Chapter I -- Part II

Overview of

Digital Communications

Chapter I Introduction

1.1 Brief History of Digital Communications

References

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3. Massey, J.L.," Information Theory : The Copernican System of Communications," IEEE Communications Magazine , Vol.22 ,No.12 , pp. 26-28, Dec. 1984.
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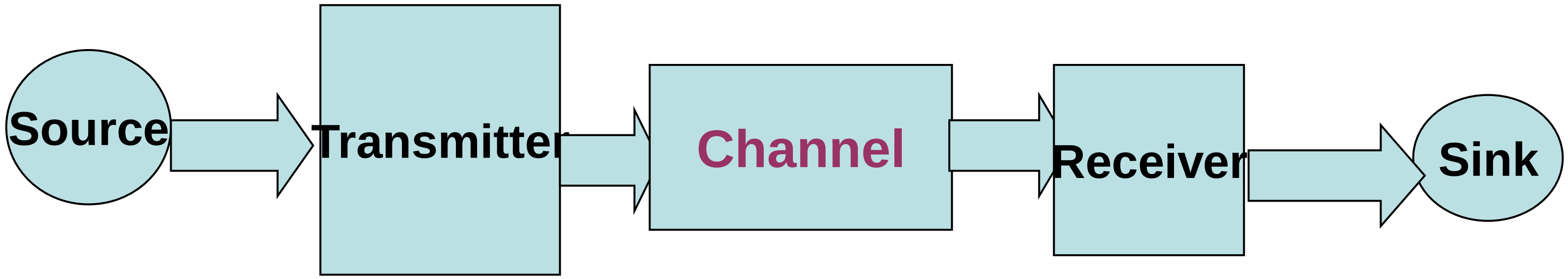
1.2 Digital Communication System Block Diagram



Transmitter End :

Coding :

Encryption

Error- Correction coding

Modulation

Transmitting Antenna

Receiver End :

Decoding

Demodulation

Receiving Antenna

1.3 Nyquist–Shannon Sampling Theorem

- The Nyquist–Shannon sampling theorem is a fundamental result in the field of information theory.
  It is often referred to as simply *the sampling theorem*.
- Sampling is the process of converting a signal (for example, a function of continuous time or space) into a numeric sequence (a function of discrete time or space).
- The theorem states that
  "Exact reconstruction of a continuous-time baseband signal from its samples is possible if the signal is bandlimited and the sampling frequency is greater than twice the signal bandwidth.".

1.3 Introduction to information Theory

Source Coding Theorem

Channel Coding Theorem

Information Capacity Theorem

A . Shannon's Information Theory

The theory provides answers to two fundamental questions (among others):

- What is the irreducible complexity below which a signal cannot be compressed?
- What is the ultimate transmission rate for reliable communication over a noisy channel?

B. Source Coding Theorem (Shannon's first theorem)

- The theorem can be stated as follows:

  Given a discrete memoryless source of entropy , the average code-word length for any distortionless source coding is bounded as

  $$L \geq H(S)$$

  where H(s) is the entropy of the source

- This theorem provides the mathematical tool for assessing data compaction, i.e. lossless data compression, of data generated by a discrete memoryless source.
- The entropy of a source is a function of the probabilities of the source symbols that constitute the alphabet of the source.

- **Memoryless source : The source is memoryless if successive symbols emitted by the source are statistically independent.**
- **Entropy of Discrete Memoryless Source L :**

Assume that the source output is modeled as a discrete random variable, , which takes on symbols from a fixed finite alphabet

$$\mathbf{S} = \{s_0, s_1, \cdots, s_{K-1}\}$$

with probabilities

$$P(S = s_k) = p_k, \quad k = 0,1,2,\cdots,K-1 \quad \text{with} \quad \sum_{k=0}^{K-1} p_k = 1$$

C. Channel Coding Theorem (Shannon's 2nd theorem)

- **Discrete Memoryless Channel :**

  A discrete memoryless channel is a statistical model with an input X and an output Y that is a noisy version of X. Both X and Y are discrete random variables .

  $X = \{ x_1, x_2, \cdots, x_N \}$, $Y = \{ y_1, y_2, \cdots, y_N \}$

  It is memory if the current output symbol depends only the current input symbol and not any previous ones.

- The channel coding theorem for a discrete memoryless channel is stated in two parts as follows:
- Let a discrete memoryless source with an alphabet have entropy and produce symbols once every T_s seconds. Let a discrete memoryless channel have capacity C and be used once every T_c seconds.

- **Then if**

$$H(s)/T_s \leqq C / T_c$$

there exists a coding scheme for which the source output can be transmitted over the channel and be reconstructed with an arbitrarily small probability of error.

- **Conversely, if**

$$H(s)/T_s \geqq C / T_c$$

It is not possible to transmit information over the channel and reconstruct it with an arbitrarily small probability of error.

- **The theorem specifies the channel capacity as a fundamental limit on the rate at which the transmission of reliable error-free message can take place over a discrete memoryless channel.**
- **Note : C** ***is defined as the maximum rate of reliable transmission over the channel.***

- The amount of information gained after observing the event is defined as the logarithmic function

$$I(s_k) = log_2(\frac{1}{p_k}) \text{ bits}$$

The entropy of the source is defined as the mean of over source alphabet given by

$$\begin{aligned} H(\mathbf{S}) &= E[I(s_k)] \\ &= \sum_{k=0}^{K-1} p_k I(s_k) \\ &= \sum_{k=0}^{K-1} p_k log_2(\frac{1}{p_k}) \quad \text{bits} \end{aligned}$$

- The entropy is a measure of the average information content per source symbol.

D. Information Capacity Theorem

(also known as Shannon-Hartley law or Shannon's 3rd theorem)

- It can be stated as follows:

The information capacity of a continuous channel of bandwidth *B* Hz , perturbed by additive white Gaussian noise of power spectral density $N_0/2$ and limited in bandwidth to *B* , is given by

$$C = B \log_2 \left(1 + \frac{P}{N_0 B}\right) \quad \text{bits/second}$$

where *P* is the average transmitted power.

- This theorem implies that, for given average transmitted power and channel bandwidth , we can transmit information at the rate *C* bits per second, with arbitrarily small probability of error by employing sufficiently complex encoding systems..

- **Shannon's Information Capacity Theorem can also expressed as , for digital communications) as**

  $C = B \log_2 (1+ E_b R / N_0 B)$

  where E_b **= bit energy of the transmitted signal in *joules***

  R **= data rate in *bits/s***

  N_0 **= single-sided noise power spectral density**

- **In digital communications , we more often use** E_b / N_0 **, *a normalized version of SNR* , as a figure of merit .**

  $E_b / N_0 = S T_b / (N/B) = (S/N) (B /R)$

Shannon limit:

- **For an ideal system that transmits data at a rate**

$$R_b = C$$

Then $$P = E_b \bullet R_b = E_b \bullet C$$

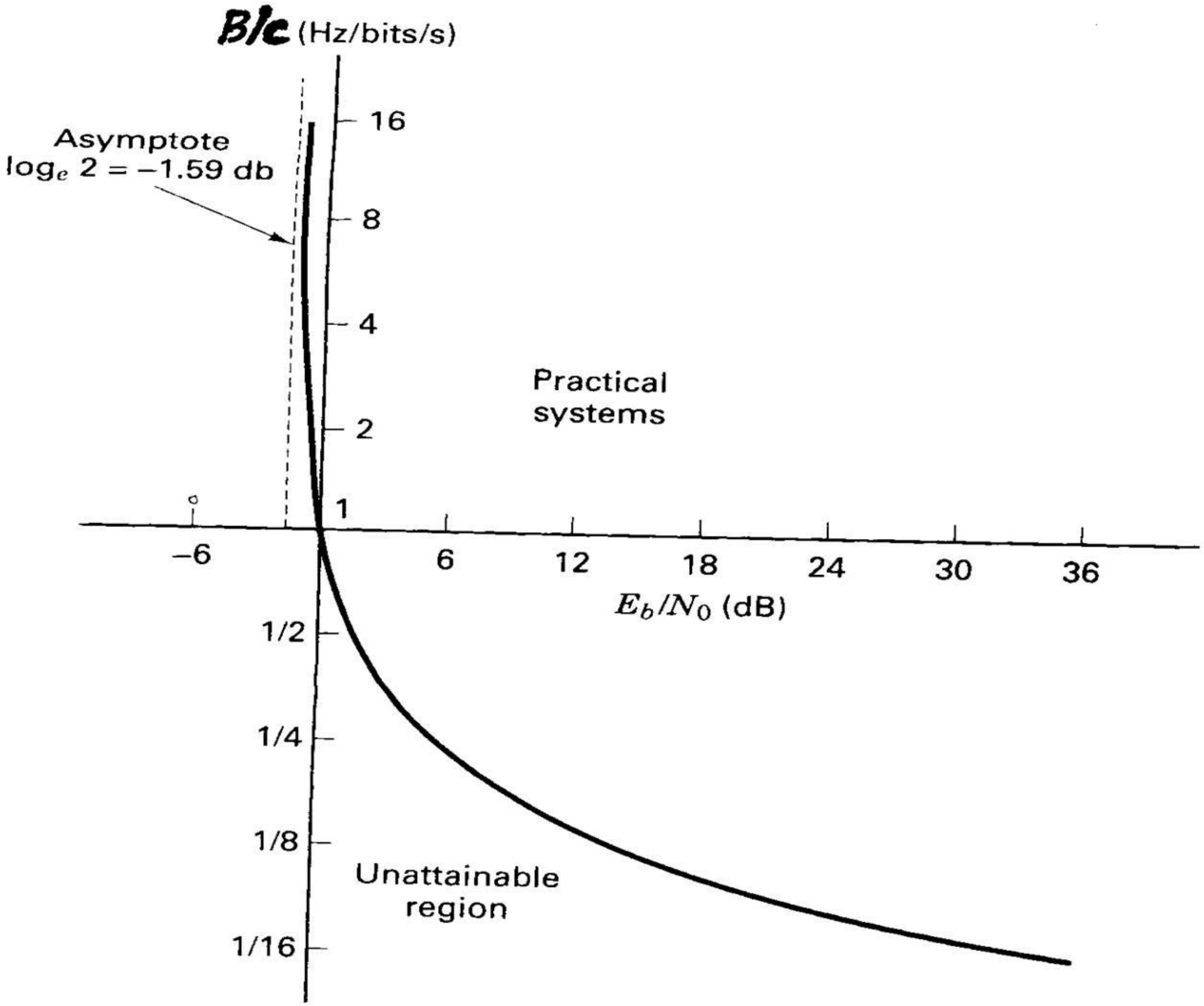
$$\frac{C}{B} = \log_2(1 + \frac{E_b}{N_o} \bullet \frac{C}{B})$$

Therefore $$\therefore \frac{E_b}{N_o} = \frac{2^{C/B} - 1}{\frac{C}{B}}$$

- **For infinite bandwidth, the approaches the limiting value**

$$(\frac{E_b}{N_o})_\infty = \lim_{B\to\infty}(\frac{E_b}{N_o}) = \ln 2 = 0.693 = -1.6\,dB$$

- **This value is called Shannon limit.**
- **There exists a limiting value of E_b/N_0 below which there can be no error-free communication at any information rate.**



Normalized channel bandwidth versus channel E_b/N_0.