (b) Convolutional deinterleaver.

5.4 Concatenated Coding Scheme

- **Concatenation is a very effective method of constructing long powerful codes from shorter codes.**
- **It was introduced by Forney in 1966.**

and is often used to achieve high reliability with reduced decoding complexity.

- **A simple serial concatenated code is formed from two codes： an (n_1 ,k_1) binary code C_1 and an (n_2,k_2) nonbinary code C_2 with symbols from GF(2^m), say a RS code.**
- **Concatenated codes are effective against a mixture of random errors and burst errors. Scattered random errors are corrected by C_1. Bursts may affect relatively few bytes, but probably so badly that C_1 cannot correct them. These few bytes can then be corrected by C_2.**



Outer Code Encoder (n_2,k_2)
Inner Code Encoder (n_1,k_1)
Channel
Inner Code Decoder
Outer Code Decoder

Encoding

- Encoding consists of two stages, the outer code encoding and the inner code encoding .

  First a message of $k_1 k_2$ bits are divided into k_2 bytes of k_1 bits each. Each k_1-bit byte is regarded as a symbol in GF(2^m).

  This k_2-byte message is encoded into an n_2-byte codeword in C_2.

  Each k_1-bit byte of   is then encoded into an n_1-bit codeword in C_2. This results in a string of n_2 codewords in C_2 , a total of $n_1 n_2$ bits.

  There are a total of   such strings which form an ($n_1 n_2$,$k_1 k_2$) binary linear code, called a concatenated code.

- C_1 is called the inner code and C_2 is called the outer code.

  If the minimum distances of the inner and outer codes are d_1 and d_2 , respectively, the minimum distance of their concatenation is at least $d_1 d_2$.

Decoding

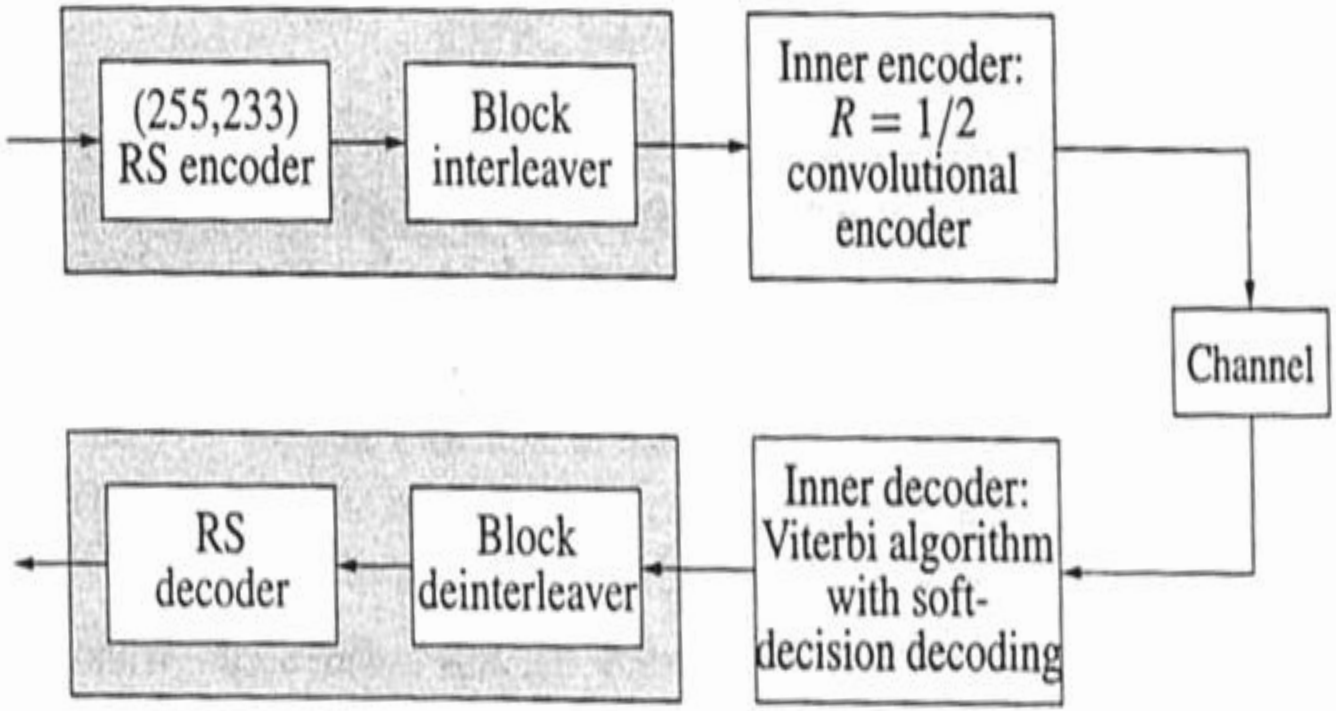
- **Decoding of a concatenated code also consists of two stages, the inner code decoding and the outer code decoding, as shown in the above figure.**
- **First, decoding is done for each inner codeword as it arrives, and the parity bits are removed. After n_2 inner codewords have been decoded, we obtain a sequence of n_2 k_1-bit bytes.**
- **This sequence of n_2 bytes is then decoded based on the outer code C_2 to give k_1k_2 decoded message bits.**
- **Decoding implementation is the straightforward combination of the implementations for the inner and outer codes.**

Error Correction Capability

- **Concatenated codes are effective against a mixture of random errors and bursts.**
- **In general, the inner code is a random-error-correcting code and the outer code is a RS code.**
- **Scattered random errors are corrected by the inner code, and bursts are then corrected by the outer code.**
- **Various forms of concatenated coding scheme are being used or proposed for error control in data communications, especially in space and satellite communications.**
- **In many applications, concatenated coding offers a way of obtaining the best of two worlds, performance and complexity.**

- **The most common concatenated code in practice is one developed in 1970s as a NASA standard . It consists of an inner rate-1/2 , 64 state convolutional code with minimum distance d=10 along with an outer (255,223, 33) RS code over GF(2^8). The inner decoder uses soft-decision Viterbi decoding , while the outer decoder uses the hard-decision Berlekamp-Massey algorithm .**
- **The RS code is capable of correcting up to 16 8-bit symbols**

To provide for the possibility of decoding bursts exceeding $16 \times 8 = 128$ bits, a symbol interleaver is placed between the RS encoder and the convolutional encoder. Since it is a symbol interleaver, burst errors which occupy a single byte are still clustered together. But bursts crossing several bytes are randomized. Block interleavers holding from 2 to 8 Reed-Solomon codewords have been employed. By simulation studies [133], it is shown that to achieve a bit error rate of 10^{-5}, with interleavers of sizes of 2, 4, and 8, respectively, an E_b/N_0 of 2.6 dB, 2.45 dB, and 2.35 dB, respectively, are required. Uncoded BPSK performance would require 9.6 dB; using only the rate 1/2 convolutional code would require 5.1 dB, so the concatenated system provides approximately 2.5 dB of gain compared to the convolutional code alone.


(255,233) RS encoder
Block interleaver
Inner encoder: R = 1/2 convolutional encoder
Channel
Inner decoder: Viterbi algorithm with soft-decision decoding
Block deinterleaver
RS decoder

Notes :

Code concatenation is a multilevel coding method allowing codes with good asymptotic as well as practical properties to be constructed. The idea date back to Elias's product code construction in the mid 1950s . Amajor advance was performed by Forney in his thesis work on concatenated codes ten years later.

As stated by Forney , concatenation is a method of building long codes out of short ones in order to resolve the problem of decoding complexity by breaking the required computation into manageable segments according to the divide and conquer strategy.

The principle of concatenation is applicable to any types of codes, convolutional or block codes.

Before the invention of turbo codes, the most famous example of concatenated codes was the concatenation of an outer algebraic c , such as RS code , with an inner convolutional code , which has been used in numerous applications, ranging from space communications to digital television broadcasting .

- **Turbo codes are parallel concatenated codes using recursive systematic convolutional codes as their constituent codes.**

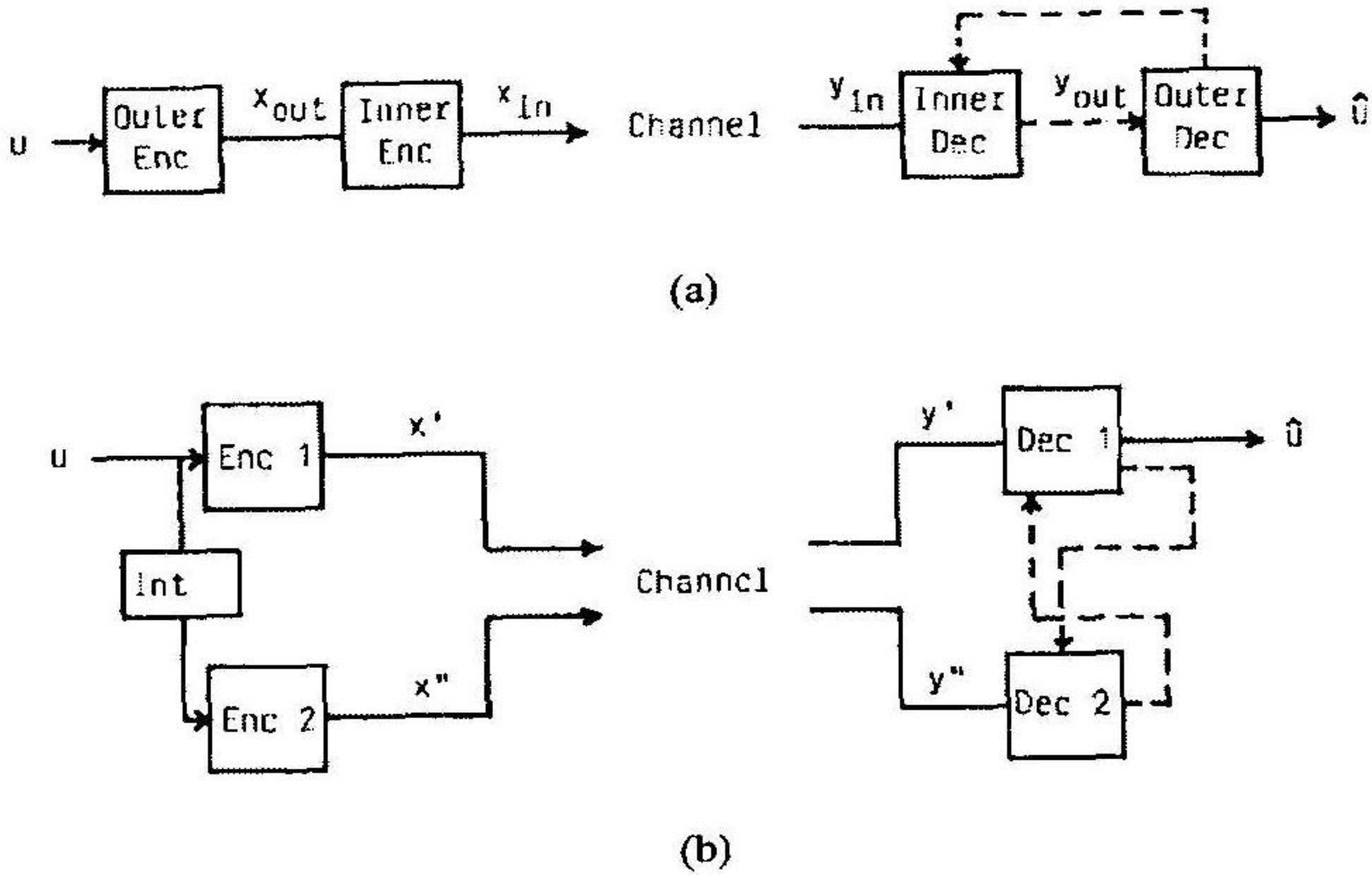


Fig. 1. (a) Serial and (b) parallel concatenation.

5.5 Product Codes

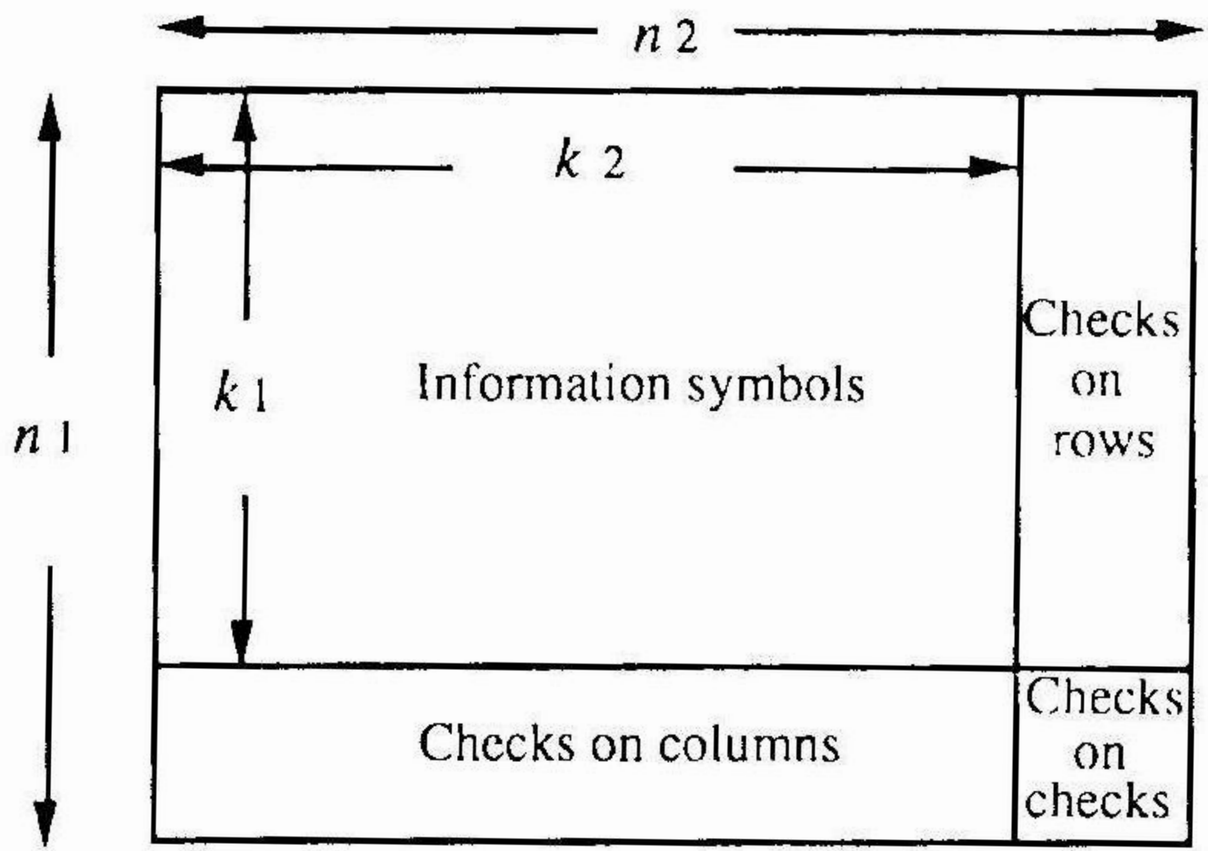
- **A product code is obtained by using an (n_1 , k_1) inner block code C_1 , symbol-interleaving it to a degree k_2 , and then applying an (n_2 , k_2) outer block code C_2 . This yields an**

  ($n_1 n_2$, $k_1 k_2$) linear block code .

  The $k_1 k_2$ digits in the upper left corner of the array are information symbols . The digits in the upper right corner of the array are computed from the parity-check rules for C_1 on rows , and the digits in the lower left corner are computed from the parity-check rules for C_2 .

  Transmission can be carried out either column by column or row by row.

  Decoding can be performed by decoding the block code in one dimension , followed by the decoding in the other direction.



Construction of product code $\mathcal{P} = C^1 \otimes C^2$.

- **Error-correcting capability :**

  If code C_1 has minimum distance d_1 and code C_2 has minimum weight d_2 , capable of correcting t_1 and t_2 errors, respectively.

  In general, the minimum distance of the product code is exactly $d_1 d_2$, capable of correcting $2 t_1 t_2 + t_1 + t_2$ *errors.*

Reference : **J.G Proakis, Digital Communications, pp. 477-478**