

# Chapter 3 -- Part-2

## Noises, Interferences and Link Budget

### 3.6 Noise and Interference

#### 3.6.1 Thermal Noise

#### 3.6.2 Equivalent Noise Temperature and Noise Figure

#### 3.6.3 Noise in Cascaded System

#### 3.6.4 Man-Made Noise

#### 3.6.5 Multiple Access Interference

### 3.7 Link Budget

#### 3.7.1 Free-Space Link

#### 3.7.2 Terrestrial Link

## 3.6 Noise and Interference

- In general, noise can be defined as unwanted ( and usually uncontrolled ) electrical signals interfering with the desired signal.
- Unwanted signals arise from a variety of sources, both natural and artificial.
- Artificial sources include noise from automobile ignition circuits, commutator sparking in electric motors, 60- cycle hum, and signals from other communications system.
- Natural sources of noise include circuit noise, atmospheric disturbances, and extraterrestrial radiation.

### 3.6.1 Thermal Noise

- Thermal noise can be considered a fundamental property of matter above the absolute temperature of  $0^{\circ}\text{K}$  .

- In any conductor at a temperature above 0°K , the free electrons move about with random velocities in all directions. These random motions have an average velocity that is zero in any direction over a long period of time. Over short intervals, there are statistical fluctuations from this zero average. We call these statistical fluctuations **thermal noise** .
- We are interested in the distribution of thermal noise power across frequency, or  $S_n(f)$ --- that is the **noise spectral density**.
- From quantum physics ,for frequency up to  $10^{12}$  Hz ( near infrared ) , the **available thermal noise spectral density** ( watts/Hz ) is approximately constant and is given by

$$\begin{aligned}
 S_n(f) &= kT/2 \\
 &= N_0/2 \quad - \infty < f < \infty
 \end{aligned}$$

(3.51)

where  $k$  is the Boltamann's constant,  $N_0 = kT$   
 $k = 1.38 \times 10^{-23} \text{ w-s} / ^\circ \text{K}$  .

- Fig.2.25 shows the Thevenin equivalent model of a thermal resistor .

- On the basis of Eq.(2.118), the two-sided spectral model for thermal noise is shown in Fig.2.26. This model shows that thermal noise has a flat spectrum across all frequencies ( positive as well as negative) , with density  $N_0/2$  .

This type of noise is referred to as **white noise**.

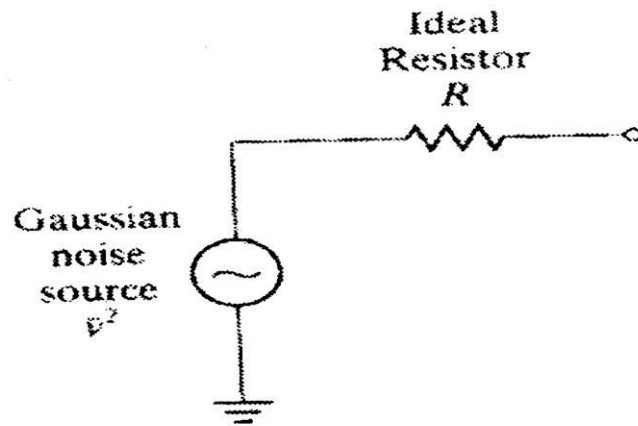
- The amplitude distribution of thermal noise at any frequency is Gaussian .
- The **noise power** in a bandwidth  $B$  is

$$P = (N_0/2) (2B) = N_0 B \quad (3.52)$$

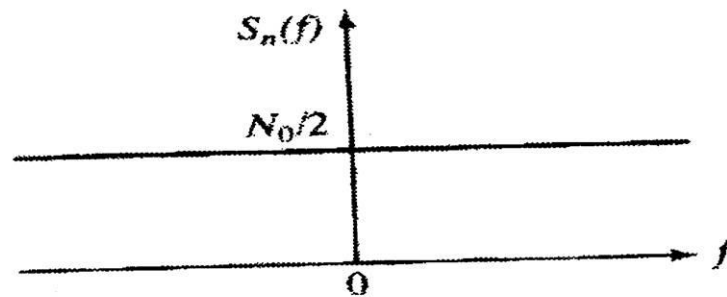
- If a filter has a frequency response  $H(f)$  , then we define the noise-equivalent bandwidth ,or **noise bandwidth**, of the filter as

$$B_{eu} = \left\{ \int_0^\infty | H(f) |^2 (N_0/2) df \right\} / (N_0/2) \quad (3.53)$$

**Fig.3.21**



**FIGURE 2.25** Thévenin model of a resistor.



**FIGURE 2.26** Spectral model for white noise.

- The noise bandwidth is equivalent to the bandwidth of an ideal rectangular filter that would pass the same amount of noise power. For many filter , the noise bandwidth is approximately equal to their 3-dB bandwidth.

## 3.6.2 Equivalent Noise Temperature and Noise Figure

- **Noise Temperature**

For any thermal noise source, if  $P_{n,max}$  is the maximum noise power the source can deliver in bandwidth, we define the equivalent noise temperature as

$$T_e = P_{n,max} / k B \quad (3.54)$$

$P_{n,max}$  is also called **available power** of the noise source.

## ■ Noise Figure

The noise figure represents the increase in noise at the output of the amplifier, referenced to the input. Let  $G(f)$  be the available power of the device as a function of frequency.

$$F = S_{no}(f) / G(f) S_{ni}(f) \quad (3.55)$$

where  $S_{no}(f)$  is the spectrum of the output noise power and  $S_{ni}(f)$  is the spectrum of the input noise power.

For an ideal noiseless amplifier, the noise figure is unity ; that is, the amplifier simply amplifies the input noise , but adds no noise itself.

The noise figure can also be expressed as the ratio of the output SNR to the input SNR ,i.e.,

$$F = ( \text{SNR} )_{in} / ( \text{SNR} )_{out} \quad (3.56)$$

- Relation between noise figure and noise temperature :**  
 Assuming that the amplifier has a power gain  $G$  that is constant, a metallic resistor is connected across the input terminals. The available noise power at the output due to the input noise is  $GkT_0$ .  
 The total output noise power density is defined as  $G ( kT_e + kT_0 )$ , where  $T_e$  is the equivalent noise temperature of the device .  
 This noise model is shown in Fig.2.27.  
 Consequently, the noise figure can be written as

$$\begin{aligned}
 F &= G ( kT_e + kT_0 ) / GkT_0 \\
 &= ( T_e + T_0 ) / T_0 = 1 + ( T_e / T_0 )
 \end{aligned}
 \tag{3.57}$$

and then we obtain the relation

$$T_e = ( F - 1 ) T_0
 \tag{3.58}$$



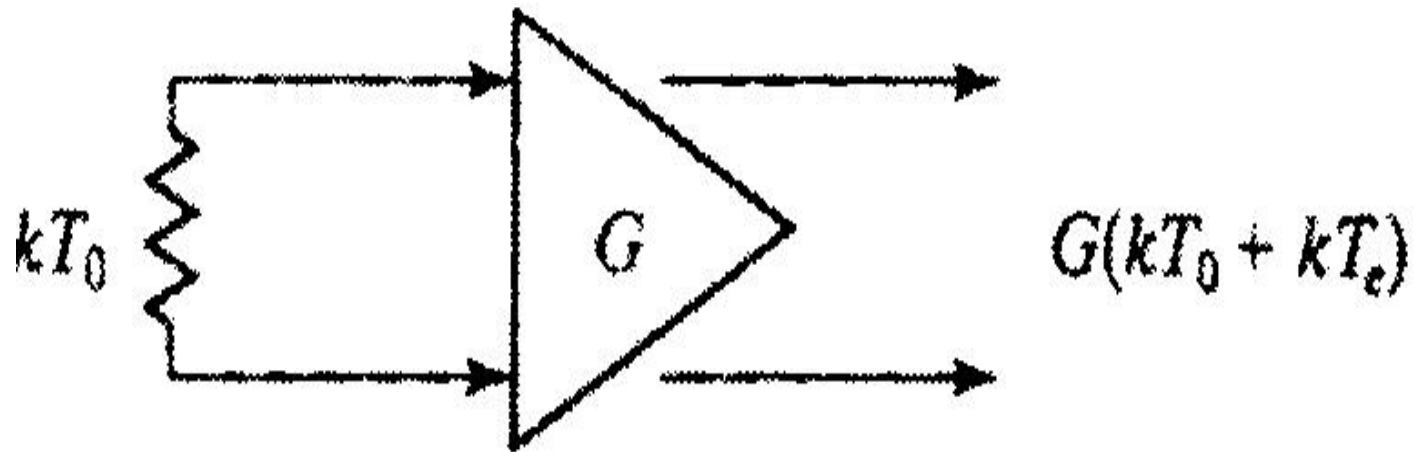
- The noise figure is usually expressed in decibels , but for **very low-noise amplifiers** such as those used as receiver front in satellite communications, the noise figure is often very close to 0 dB. The decibel scale makes it difficult to distinguish between the performance of the different amplifiers in this case.

In these situations , it is often preferable to use the equivalent noise temperature of the amplifier.

- At room temperature  $T_0 = 290^\circ K$

$$\begin{aligned} k T_0 &= 4 \times 10^{-21} \text{ W / Hz} \\ &= -174 \text{ dBm / Hz} \end{aligned} \quad (3.59)$$

**Fig.3.22 Noise model of amplifier**



## **Example : Noise Figure and Receiver Sensitivity ( Haykin, p.67)**

**Suppose a wireless receiver has a noise figure of 8 dB and includes a modem that has requires an SNR of 12 dB for prper operation in a 5 KHz bandwidth.**

**What is the receiver sensitivity?**

**Solution :**

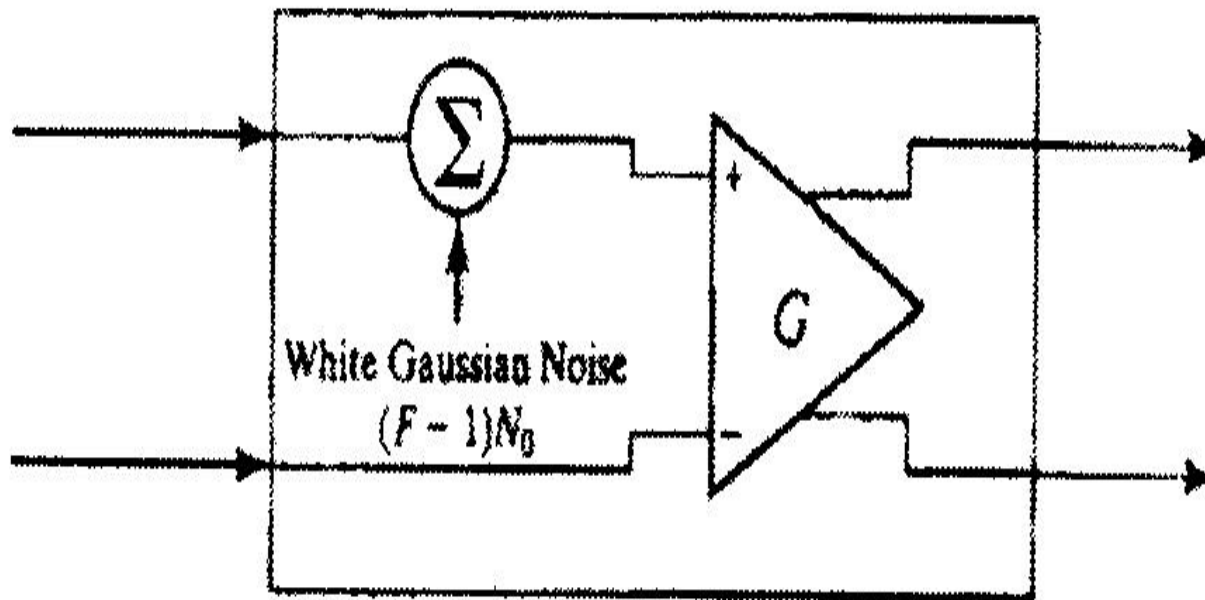
**The noise power due to the receiver in a bandwidth B is given by**

$$\begin{aligned} N &= F k T B \\ &= k + T + F + B \quad \text{all in dB} \\ &= -196.8 + 10 \log (290) + 8 + 10 \log 5000 \\ &= -129.0 \text{ dBm} \end{aligned}$$

**The corresponding receiver sensitivity is given by**

$$\begin{aligned} S &= (\text{SNR}) \times N \\ &= 12 + ( -129 ) \text{ dBm} \\ &= -117 \text{ dBm} \end{aligned}$$

**Fig.3.23 Two-port model of a communication system component**



## Fig.3.25 Illustration of Cascade noise calculation for two –device system

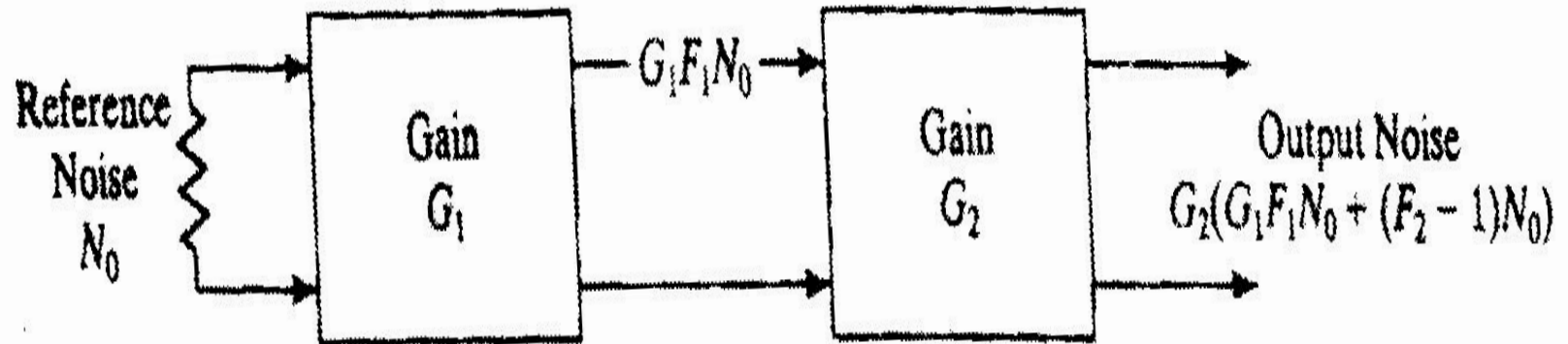


FIGURE 2.29 Illustration of cascaded noise calculation for two-device system.

### 3.6.3 Noise in Cascaded System

- A communication receiver is composed of a number of components, including an antenna, amplifiers, filters, and mixers and these components may be connected by transmission lines.
- The overall system's noise temperature involves contributions from these components in a weighted manner.
- We can model the aforementioned devices with the two-port model shown in Fig.2.28 , where each device has a noise figure  $F$  and a gain  $G$  . With this model, the input to the device is summed with an internal noise source .

The spectral density of the internal noise source is

$$(F - 1) kT_0.$$

The combined signals then processed by an ideal ( noiseless) amplifier with gain  $G$ .

- If  $S(f)$  is the device input , then the output of the circuit is given by

$$\begin{aligned}
 Y(f) &= G S ( f ) + G (F -1) kT_0 \\
 &= G [ S(f) + (F-1) kT_0 ] \qquad (3.60 )
 \end{aligned}$$

That is , the output has both a signal component and a noise component.

- Now consider a system with two devices, having noise figures  $F_1$  and  $F_2$  , and gains  $G_1$  and  $G_2$  , respectively as shown in Fig. 2.29.

The output of the first stage and input to tne second stage

is  $F_1 G_1 N_0$ .

The output of the second stage is

$$G_2 \{ ( F_2 - 1 ) N_0 + F_1 G_1 N_0 \}$$

**We can find that the overall noise figure is given by**

$$F = G_2 \{ (F_2 - 1) N_0 + F_1 G_1 N_0 \} / G_1 G_2 N_0$$
$$= F_1 + (F_2 - 1) / G_1 \quad (3.61)$$

- **In general, for a system with n cascaded devices, the overall noise figure is given**

$$F = F_1 + (F_2 - 1) / G_1 + (F_3 - 1) / G_1 G_2 + \dots \quad (3.62)$$

**and the equivalent noise temperature is given by**

$$T_{sys} = T_1 + T_2 / G_1 + T_3 / G_1 G_2 + \dots \quad (3.63)$$

- **If we include the equivalent noise temperature of the antenna in Eq. (3.63), the equation becomes**

$$T_{sys} = T_A + T_1 + T_2 / G_1 + T_3 / G_1 G_2 + \dots \quad (3.64)$$



## **Example :**

### **Noise figure of a lossy transmission line ( or waveguide)**

**The purely resistive attenuator imposes a loss of a factor  $L$  in available power between input and output. Assuming that the physical temperature of the lossy line is  $T_g$ .**

- The lossy transmission, such as cable or waveguide, can be viewed as a network with gain  $G$  equals  $1/L$ .
- Let all the component of the line be at thermal equilibrium with the environment at temperature  $T_g$ .

The total output noise power flowing from the network ( lossy line ) into the matched load is

$$N_{out} = kT_g B \quad (3.65)$$

where  $B$  is the bandwidth.

- The total power flowing from the load back into the network (lossy line ) must also equal  $N_{out}$ .
- $N_{out}$  can be considered as being comprised of two components ,  $G kT_g B$  and  $G N_{Line}$ ,

$$\begin{aligned} N_{out} &= G kT_g B + G N_{Line} \\ &= ( 1/L ) ( kT_g B + k T_e B_e ) \end{aligned} \quad (3.66)$$

- From (3.65) and (3.66) , we can find that

$$(L-1) kT_g B = kT_e B$$

then

$$T_e = (L-1) T_g \quad (3.67)$$

Thus, the noise figure of the lossy line

$$F = L \quad (3.68)$$

Composite of feed line and amplifier

$$\begin{aligned} F_{\text{comp}} &= F_{\text{feedline}} + (F-1) / (1/L) \\ &= L + (F-1) L \\ &= LF \end{aligned}$$

$$\begin{aligned} T_{\text{comp}} &= T_{\text{feedline}} + (F-1) / (1/L) T_0 \\ &= (LF-1) T_0 \end{aligned}$$

- **Example :**

A receiver front end has a noise figure of 10 dB , a gain of 80 dB, and a bandwidth of 6 MHz . The input signal power ,  $S_i$ , is  $10^{-11}$  w . Assume that the feedline is lossless and the antenna temperature is 150°K. Find the  $(\text{SNR})_{\text{out}}$

*Solution :*

$$T_R = (F-1) 290^\circ\text{K} = 2610^\circ\text{K}$$

$$T_S = T_A + T_R = 150 + 2610 = 2760 \text{ }^\circ\text{K}$$

$$\text{Bandwidth } B = 6 \times 10^6$$

$$N_{\text{out}} = GkT_S = 10^8 \times 1.38 \times 10^{-23} \times 2760 = 22.8 \text{ } \mu\text{W}$$

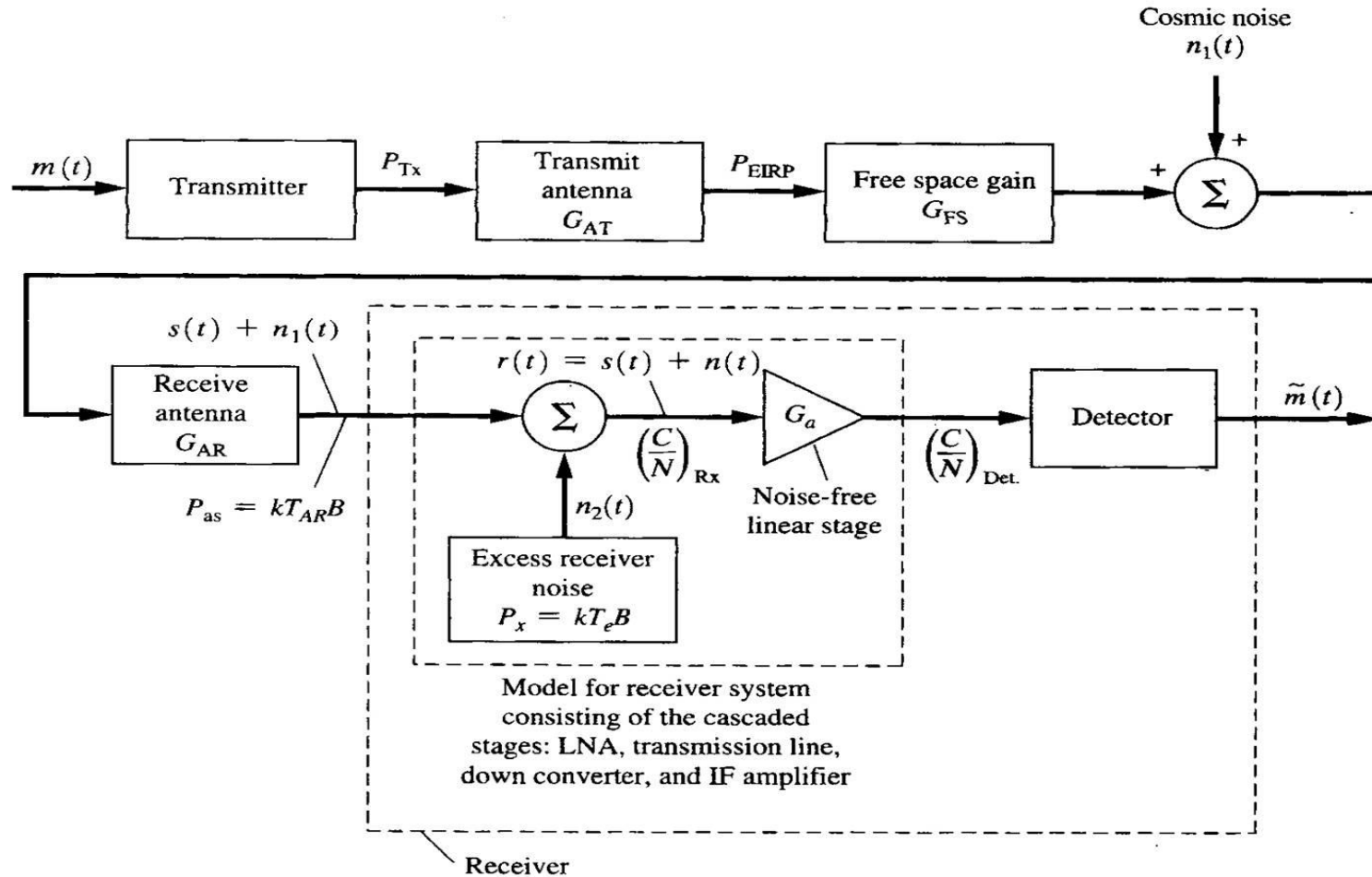
$$(\text{SNR})_{\text{out}} = S_{\text{out}} / N_{\text{out}} = 10^8 \times 10^{-11} / GkT_S = 43.9 \text{ (16.4 dB)}$$

### 3.6.4 Man-Made Noise

- Artificial ( or man-made ) noise is due to electrical machinery and discharges that produce harmonics or impulse noise in the radio frequencies used for communications.
- The source of **impulse noise** include the following :
  - a. Noised from electrical machinery ( particularly commutating motors)
  - b. Noise from sparking ignition system in automobile or other internal combustion
  - c. Switching transients
  - d. Discharge lighting
- The out-of-band transmission from other communicationlike service include the **harmonics** of high-power radars and televisions.

# 3.7 Link Budget

## Communication System Model (From Couch II, page 596)



Communication system model for link budget evaluation.

## 3.7.1 Free- Space Link

- Friis equation for free-space propagation

$$P_R = P_T * G_t * G_r / L_p$$

where  $G_t$  is the transmitting antenna gain,  $G_r$  is the receiving antenna gain,  $L_p$  is the path loss . Then we obtain

$$P_R / N_0 = P_T * G_t * G_r / kT_e L_p$$

For satellite applications, this equation is often written as

$$C / N_0 = \text{EIRP} - L_p + ( G_r / T_e ) - k \quad (3.68)$$

where all quantities are expressed in **decibels** and

$C / N_0 = P_R / N_0$  is the received **carrier-to-noise density ratio** (dB-Hz)

**EIRP** =  $P_T G_t$  is the **equivalent isotropic radiated power** of the transmitter ( dBW)

$G_r/T_e$  is the ratio of the receiver antenna gain to noise temperature.

$k$  is Boltzmann's constant (  $-228.6 \text{ dBWatt-sec}/^\circ\text{K}$  or  $1.38 \times 10^{-23} \text{ joules}/^\circ\text{K}$  )

Note that the  $C/N_0$  ratio is a popular way of expressing the SNR . It makes no assumption about the underlying modulation strategy.

### ■ $C/N$ vs. $E_s/N_0$

The **carrier-to-noise ratio** (CNR) is given by

$$C/N = P_c / ( N_0 B_{RF} )$$

Note that  $P_c T_s = E_s$  is the **symbol energy**.  $R_s = 1/T_s$

We obtain  **$C/N = ( E_s R_s ) / ( N_0 B_{RF} )$**  .

Given a **modulation scheme** , we can use a suitable **bandwidth criterion** to determine  **$R_s/B_{RF}$**  .



## 3.7.2 Receiver Sensitivity

- **Receiver sensitivity** is a parameter that indicates the minimum signal level required at the antenna terminals in order to provide **reliable communications**.

It depends on the factors : receiver design, modulation format, and transmission rate.

Receiver sensitivity is often expressed in dBm

- **Illustrative example : ( Haykin p.15)**

A commercial mobile receiver for data transmission may be specified with a sensitivity of -90 dBm, what is the radius of the service area of the receiver at a transmission frequency of 800 MHz ?  
Assuming that the transmitter antenna transmits 100 mw signal.

Ans . 9.2 Km ( free-space path model)

## ***Solution of Example 2.1 [Haykin]***

***Receiver sensitivity = -90 dBm =  $10^{-9}$  mw***

***Path loss  $L_p = P_T / P_R = (4\pi d / \lambda)^2$***

***$P_R \geq 10^{-9}$  mw is required.***

***$P_T = 100$  mw ,***

***$f = 800$  MHz , that is ,  $\lambda = 3/8$  m***

***Thus the maximum range of service is given by***

***$d = (\lambda / 4\pi) (\sqrt{L_p}) = 9200$  m = 9.2 km***

## 3.7.3 Terrestrial Link

### **Example** ( Haykin ,2.20)

- A mobile radio service that is intended to service the city core and the surrounding suburban and rural areas. The base station is assumed to be located close to the city core , with an antenna mounted on a high tower that provides a relatively unobstructed view of the surrounding rural area . The link parameters can be broken into four subsections :

**Base station transmitter, transmission loss, received signal, receiver characteristics**

## ■ Base station transmitter

Transmit frequency : 705 MHz ( Note: mobile public safety band )  
Transmit power : 15 dBW  
Transmit antenna gain : 2 dBi ( Note : Uniform radiation in azimuth )  
Transmit EIRP : 17 dBW  
Power at 1 m : 17.6 dBm (Note :  $P_0 = P_T / (4\pi/\lambda)^2$  )

## ■ Losses

Path-loss exponent : 2.4 ( Note : applicable at edge of coverage )  
Range : 10 km (Note : range at edge of coverage)  
Medium path loss : 96 dB (Note:  $2.4 \log_{10}(r/r_0)$ )  
Lognormal shadowing : 8 dB ( Note : standard deviation of shadowing)  
Shadowing margin : 13.2 dB (Note : for 95% availability)

## ■ Received signal

Receive antenna gain : 1.5 dBi (Note : Vertically polarized whip antenna)  
Received signal strength : -90.1 dBm (Note:  $P_R = P_0 + G_R - 21\log_{10}(r/r_0) - M_{\text{shad}}$ )

## ■ Receiver characteristics

Required  $C/N_0$  : 69.8 dB-Hz ( Note: from modem characteristics)

Boltzmann's constant : -198.6 dBm-K

Receiver noise figure : 6.0 dB (Note : provided)

Receiver sensitivity : -98.2 dBm ( Note :  $S = C/N_0 + NF + kT_0$  )

■ Margin : 8.1 dB ( Note : Margin =  $P_R - S$  )

## Remarks :

Receiver Sensitivity :  $S \text{ (dBm)} = C/N_0 \text{ (dBHz)} + kT_0 \text{ (dBHz)} + F \text{ (dB)}$

*The above equation can be obtained as follows.*

$$N_0 = F \cdot kT_0$$

$$N = 2B \cdot N_0/2 = B \cdot N_0 = B \cdot F \cdot kT_0$$

Received signal strength  $S$  = Received carrier power  $C$

$$= C \cdot (F kT_0 / N_0)$$

$$= (C / N_0) (F kT_0)$$

## Example : Wireless LAN ( Haykin pp. 85-86 )

- IEEE 802.11a wireless LAN standard recommends that the receiver have a noise figure of 10 dB or better. Thus the nominal value of the noise floor of the receiver is

$$\begin{aligned} N_0 &= FkT = 10 kT = 10 + ( -174 ) \text{ dBm / Hz} \\ &= -164 \text{ dBm / Hz} \end{aligned}$$

- For 6-Mbps service , BPSK modulation is used. In practice , the requirement of  $E_b/N_0$  for a BPSK modulation is 14 dB ( including the implementation loss) to achieve a BER of  $10^{-6}$ .  
Find the sensitivity of the receiver ( that is, the minimum received power for reliable communication) ?

**Solution :**

*From (2-140) , the received signal strength  $S$  and the carrier-to-noise ratio are related by*

$$\begin{aligned} S(\text{ dBm}) &= C/N_0 (\text{ dBHz}) + kT (\text{ dB Hz}) + F (\text{ dB}) \\ &= E_b/N_0 (\text{ dB}) + R_b (\text{ dBHz}) + N_0 (\text{ dBm/ Hz}) \\ &= 14 + 10 \log ( 6 \times 10^6 ) + ( - 164 \text{ dBm / Hz}) \\ &= - 82 \text{ dBm} \end{aligned}$$

- Radio for 5-GHz wireless LAN applications must transmit 200 milliwatts or less.

Assuming that the transmission is over an open office environment with no walls, what is the expected range of this service ?

**Solution :**

*The propagation model ( 2.146 ) simplifies to*

$$P_R = P_T - 41 \text{ (dB)} - 31 \log r$$

$$\begin{aligned} \text{Thus } 31 \log r \text{ (m)} &= P_T - 41 \text{ (dB)} - P_R \\ &= 23 - 41 - (-82) \\ &= 64 \text{ dB} \end{aligned}$$

*Solving the equation , we obtain  $r = 116$  meters*

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