Homework #2

Principles of Communications, Fall 2009

Due on Oct. 19, 2009

1. Problem 2.28 on page 104. (11%)

2.28.

a. Given $\Pi(t) \longleftrightarrow \text{sinc}\, f$, find the Fourier transforms of the following signals using the frequency translation followed by the time delay theorem.

i. $x_1(t) = \Pi(t-1)\exp[j4\pi(t-1)]$.

ii. $x_2(t) = \Pi(t+1)\exp[j4\pi(t+1)]$.

b. Repeat the above, but now applying the time delay followed by the frequency translation theorem.

2. Problem 2.46 on page 105. (11%)

2.46. A system is governed by the differential equation (a, b, and c are nonnegative constants)

$$\frac{dy}{dt} + ay = b\frac{dx}{dt} + cx$$

a. Find $H(f)$.

b. Find and plot $|H(f)|$ and $\underline{/H(f)}$ for $c = 0$.

c. Find and plot $|H(f)|$ and $\underline{/H(f)}$ for $b = 0$.

3. Problem 2.54 on page 107. (11%)

2.54. Given a filter with frequency-response function

$$H(f) = \frac{5}{4 + j(2\pi f)}$$

and input $x(t) = e^{-3t}u(t)$, obtain and plot accurately the energy spectral densities of the input and output.

4. Problem 2.56 on page 107. (11%)

2.56. A filter has amplitude response and phase shift shown in Figure 2.40. Find the output for each of the inputs given below. For which cases is the transmission distortionless? Tell what type of distortion is imposed for the others.

a. $x_1(t) = \cos(48\pi t) + 5\cos(126\pi t)$.

b. $x_2(t) = \cos(126\pi t) + 0.5\cos(170\pi t)$.

c. $x_3(t) = \cos(126\pi t) + 3\cos(144\pi t)$.

d. $x_4(t) = \cos(10\pi t) + 4\cos(50\pi t)$.

5. Problem 2.58(a) on page 107. (11%)

2.58. Determine and accurately plot, on the same set of axes, the group delay and the phase delay for the systems with unit impulse responses:

a. $h_1(t) = 3e^{-5t}u(t)$.

b. $h_2(t) = 5e^{-3t}u(t) - 2e^{-5t}u(t)$.

6. Problem 2.69 on page 109. (11%)

2.69. Given the bandpass signal spectrum shown in Figure 2.43, sketch spectra for the following sampling rates f_s and indicate which ones are suitable: (a) $2B$, (b) $2.5B$, (c) $3B$, (d) $4B$, (e) $5B$, (f) $6B$.

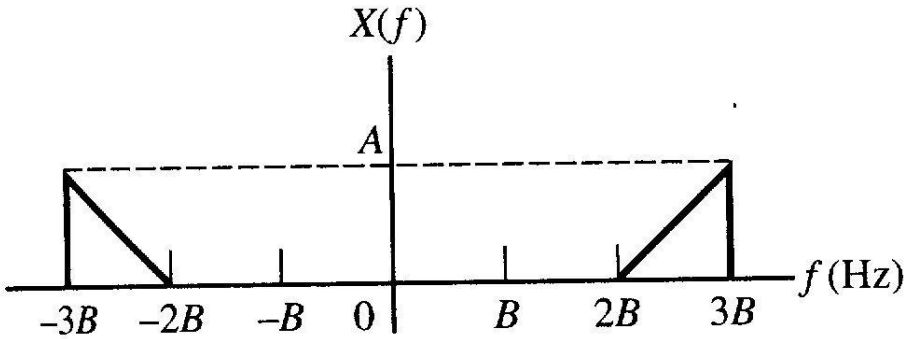

Figure 2.43

7. Problem 2.70 on page 109. (11%)

2.70. Determine the range of permissible cutoff frequencies for the ideal lowpass filter used to reconstruct the signal

$$x(t) = 10\cos(600\pi t)\,\cos^2(2400\pi t)$$

which is sampled at 6000 samples per second. Sketch $X(f)$ and $X_\delta(f)$. Find the minimum allowable sampling frequency.

8. Compute the FFT (DFT) of a rectangular pulse and plot its magnitude and phase spectra for the following cases. (Ref: Computer Example 2.3 on page 93) The rectangular pulse has unit height (height =1) and unit duration (width=1) and is centered at 0 (coordinate). (23%)

(a) Take 8 samples at nT, T=1/4, n=-3,…,4.

(b) Take 256 samples at nT, T=1/128, n=-127,…,128.

(c) Take 256 samples at nT, T=1/64, n=-127,…,128.

COMPUTER EXAMPLE 2.3

The MATLAB program given below computes the fast Fourier transform (FFT) of a double-sided exponentially decaying signal truncated to $-15.5 \leq t \leq 15.5$ sampled each $T_s = 1$ s. The periodicity property of the FFT means that the resulting FFT coefficients correspond to a waveform that is the periodic extension of this exponential waveform. (Figure 2.33). The frequency extent of the FFT is $[0, f_s(1-1/N)]$ with the frequencies above $f_s/2$ corresponding to negative frequencies.

```
% file: c2ce3
%
clf
tau = 2;
Ts = 1;
fs = 1/Ts;
ts = -15.5:Ts:15.5;
N = length(ts);
fss = 0:fs/N:fs-fs/N;
xss = exp(-abs(ts)/tau);
Xss = fft(xss);
t = -15.5:.01:15.5;
f = 0:.01:fs-fs/N;
X = 2*fs*tau./(1+(2*pi*f*tau).^2);
subplot(2,1,1), stem(ts, xss)
hold on
subplot(2,1,1), plot(t, exp(-abs(t)/tau), '-'), xlabel('t, s'), ylabel
('Signal & samples'),...
legend('x(nT_s)', 'x(t)')
```

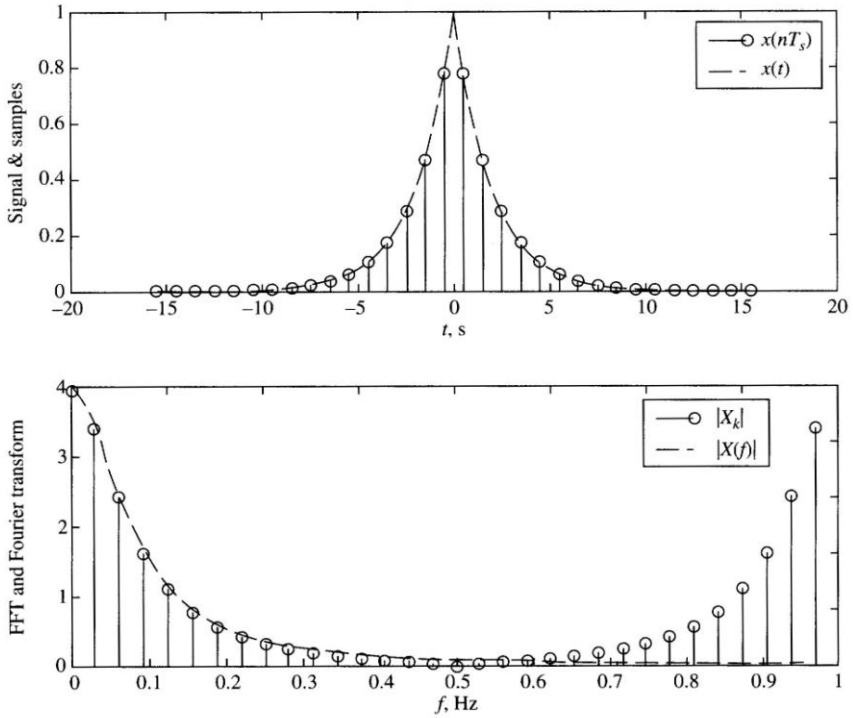

Figure 2.33
(a) $x(t) = \exp(-|t|/\tau)$ and samples taken each $T_s = 1$ s for $\tau = 2$ s. (b) Magnitude of the 32-point FFT of the sampled signal compared with the Fourier transform of $x(t)$. The spectral plots deviate from each other around $f_s/2$ due to aliasing.

```
subplot(2,1,2), stem(fss, abs(Xss))
hold on
subplot(2,1,2), plot(f, X, '-'), xlabel('f, Hz'), ylabel('FFT and Fourier
transform')
legend('|tX_k|'. '|X(f)|')
```

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