# 7.6 Extrinsic Information Transfer (EXIT)

## 7.6.1. Introduction to EXIT

- **In each iteration of the turbo code decoding , the extrinsic information is updated while the channel LLRs remain fixed. It is obvious that , for the decoder to converge , the extrinsic information should be providing improved information about the transmitted bits at each iteration.**

- **The mutual information $I(X;E)$ between the extrinsic information E and the transmitted symbols x can be used to measure the usefulness of the extrinsic information A.**

- **Also it is known that the bit error rate curves for turbo codes have a distinct waterfall region : once $E_b / N_0$ crosses a threshold , the BER decays sharply.**

- **Extrinsic information transfer (EXIT) chart provide a computationally efficient tool for visualizing the progress of iteration of iterative decoding , showing what happen before and after the waterfall region.**

## 7.6.2 Mutual Information

- **The mutual information between two random variables, $X$ and $Y$, is expressed by**

$$I(X;Y) = H(X) - H(X \mid Y) \qquad (7.44)$$

**The mutual information $I(X;Y)$ gives the amount of uncertainty in $X$ that is removed by knowing $Y$ . It can be calculated as**

$$I(X;Y) = \sum_x \sum_y p(x,y) \log_2 p(x,y) / p(x)p(y) \qquad (7.45)$$

**When $p(x,y) = p(x)\, p(y)$ , $I(X;Y) = 0$**

- **The binary-input additive white Gaussian noise (BI-AWGN ) channel can be described by the equation**

$$y_i = \mu x_i + n_i$$

**where $x_i$ is the transmitted symbol , $y_i$ is the received symbol and $n_i$ is the additive Gaussian noise with zero-mean and variance $\sigma^2 = 2\mu$ .**

**The probability density function ( pdf ) for $n_i$ is**

$$p(n_i) = \{1/\sqrt{(2\pi\sigma^2)}\}\ \exp(-n_i^{\,2} / 2\sigma^2)$$

$$= \{1/\sqrt{(2\pi\sigma^2)}\}\ \exp(-n_i^{\,2} / 2\sigma^2) \qquad (7.46)$$

**The mutual information $I(X;Y)$ is then**

$$I(X;Y) = \Sigma_x \int p(x,y) \log_2 \{ p(x,y) / p(x)p(y)\} dy$$

$$= \Sigma_x \int p(y \mid x)\, p(x) \log_2 p(y \mid x) / p(y)\} dy \qquad (7.47)$$

**the conditional pdf $p(y \mid x)$ is given by**

$$p(y \mid x) = 1/\sqrt{(2\pi\sigma^2)}\ \}\ \exp\{-[y - (\sigma^2 x/2)]^2 / 2\sigma^2\}$$

**(7.48)**

**Then we obtain**

$$I(X;Y) = 1- \{1/ \sigma\sqrt{( 2\pi)} \int \exp[- (y-\sigma^2/2)^2 / 2\sigma^2 ] \log_2(1+e^{-y}) \, dy \} \qquad (7.49)$$

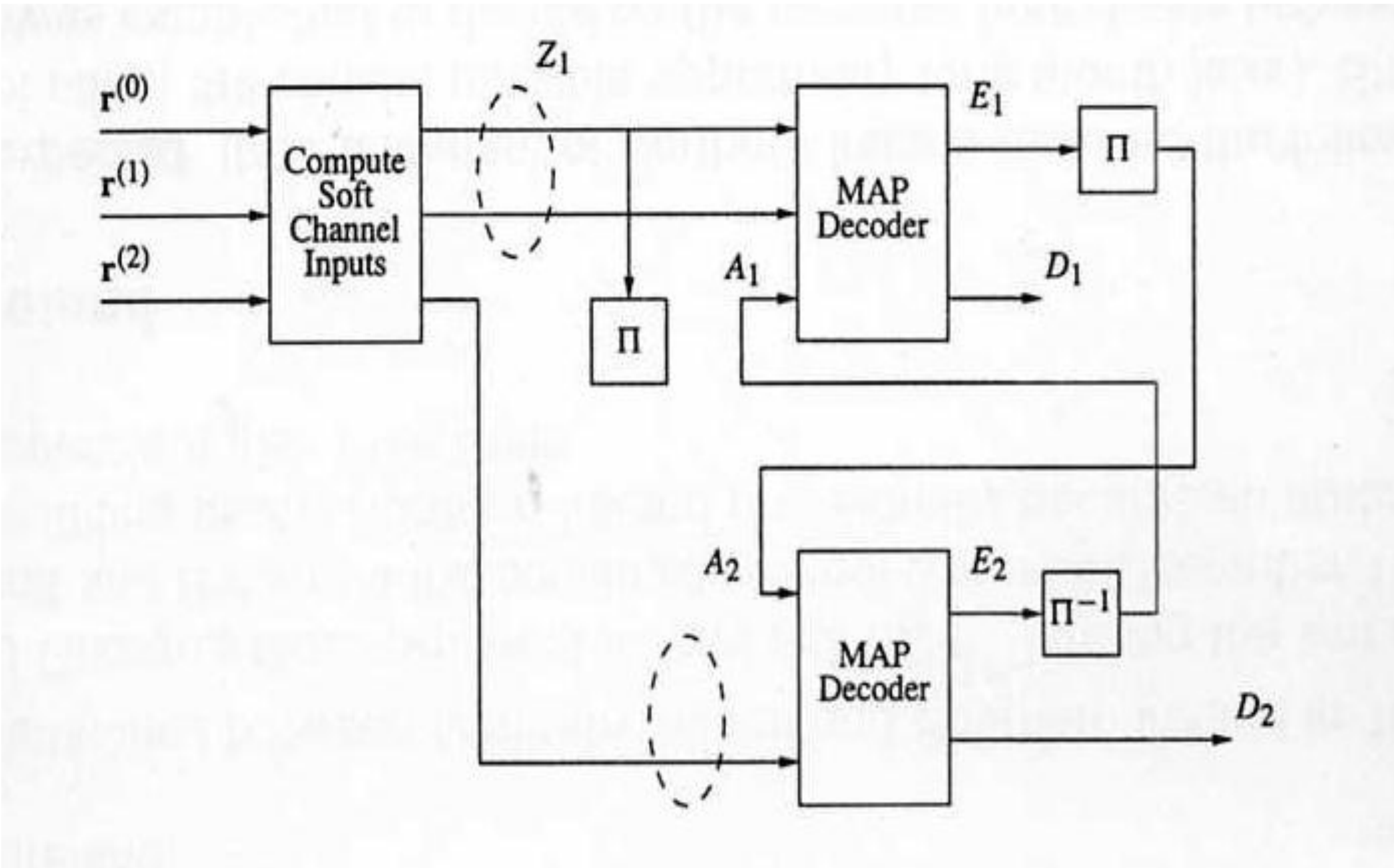
## 7.6.3. EXIT Chart

- **Fig.7.xx shows an iterative decoder for parallel-concatenated codes. The variables used for EXIT chart analysis are also shown in the figure. Two MAP decoders are used in the iterative decoder.**
- **The a priori information is labeled as $A_i$ , $i = 1,2$ .**

**The extrinsic information is $E_i$ , the decoder output information is $D_i$, and the soft input information is $Z_i$, with**

$$D_i = Z_i + A_i + E_i \qquad (7.50)$$

# Fig. 7.x1 iterative decoder for parallel-concatenated codes



- **The key concept of the EXIT chart is measuring the amount of information that the prior A conveys about the transmitted data X and that the extrinsic information conveys about the data information X . This information is measured using mutual information in the form of**

  **$I\,(X;\,A_i)$ and $I\,(X;\,E_i)$ .**

- **The received discrete-time signal from the AWGN channel is expressed as**

  $$z = x + n$$

  **The conditional probability density function $p\,(z \mid x)$ is given by**

  $$p\,(z \mid x) = \{1/\sqrt{(2\pi\sigma_n^{\,2})}\}\,\exp\,(-\,(z-x)^2 / 2\sigma_n^{\,2})$$

- **The soft output from the channel is obtained from the LLR of z is**

  $$Z = \ln\,\{\,p\,(z \mid x = 1) / p\,(z \mid x = -1)\,\} = 2\,(x + n) / 2\sigma_n^{\,2}$$

- **The above equation can also be formulated as**

$$Z = \mu_z x + n_z$$

**with $\mu_z = 2 / \sigma_n^{\,2}$**

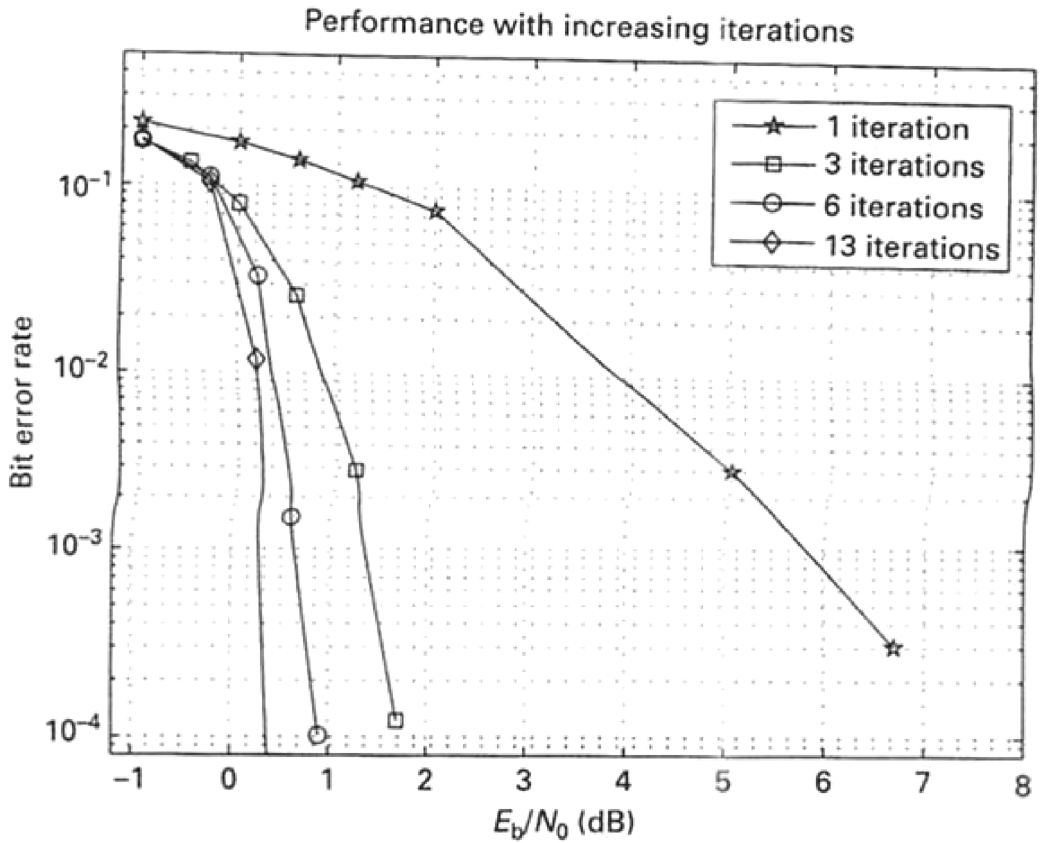
**and $n_z$ being Gaussian distributed with zero-mean and variance $\sigma_z^{\,2} = 4 / \sigma_n^{\,2} = 2\,\mu_z$**

- **The following assumptions are made for the analysis :**

**(1) For sufficiently large interleavers , the a priori values $A_i$ are fairly uncorrelated from their respective channel observations $Z_i$ over many iterations .**

**(2)The probability density function function of the extrnsic information $E_i$ –which are the prior inoputs for the next MAP decoder – approach Gaussian distribution with increasing iterations.**

**Bit error rate as a function of number of iterations for a rate 1/3 turbo code**



- **The mutual information between X and E can be computed by**

$$I_E = I(X;E)$$
$$= (1/2) \int_{-\infty}^{\infty} p_E(y \mid X = x)$$
$$\log_2 \{2\, p_E(y \mid X = x) /$$
$$[\, p_E(y \mid X = +1) + p_E(y \mid X = -1)]\}$$

$$0 \leqq I_E \leqq 1$$

- **There is some functional relationship between $I_A$ , $E_b$ , and $I_E$ ,denoted abstractly as**

$$I_E = T(I_A , E_b / N_0)$$

**For a fixed $E_b / N_0$ , *we have***

$$I_E = T(I_A)$$

**The function *T* denotes the " transfer " of information from the prior information *A* at the input of a decoder to the extrinsic information *E* at the output of the decoder.**

- **Mutual information I(X;A) :**

**We can model the a priori probability input to a decoder as**

$$A_i = \mu_A x_i + n_A$$

**The conditional pdf belonging to the LLR of *A* is**

$$p_A ( y \mid X = x )$$

$$=\{1/ \sqrt{( 2\pi\sigma_A^2 )} \} \exp [- (y- \sigma_A^2 /2 )^2 / 2\sigma_A^{2} ]$$

**Then the mutual information between X and A can be computed by**

$$I(X;A) = I_A (\sigma_A )$$

$$= 1- \{1/ \sigma\sqrt{( 2\pi)} \cdot$$

$$\int_{-\infty}^{\infty} \exp[- (y- \sigma_A^2 /2)^2 / 2\sigma_A^{2} ] \log_2(1+e^{-y} ) dy \}$$

**For abbreviation wedefine** $J(\sigma) = I_A (\sigma_A = \sigma )$

- **Consider the iterative decoder shown in Fig. 7.xx .**

**The extrinsic information $E_1$ at the output of the first decoder is permuted and used as the prior information $A_2$ at the next decoder .**

**Let $I_{A1}^{[n]}$ denote the mutual information $I(X;A_1)$ at the *n*th iteration of the first decoder , starting with zero a priori knowledge $I_{A1}^{[0]} = 0$ .**

**Similarly let $I_{E1}^{[n]} = I(X;E_1)$ denote the mutual information $I(X;E_1)$ at the output of the first decode at the nth iteration , $I_{E1}^{[n]} = T_1(I_{A1}^{[n]})$ . This is forwarded to to the next decoder to become $I_{A2}^{[n]} = I_{E1}^{[n]}$ . This passes through the second decoder to become $I_{E2}^{[n]} = T_2(I_{A2}^{[n]})$, which in turn is passed back to the first decoder as the prior , $I_{A1}^{[n+1]}$ .**

**Fig.7.x2shows the mutual information $I_E$ at the output of a single decoder as a function of the mutual information $I_A$ at the input pf the decoder .**

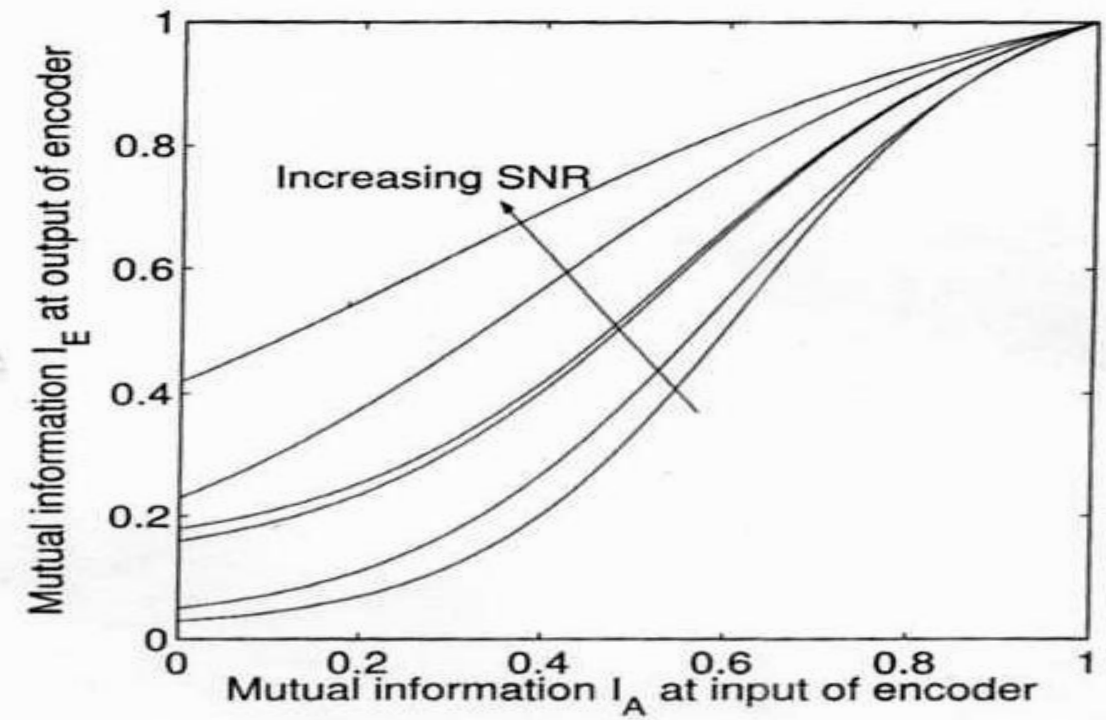
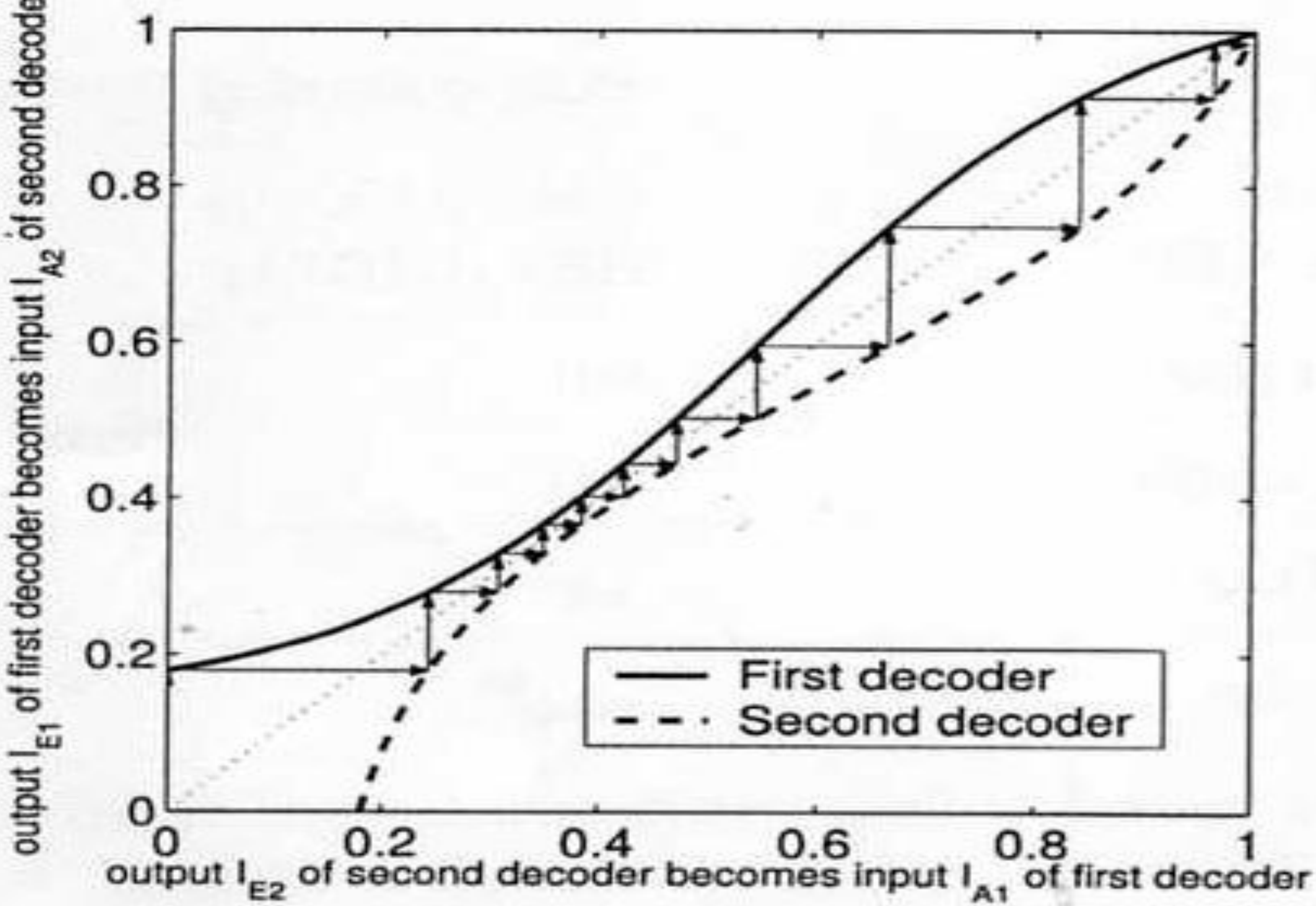


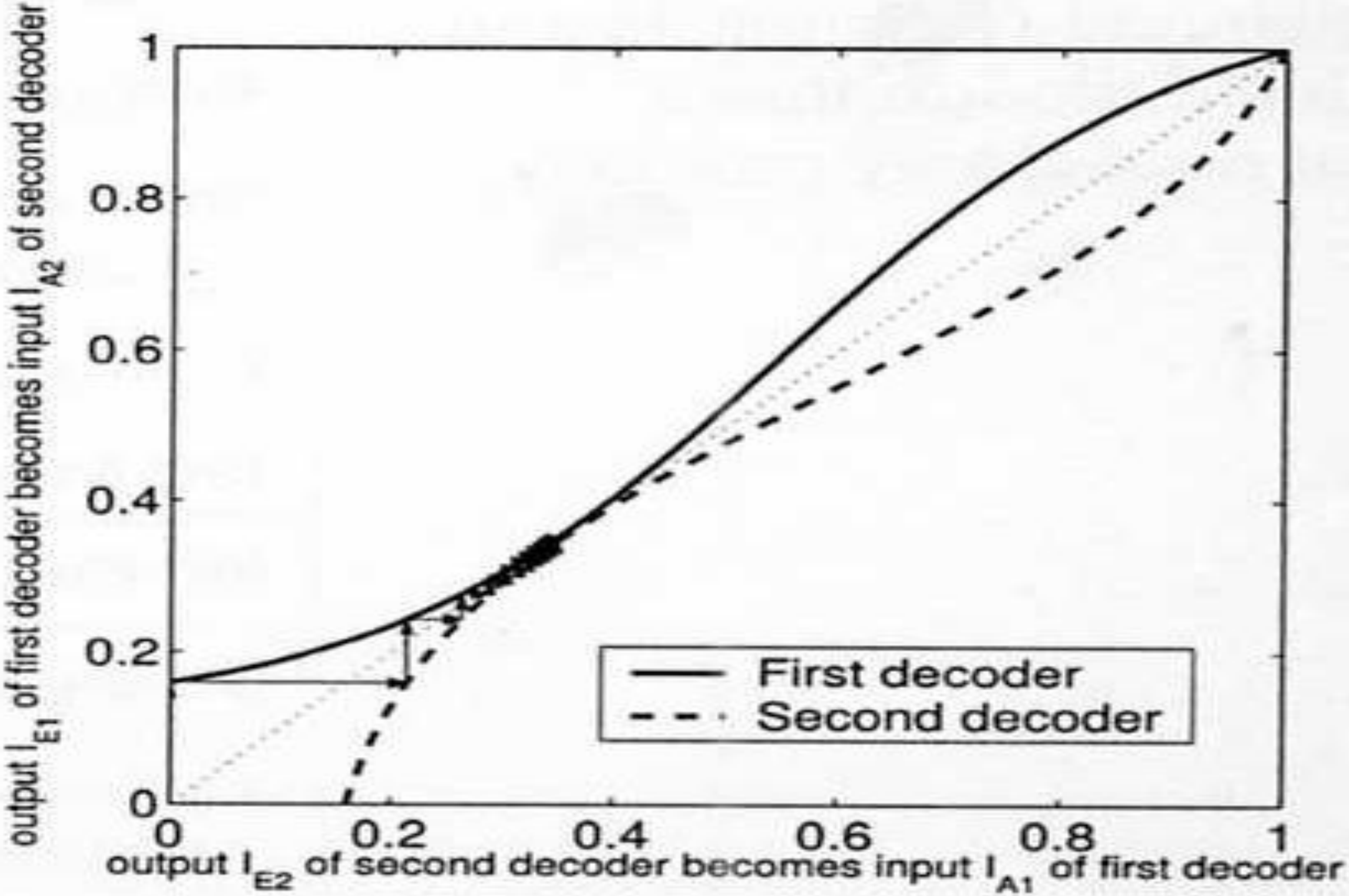
**Fig.7.x2a) shows the information decoding ina swquence of decoding steps, following the arrows. Ultimately , a point is reached where $I_A$ = *1* . If the prior information about a bit is sufficiently close to 1 , then we conclude that the prior information is sufficien tto accurately decode the bit . There is a “channel “ or “gap” between the two curves . The decoding proceeds by walking through this channel.**

**Fig.7.x2(b) shows that the iterative decoding process can break down for a lower SNR . In trhis case , the function $T(I_A)$ crosses the y =x line . As a result , the iterations get Stuck at the crossover point.**

**Thus, there is a threshold of SNR for the decoder to be able to decode correctly. The decoder can not correctly decode , no matter how many times the decoder iterates, if the SNR is below the threshold.**




(a) Decoding above threshold.




(b) Decoding below threshold. The decoder cannot get past the pinchoff point.

**Reference**

**ten Brink,S., “ Convergence Behaviors of Iteratively Decoded Parallel Concatenated Codes ,” IEEE Trans. Commun. Vol.49, no.10 ,pp.1727-1737, Oct. 2001.**