Chapter 4

Baseband Digital Transmission

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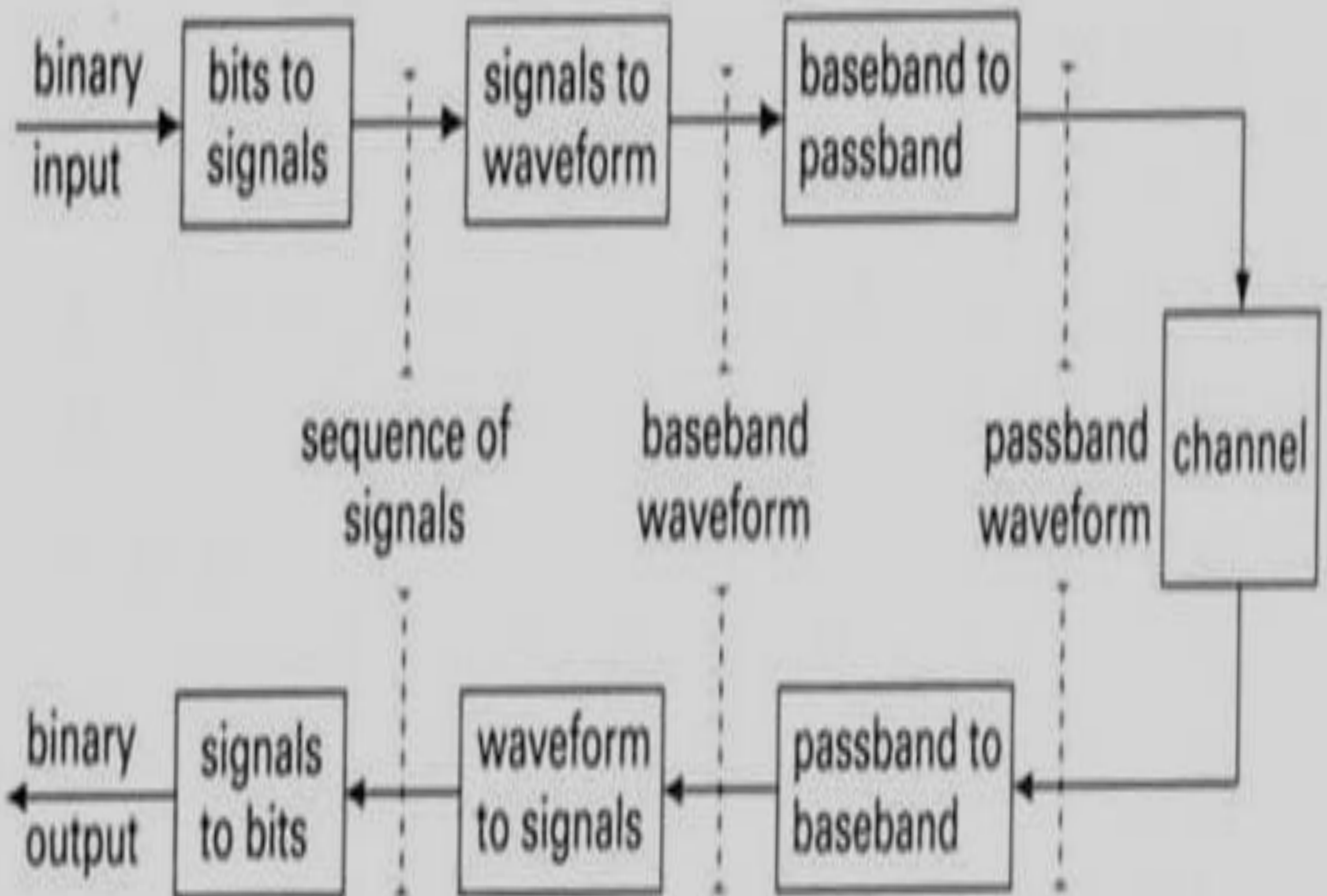


Layers of a modulator (channel encoder) and demodulator (channel decoder).

4.1 Baseband Signals

4.1.1 Waveform Representation of digital signal

- **The binary digits (bits) must be represented with electrical pulses inorder to be transmitted through a baseband channel.**

  This kind of process is also known as binary line coding, or binary signaling.

- **Some of the most common formats are shown in Fig.4.1, including unipolar NRZ (non-return-to –zero), unipolar RZ (return-to-zero), polar RZ, polar NRZ, bipolar RZ , Manchester NRZ , etc..**
- **For RZ signaling, the waveform returns to a zero-level for a portion (usually one-half) of the bit interval.**

- **Unipolar signaling :**

  Binary 1 is represented by a high-level (+A volts) and binary 0 is represented by a zero-level of the bit interval .

- **Polar signaling :**

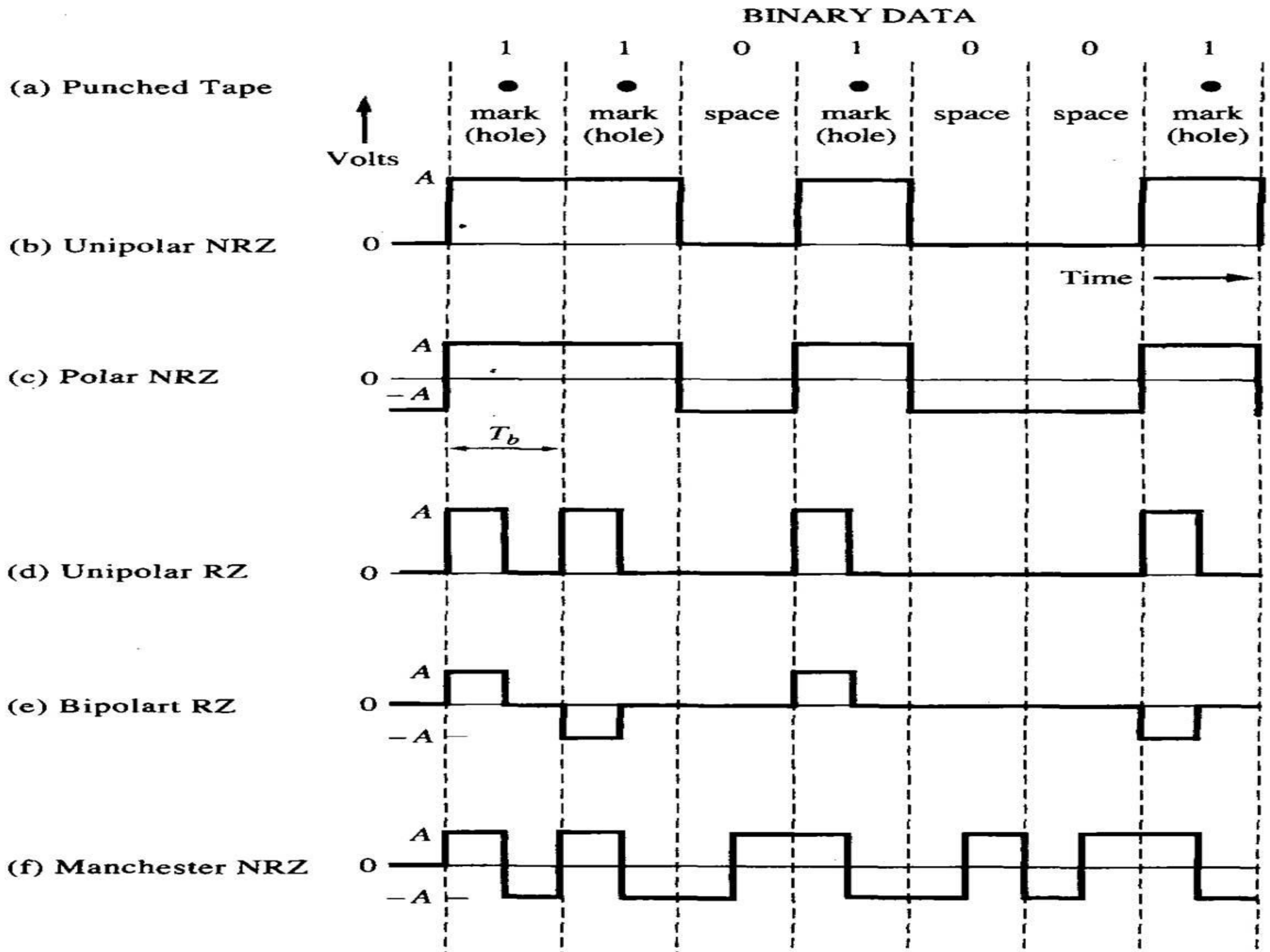
  Binary 1's and 0's are represented by equal positive and negative levels, respectively. Bipolar signaling :

  Binary 1's are represented by alternately positive or negative values. The binary 0 is represented by a zero level.

- **Manchester signaling :**

  Each binary 1 is represented by a positive half-bit period pulse followed by a negative half-bit period pulse. Similarly, a binary 0 is represented by a negative half-bit period pulse followed by a positive half-bit period pulse.

Fig.4.1 Binary signaling formats



Binary signaling formats.

4.1.2 Power Spectra for Binary Signals

- **The signal waveform of a (random) binary data stream can be expressed as**

$$x(t) = \Sigma_{n=-\infty}^{\infty} a_n \, g(t-nT_s) \qquad (4.1)$$

where $g(t)$ is the signal pulse shape , T_s is the duration of one symbol.

In general the pulse is rectangular in shape.

- **A general expression for the power spectral density (PSD) of a binary data stream is given by**

$$S_x(f) = [\,|\,G(f)\,|^2 / T_s\,] \, \Sigma_{k=-\infty}^{\infty} R(k)\, e^{j2\pi k f T_s} \qquad (4.2)$$

where $G(f)$ is the Fourier transform of $g(t)$, R(k) is the autocorrelation of the data stream.

The autocorrelation R(*k*) is given by

$$R(k) = \Sigma_{m=1}^{M} (a_n a_{n+k})_m P_m \qquad (4\text{-}3)$$

where a_n and a_{n+k} are the (voltage) level of the data pulse of the *n-th* and (*n*+*k*)*-th* symbol positions, respectively.

P_m is the probability of having *the m-th* ($a_n a_{n+k}$) product .

T_b denotes the time that it takes to send one bit , and *A* denotes the amplitude of the pulse.

- **Remarks : Calculation of R(*k*)**

Taking the case of polar NRZ signaling as example. The possible levels for the *a'* s are +*A* and –*A* . For equally likely occurrence of +*A*

and -A , and assuming that the data are independent from bit to bit, we get

$$R(0) = \Sigma_{n=-\infty}^{\infty} (a_n a_{n+k})_m P_m = A^2 (1/2) + (-A)^2 (1/2)$$
$$= A^2$$

For k = 0

$$R(k) = \Sigma_{n=-\infty}^{\infty} (a_n a_{n+k})_m P_m$$
$$= A^2 (1/4) + (-A) A (1/4) + A (- A) (1/4) + (-A)(- A) (1/4)$$

- **PSD of unipolar NRZ binary signal**

$$S\,(f) = (A^2 T_b)/4 \; [\,(\sin \pi f T_b)\; / \; \pi f T_b\,]^2 \; [\,1+ \delta(f)\,/\,T_b\,] \qquad (4.4)$$

- **PSD of unipolar RZ binary signal**

$$S\,(f) = [\,(A^2 T_b)/16\,]\;[\,(\sin \pi f T_b/2)\; / \; \pi f T_b/2\,]^2$$

$$\{\,1 + (1/T_b)\,\Sigma_{n=-\infty}^{\infty}\,\delta(f- n/T_b)\,\} \qquad (4.5)$$

- **PSD of polar NRZ binary signal**

$$S\,(\mathrm{f}) = (A^2 T_b)\;[\,(\sin \pi f T_b)\; / \; \pi f T_b\,]^2 \qquad (4.6)$$

- **PSD of Bipolar RZ binary signal**

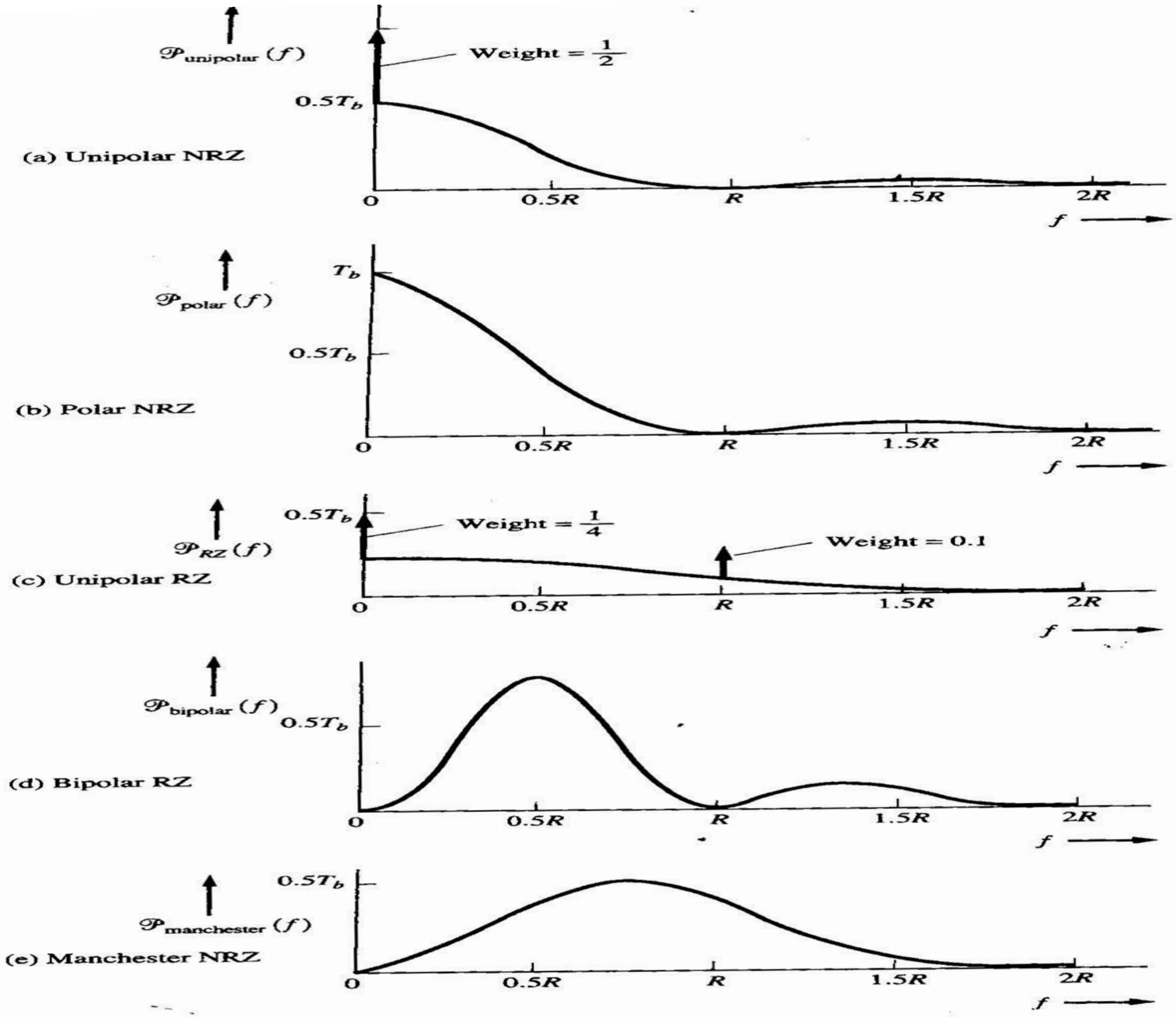
$$S(f) = (A^2 T_b/4)\left[(\sin \pi f T_b/2) / \pi f T_b/2 \right]^2 \sin^2(\pi f T_b) \qquad (4.7)$$

- **PSD of Manchester NRZ binary signal**

$$S(f) = (A^2 T_b) \left[(\sin \pi f T_b/2) / \pi f T_b/2 \right]^2 \sin^2(\pi f T_b/2) \qquad (4.8)$$

Note that the power spectral density of the binary data stream is a function of the bit pattern as well as the pulse shape.

Fig.4.2 Power spectral density of the line codes

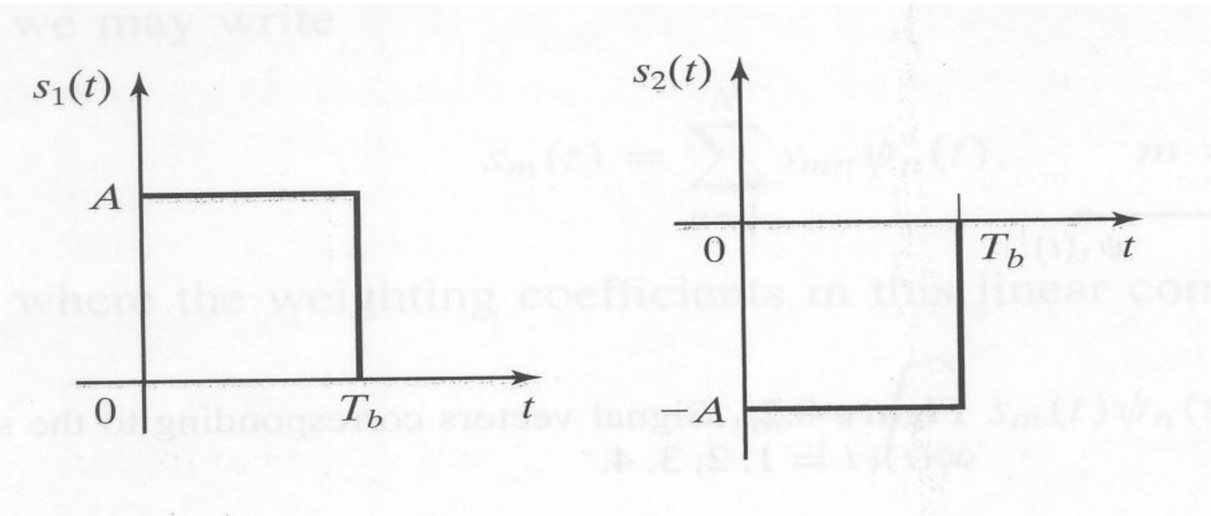


PSD for line codes (positive frequencies shown).

4.2 Baseband Modulation

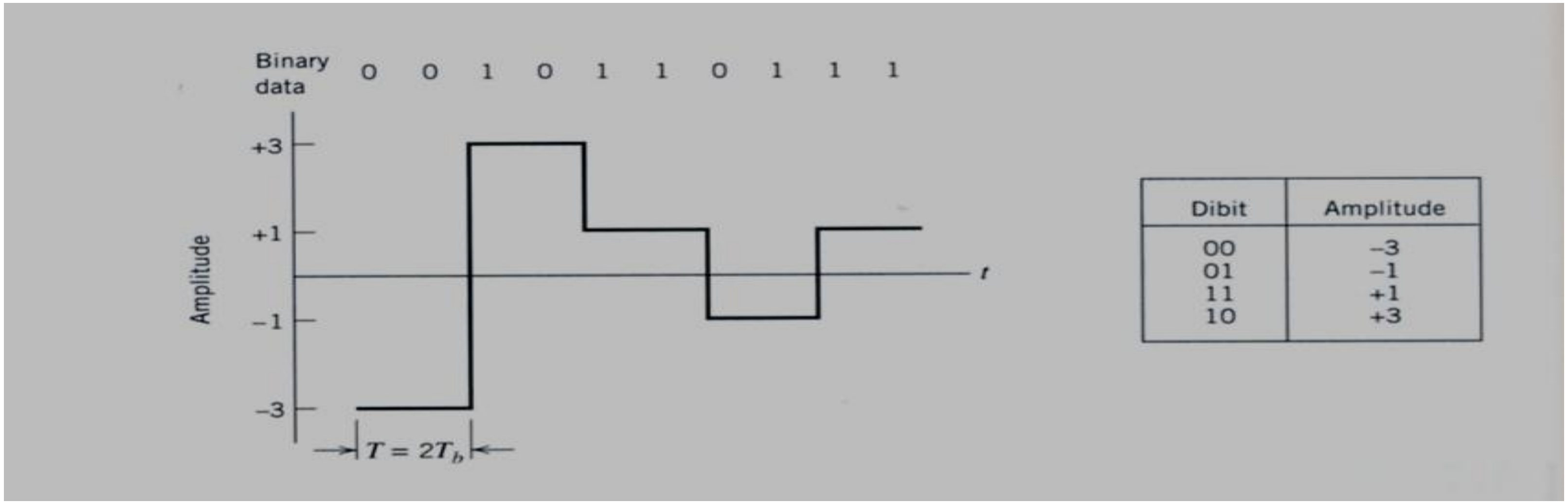
4.2.1 Binary Pulse Amplitude Modulation

- Binary PAM is the simplest digital modulation method. In binary PAM , the message bit 1 may be represented by a pulse of amplitude *A* , and the message 0 may be represented by a pulse of amplitude - *A* , as shown below . Pulses are transmitted at a bit rate $R_b = 1/T_b$ bits/sec , where T_b is called the bit interval.



4.2.2 M-ary Pulse Amplitude Modulation

- **The generalization of binary PAM to M-ary PAM is relatively straightforward . The k-bit symbols are used to select $M = 2^k$ signal amplitudes. The following figure illustrates a quaternary ($M = 4$) system and the binary data sequence 0010110111 .**
- **The symbol duration T of the M-ary PAM system is related to the bit duration T_b of the equivalent binary PAM system by** $T = T_b \log_2 M$



4.2.3 Pulse Code Modulation

- **Pulse-code modulation (PCM) is a method used to digitally represent sampled analog signals.**
- **A PCM bit-stream is a digital representation of an analog signal, in which the magnitude of the analogue signal is sampled regularly at uniform intervals, with each sample being quantized to the nearest value within a range of digital steps.**
- **In PCM ,we make the following assumptions :**

1. the waveform (signal) is bandlimited with a maximum frequency of W . Therefore , it can be reconstructed from samples taken at a rate of $f_s = 2W$ or higher .

2.The signal is of finite amplitude with maximum value x_{max} .

3. The quantization is done with a large number of quantization levels N which is a power of 2 ($N = 2^K$)

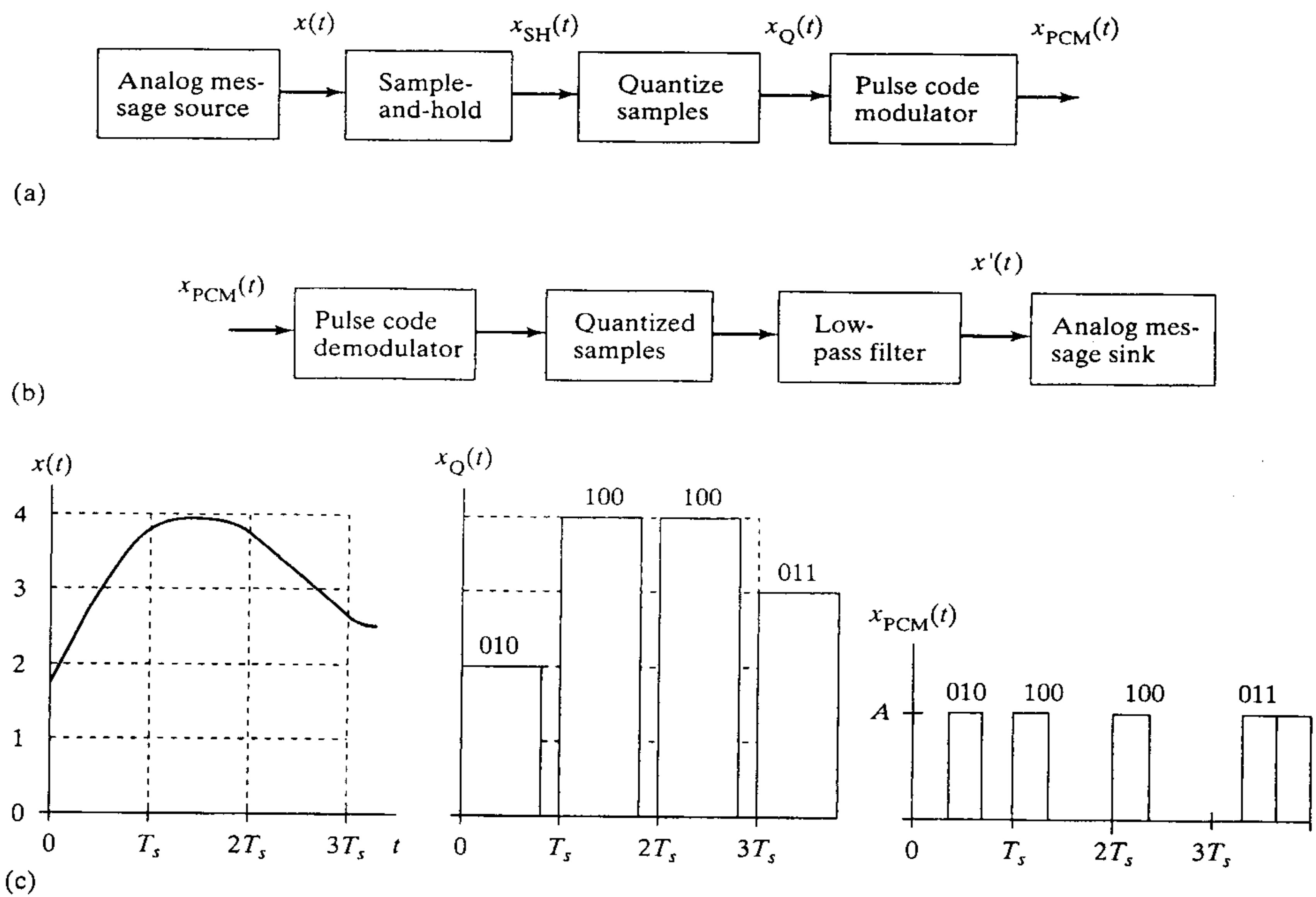


FIGURE 2–19 A pulse code modulator (a), demodulator (b), and waveforms (c).

4.3 Intersymbol Interference and Pulse Shaping

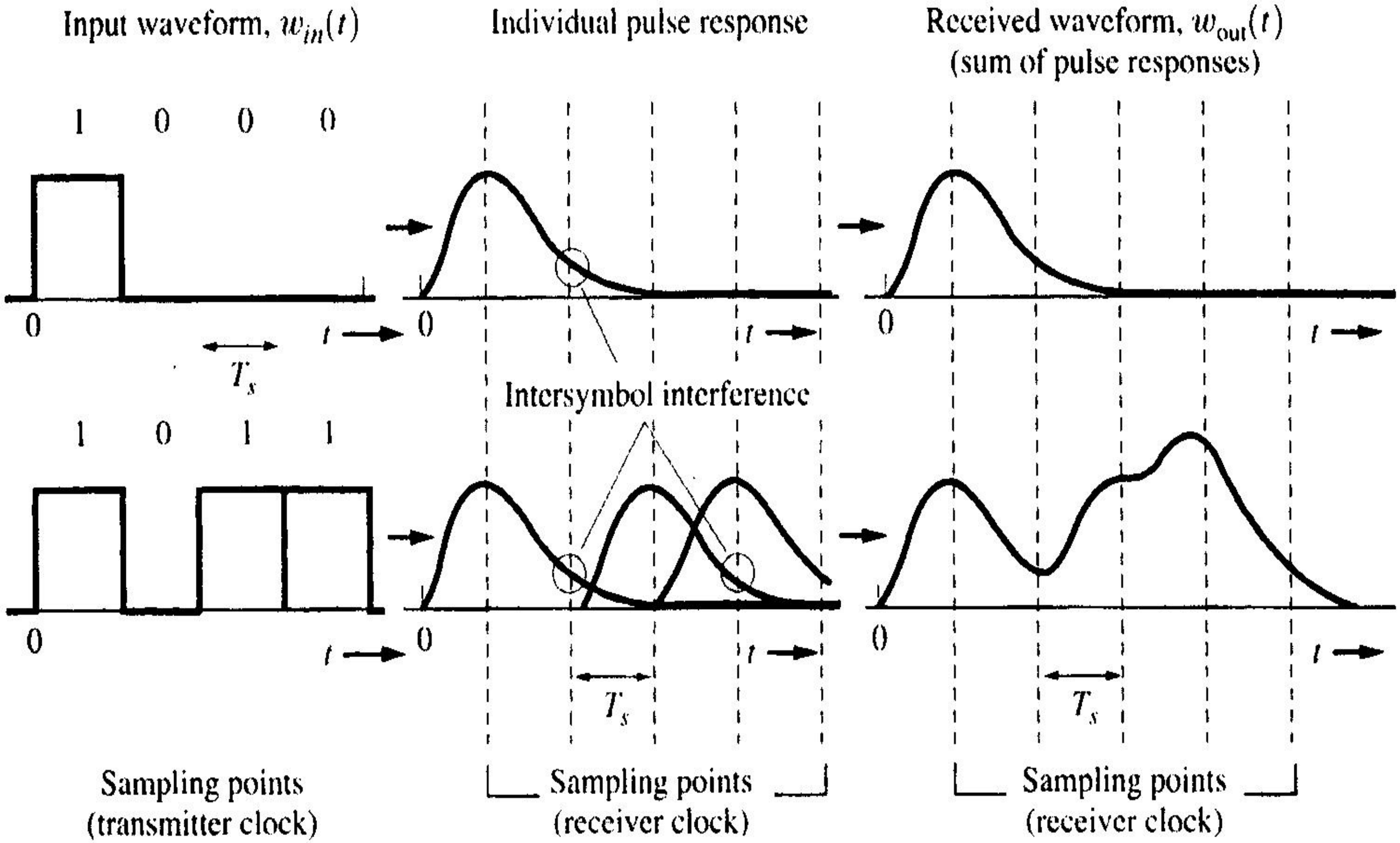
- **The pulse defined in the previous section for representing binary symbol 1 or 0 is rectangular in shape.**

  In practical perspective, the use of rectangular pulse shape is undesirable because the spectrum of a rectangular pulse is infinite in extent.

- **When the rectangular pulses are passed through a bandlimited channel , the pulse will spread in time , and the pulse for each symbol will smear into the time intervals of succeeding symbols , as illustrated in Fig.4.3 .**

  This causes intersymbol interference (ISI) and leads to an increase of bit error rate in receiver detection.

Fig.4.3 Illustration of ISI in binary transmission system



4.3.1 Nyquist Shaping Filter

- **H.Nyquist was the first to solve the problem of overcoming ISI while keeping the transmission bandwidth low.**

- **Nyquist in 1928 at AT&T observed that the effects of ISI could be completely nullified if the overall response of the communication system (including transmitter, channel, and receiver) is designed so that at every sampling instant at the receiver, the response due to symbols except the current symbol is equal to zero.**

$$p(n\,T_b) = \begin{cases} 1 & n = 0 \\ 0 & n = 0 \end{cases}$$

where T_b is the duration of one bit . This is called the Nyquist shaping criterion.

- **Let $P(f)$ denote the overall frequency response of the system.**

  According to Nyquist, the effect of ISI can be reduced to zero by shaping $P(f)$ which satisfies the condition

$$\Sigma\, P(f + n/T_b) = T_b \qquad |f| \leqq 1/2T_b$$

  In general P (f) consist of *a flat portion and a* rolloff portion as shown in Fig. 4.4(a) .

- **Note that p(t) , the impulse response of $P(f)$, has the value of unity at the current signaling instant and zero-crossings at all other consecutive signaling instant.**

- **Specifically, for a data rate of $(1/T_b)$ bits/second , the channel bandwidth may extend from the minimum value $W = 1/2T_b$ to an adjustable value from W to $2W$ by defining $P(f)$ as follows :**

$$P(f) = \begin{cases} 1/2W & |f| \leqq f_1 \\ 1/(4W)\,[1+\cos\theta] & f_1 \leqq |f| \leqq 2W - f_1 \\ 0 & |f| \geqq 2W - f_1 \end{cases} \tag{4.9}$$

where $\theta = \pi/(2W\rho)\ [\,|f| - W(1-\rho)\,]$ **,**

and $\rho = 1 - (f_1 / W)$ **(4.10)**

The parameter ρ is called the roll-off factor, which indicates the excess bandwidth over the ideal solution corresponding to $\rho = 0$.

- **$P(f)$ define in Eq. (4.9) is called the raised-cosine (RC) spectrum and is plotted in Fig.4.4(a) . The filter with frequency response $P(f)$ is denoted as Nyquist shaping filter.**
- **The impulse response $p(t)$ is expressed by**

$$p(t) = [\cos(2\pi\rho W t) / (1-16\rho^2 W^2 t^2)] \; \mathrm{sinc}(2Wt) \qquad (4.11)$$

where the function

$$\mathrm{sinc}(x) = \sin(\pi x) / \pi x \qquad (4.12)$$

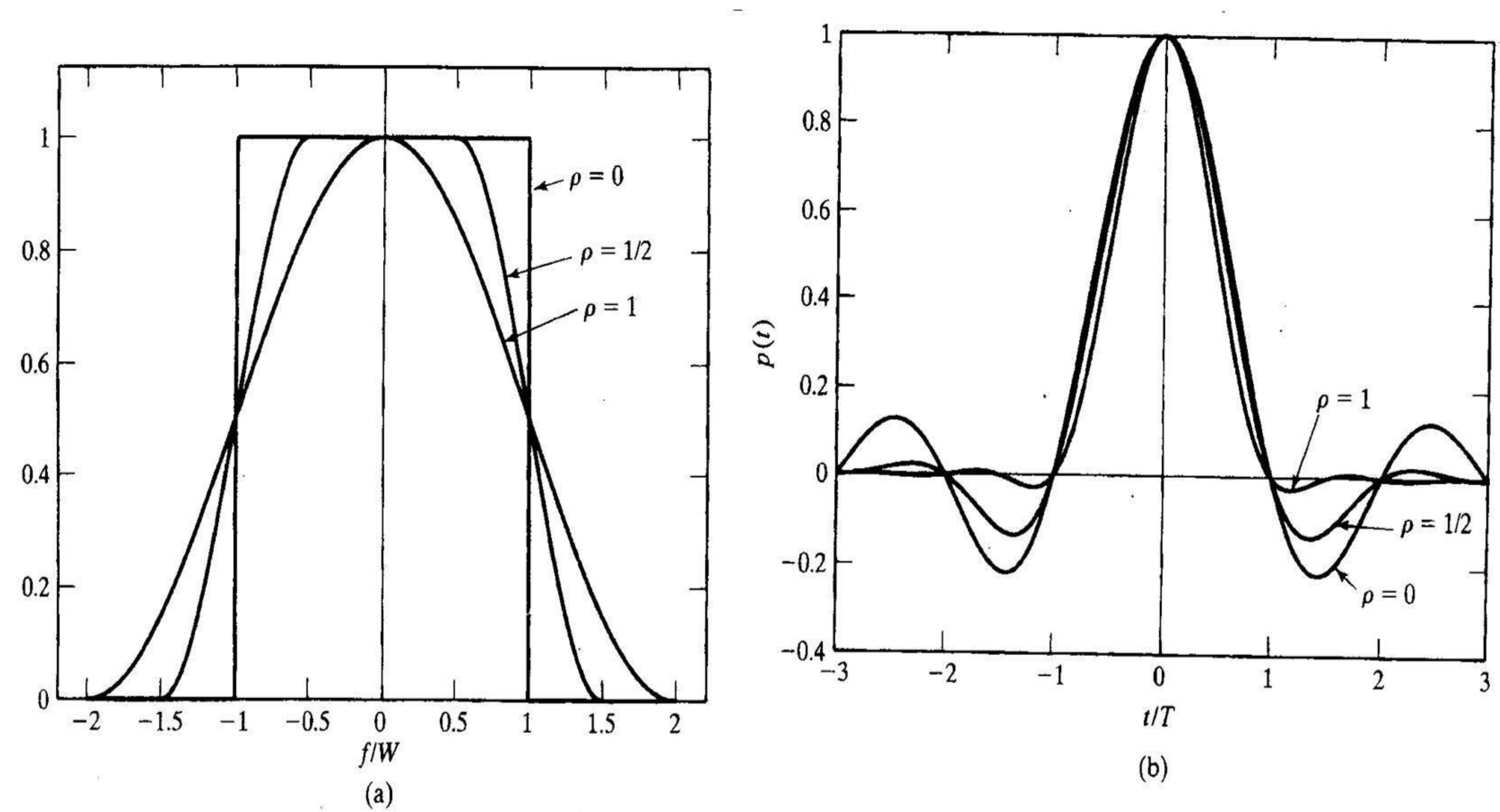
is called the sinc function.

- **For $\rho = 0$, $P(f)$ becomes a rectangular " brick- wall" filter. and $p(t) = \mathrm{sinc}(2Wt)$**

For $\rho = 1$, $P(f)$ is called full-cosine roll-off shaping filter.

Fig.4.4 (a) Raised-cosine spectrum

(b) Impulse respond of the Nyquist shaping filter



4.3.2 Root Raised-Cosine Pulse Shaping

- A more sophisticated form of pulse shaping uses the square-root raised-cosine (SRRC) spectrum rather than the regular RC spectrum of Eq. (4.9).
- The frequency response of the root raised cosine pulse is given by

$$P_r(f) = \begin{cases} 1/\sqrt{(2W)} & |f| \leqq f_1 \\ 1/\sqrt{(2W)}\,[1+\cos(\theta/2)\,] & f_1 \leqq |f| \leqq 2W - f_1 \\ 0 & |f| \geqq 2W - f_1 \end{cases} \tag{4.13}$$

where $\theta = \pi/(2W\rho)\ [\,|f| - W(1-\rho)\,]$,

and $\rho = 1- (f_1/W)$ (4.14)

- **The corresponding pulse shaping is given by**

$$p_r(t) = \sqrt{(2w)} / (1 - (8\rho W t)^2) \{ \sin(2\pi W(1-\rho)t) / 2\pi W t + (4\rho/\pi) \cos(2\pi W(1+\rho)t) \} \qquad (4.15)$$

Fig.4.5 shows the $P_r(f)$ and $p_r(t)$ for the root raised cosine shaping filter. In general, the root RC waveform, $p(t)$, occupies a larger dynamic range than the regular RC waveform , with same bandwidth.

- **The overall raised cosine spectral characteristics can be split evenly between the transmitter filter and receiving filter.**

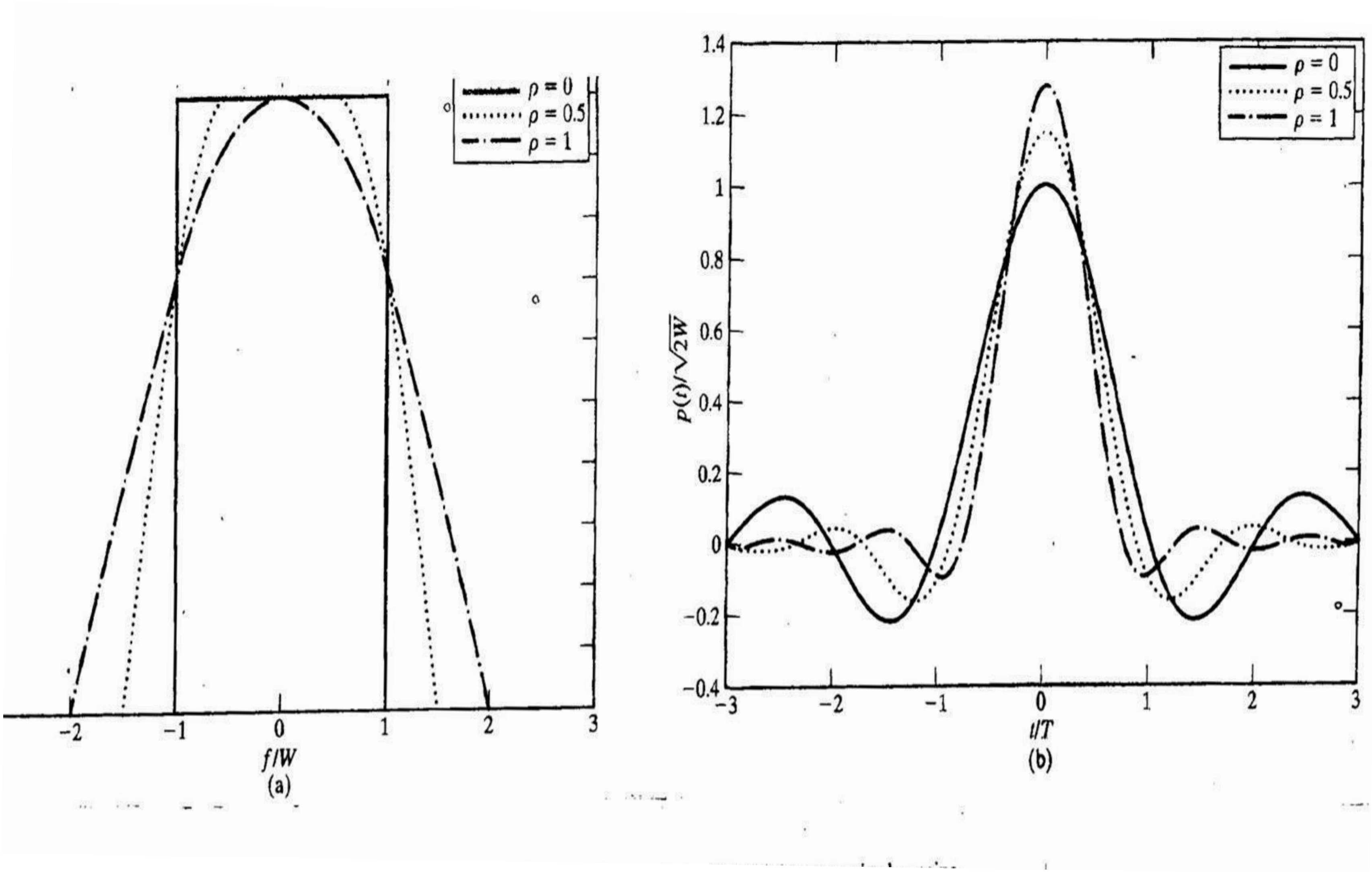
The receiving filter can be expressed by

$$G_{rec}(f) = P_r(f) \exp(-j 2f t_0) \qquad (4.16)$$

where t_0 is some nominal delay to ensure physical realizability of the filter.

Fig.4.5 (a) *P* (*f*) for root raised-cosine spectrum

(b) *p* (t) of root raised-cosine spectrum



4.4 Geometrical Representation of Signals : Signal –Space Concept

- **In 1947 , Kotel'nikov introduced the use of signal –space into communication system characterization ,expanded later by Wozencraft and Jacobs in their book (1965) .**
- **A K-dimensional generalized vector space is defined by the orthonormal basis function set $\{\psi_1(t), \psi_2(t), \ldots, \psi_K(t)\}$, where**

$$\int_0^T \psi_i(t)\,\psi_j(t)\,dt = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

The functions $\psi_i(t)$ may be complex . This vector space is also denoted as sigal space.

Any function $s_n(t)$ in this vector space can be expressed as

$$s_n(t) = \Sigma_{k=1}^{K}\ s_{nk}\,\psi_k(t)\ ,\ 0 \leqq t \leqq T$$

where $s_{nk} = \int_0^T s_n(t)\,\psi_k^*(t)$

- **The signal $s_i(t)$ can be expressed as a vector**

  $$\mathbf{s}_i = [\, s_{ik} \; s_{ik} \; \ldots \; s_{ik} \,]^T$$

  The vector $\mathbf{s}_i$ is called a signal vector .

- **By definition, the energy of a signal $s_n(t)$ of duration T is**

  $$E_i = \int_0^T \mathbf{s}_i^{\,2}(t)\, dt$$
  $$= \int_0^T [\, \Sigma_{k=1}^{K} \; s_{ik} \, \psi_k(t) \,] \, [\, \Sigma_{n=1}^{K} \; s_{in} \, \psi_n(t) \,] \, dt$$
  $$= \Sigma_{k=1}^{K} \; \Sigma_{j=1}^{K} \; s_{jk} \, s_{in} \int_0^T \psi_k(t) \, \psi_n(t) \, dt$$

  Since $\{\psi_k(t)\}$ form an orthonormal set, the above equation reduces to

  $$E_i = \Sigma_{k=1}^{K} s_{jk}^{2} = \| \, \mathbf{s}_i \, \|^2$$

- **Gram-Schmidt Orthogonalization Procedure**

  Given a finite set of signals $\{ \mathbf{s}_1(t) , \mathbf{s}_2(t) , \ldots , \mathbf{s}_M(t) \}$ defined on the time interval (0 , T) , an orthonormal basis set may be constructed according to the following procedure :

Step 1 : ***Let the first basis function be***

$$\psi_1(t) = s_1(t) / \| s_1(t) \| = s_1(t) / \sqrt{E_1}$$

Step 2 :

Using $s_2(t)$, we define the coefficient s_{21} as

$$s_{21} = \int_0^T s_2(t)\, \psi_1(t)\, dt$$

Then $s_{21}\, \psi_1(t)$ is subtracted from $s_2(t)$ to yield

$$g_2(t) = s_2(t) - s_{21}\, \psi_1(t)$$

This waveform is orthogonal to $\psi_1(t)$.

The normalized waveform that is orthogonal to $\psi_1(t)$ is

$$\psi_2(t) = g_2(t) / \sqrt{\varepsilon_2}$$

Step 3 :

In general , the orthogonalization of the *k*th function leads to

$$\psi_n(t) = g_n(t) / \sqrt{\varepsilon_n}$$

where

$$g_n(t) = s_n(t) - \Sigma_{j=1}^{n-1} s_{nj}\, \psi_j^*(t)$$

.

$$s_{nj} = \int_0^T s_n(t)\, \psi_j(t)\, dt$$

$$\varepsilon_n = \int_0^T (g_n(t))^2\, dt$$

The orthogonalization process is continued until all the K signal waveforms have been exhausted.

4.5 Optimum Receiver

4.5.1 Matched Filter

- **Consider a known signal *s(t)* is corrupted by AWGN *w(t)*, resulting in the received signal**

$$x(t) = s(t) + w(t) \qquad (4.55)$$

- **Correlation Receiver**

An optimum receiver for detecting the known signal s(t) in the received signal *x(t)* ,called correlation receiver , consists of a correlator , with two inputs, one being the noisy received signal *x(t)* and the other being a locally generated replica of the known signal *s(t)* , as shown in Fig. 4.21.

- **Another way of constructing the optimum receiver is to use a matched filter.4.11.1 Matched Filter**

- **A matched filter is a linear filter designed to provide the maximum signal-to-noise power ratio at its output for a given real-valued input waveform *s(t)* .**

If the noise added to the signal is AWGN, then the impulse response of the matched filer is given by

$$h(t) = \begin{cases} k\, s(T - t) & 0 \leqq t \leqq T \\ 0 & \text{elsewhere} \end{cases} \qquad (4.56)$$

Fig.4.21 (a) Correlation receiver   (b) Matched –filter receiver

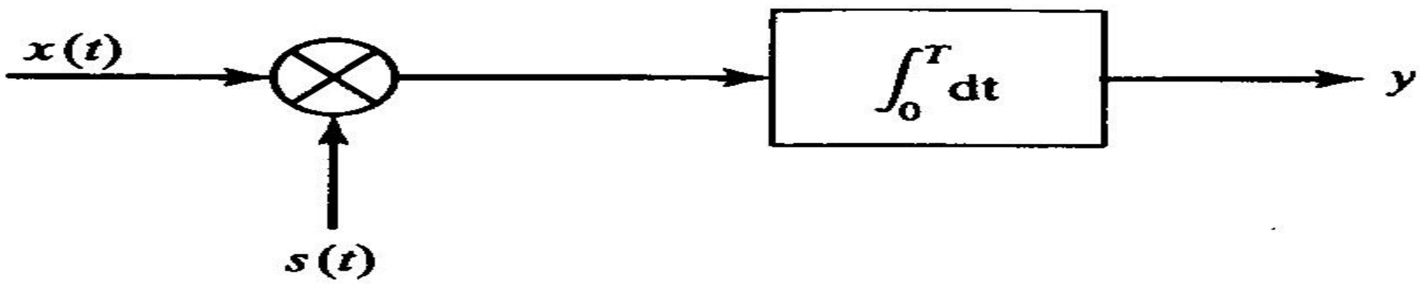
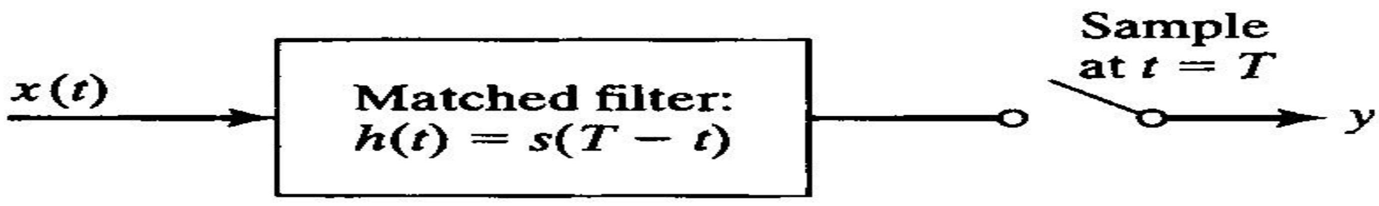


FIGURE D.2   Correlation receiver.



4.5.2 Detection of Known Signals in Additive White Gaussian Noise

- **To detect a known signal in AWGN with a correlation receiver or a matched-filter receiver, the output sample is compared against a threshold and then a decision is made by the receiver, depending on whether the threshold is exceeded or not.**
- **The receiver is subject to errors due to random behavior of the additive noise in the received signal *x(t)*.**

$$x(t) = s(t) + w(t) \qquad (4.57)$$

- **To calculate the average probability of error incurred by the receiver, *x(t)* is used as the input signal applied to the correlation receiver. The resulting output sample is**

$$y(t) = \int_0^T x(t)\, s(t)\, dt = \ldots$$

$$= E_s + \int_0^T w(t)\, s(t)\, dt \qquad (4.58)$$

where $E_s = \int_0^T s^2(t)\, dt$ **is the energy of the known signal *s(t)* .**

- **Since $w(t)$ is the sample function of a Gaussian process $W(t)$, it follows that y is the sample function of a Gaussian distributed random variable Y.**

The mean value of Y is

$$\mu_Y = E[Y] = \dots$$

$$= E_s + E\left[\int_0^T w(t)\, s(t)\, dt\right]$$

$$= E_s + E\left[\int_0^T w(t)dt\right] s(t) = E_s \qquad (4.59)$$

The variance of Y is

$$\sigma_Y^2 = E[(Y - \mu_Y)^2])$$

and $E[W(t_1)\, W(t_2)] = (N_0/2)\, \delta(t_1 - t_2)$

After some mathematical manipulation, we obtain

$$\sigma_Y^2 = N_0 E_s /2 \qquad (4.60)$$

- **In summary, we find that the correlation receiver output *y* is the sample value of a Gaussian-distributed random variable *Y* with mean $\mu_Y = E_s$ and**

  variance $\sigma_Y^{\,2} = N_0 E_s/2$.

  Accordingly, the probability density function of the random variable *Y* is given by

$$p_Y = 1 / \{\sigma_Y \sqrt{(2\pi)}\}\ \exp\{-(y-\mu_Y)^2 / 2\sigma_Y^{\,2}\}$$
$$= 1/\sqrt{(\pi N_0 E_s)}\ \exp\{-(y-E_s)^2 / N_0 E_s\} \qquad (4.62)$$

- **Let λ denote the threshold which the correlator output is compared.**

  When $y > \lambda$, a decision is made that the known signal *s(t)* is present.

  When $y < \lambda$, a decision is made that the received signal *x(t)* consists of solely of noise *w(t)*, *and s(t) is not present*.

- **The conditional probability of error, given that the known signal *s(t)* is present in the receiver input is defined by**

$$p_e = \int_{-\infty}^{\lambda} f_Y(y)\ dy$$

$$= 1/\sqrt{(\pi N_0 E_s)} \int_{-\infty}^{\lambda} exp\{-(y - E_s)^2 / N_0 E_s\}\ dy$$

Let $z = (y - E_s) / \sqrt{(N_0 E_s)}$ **, then we have**

$$p_e = 1 / \sqrt{(\pi)} \int \quad exp(-z^2)\ dz \qquad (4.63)$$

Fig.4.22 shows the pdf and conditional pdf for a typical binary receiver.

- **The Q-function is defined as**

$$Q(\mathrm{x}) = 1/\sqrt{(2\pi)} \int_x^{\infty} exp(-z^2)\, dz \qquad \textbf{(4.64)}$$

For the special case $\lambda = 0$,

$$p_e = = Q[\sqrt{(2E_s/N_0)}] \qquad \textbf{(4.65)}$$

Fig.4.22 Conditional pdf of a correlation receiver output

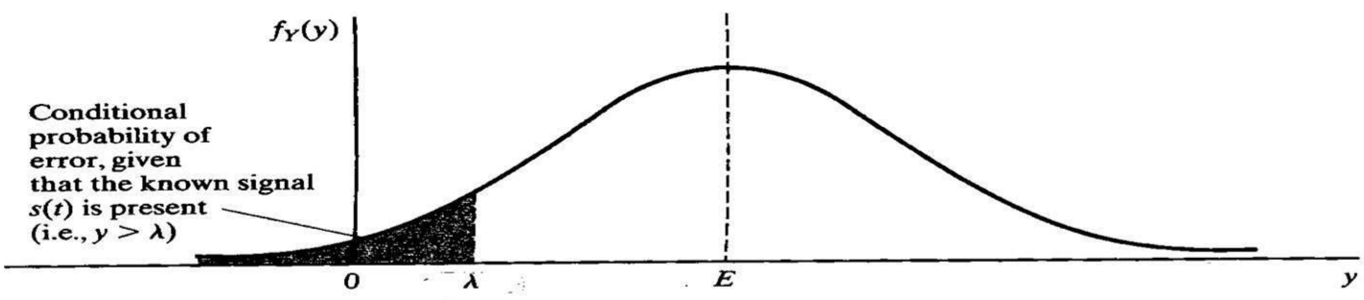


FIGURE D.4 Probability distribution of the correlation receiver output.

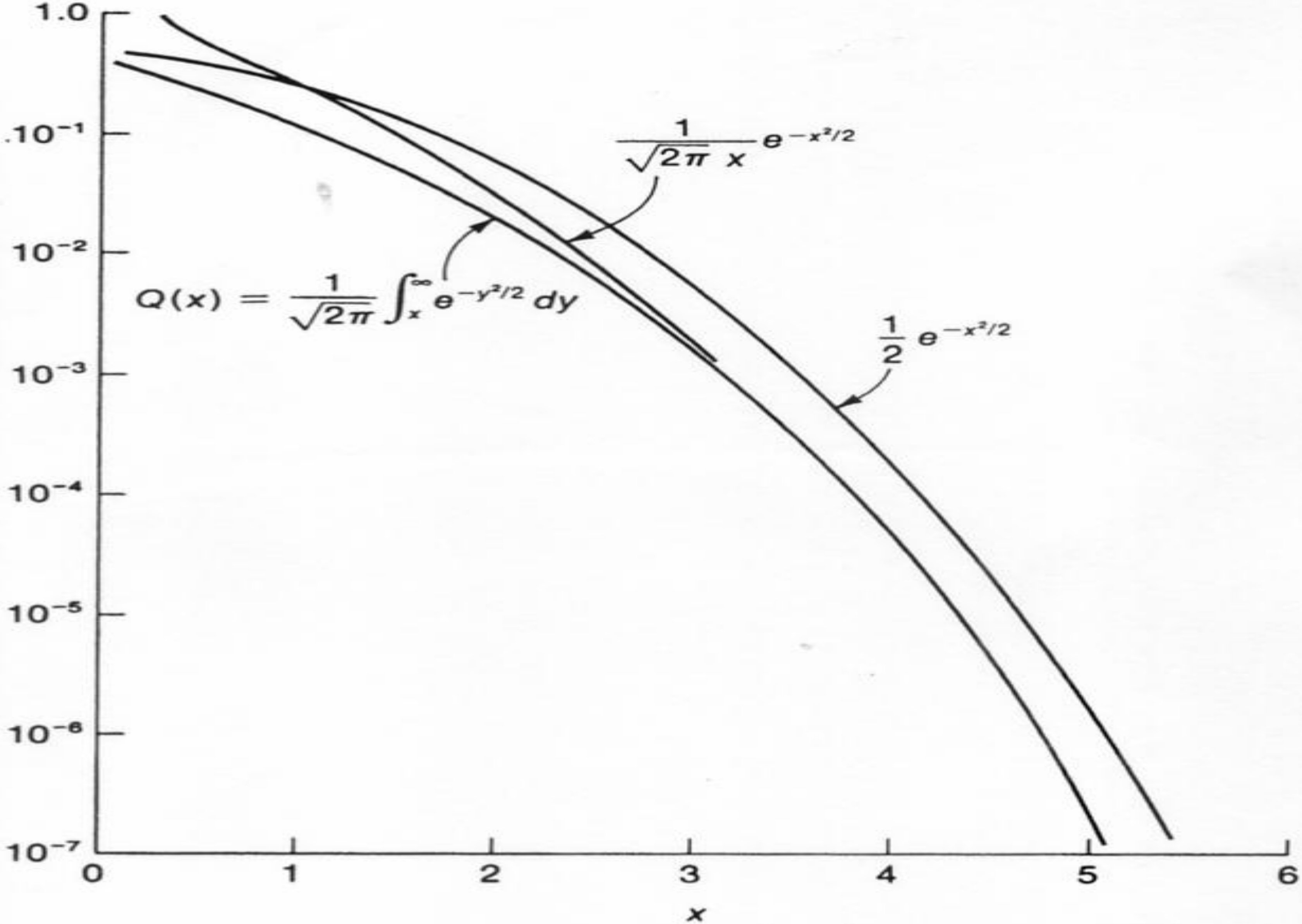


FIGURE A.1 Gaussian tail integral $Q(x)$ and some approximations.

4.5.3 Detection of Binary Antipodal Signals in AWGN

- **To develop a general formula for the BER of a detected binary signal. The transmitted signal over a bit interval (0 , T) is**

$$s(T) = \begin{cases} s_1(t), & 0 \leqq t \leqq T \\ s_2(t), & 0 \leqq t \leqq T \end{cases}$$

If $s_1(t) = -s_2(t)$, s(t) is called an antipodal signal.

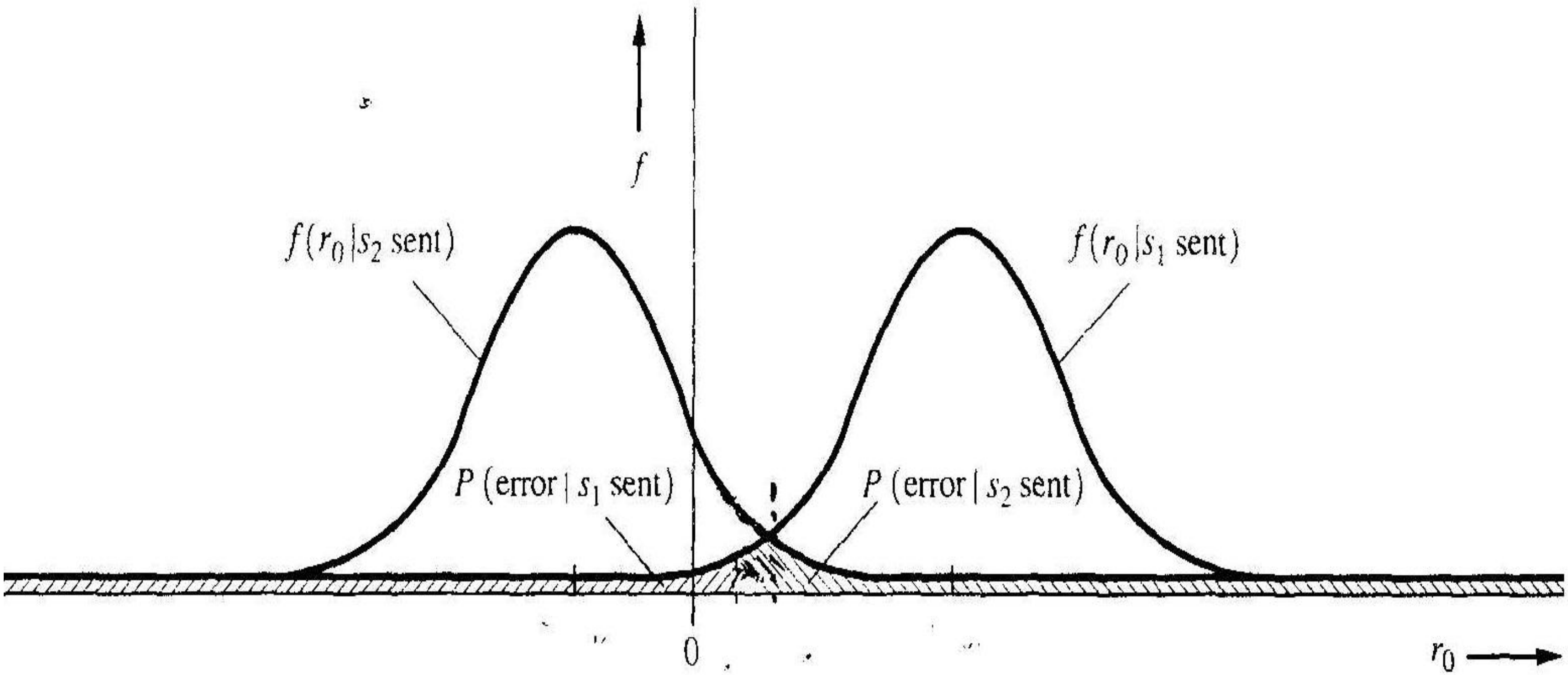
For optimal receiver, the input to the detector is the binary signal plus noise , denoted by

$$r_o(t_o) = s_o(t_o) + w_o(t_o)$$

where

$$s_o(t_o) = \begin{cases} s_{01}, & \textit{for a binary 1 sent} \\ s_{o2}, & \textit{for a binary 0 sent} \end{cases}$$

Fig.4.22 Conditional pdf of a typical binary detector



- **Assuming that the transmitted signal s(t) has equal probabilities**

  for s_1(t) and s_2(t) , the optimum choice of the decision threshold is given by

  $$\lambda = \tfrac{1}{2} (s_{01} + s_{02})$$

The average power of white Gaussian noise is $N = \sigma_0^{\,2}$.

It can be shown that the bit error rate (BER) of detection is

$$p_e = Q\left((s_{01} - s_{02}) / 2\sigma_0^{\,2}\right)$$
$$= Q\left(\sqrt{\{(s_{01} - s_{02})^2 / 4\sigma_0^{\,4}\}}\right)$$

For optimal receiver , the error probability can be expressed as

$$p_e = Q\left(\sqrt{(E_d / 2N_0)}\right)$$

where E_d *is the energy difference between the binary signals* , s_1(t) and s_2(t) , over a bit interval T ,

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 \, dt$$

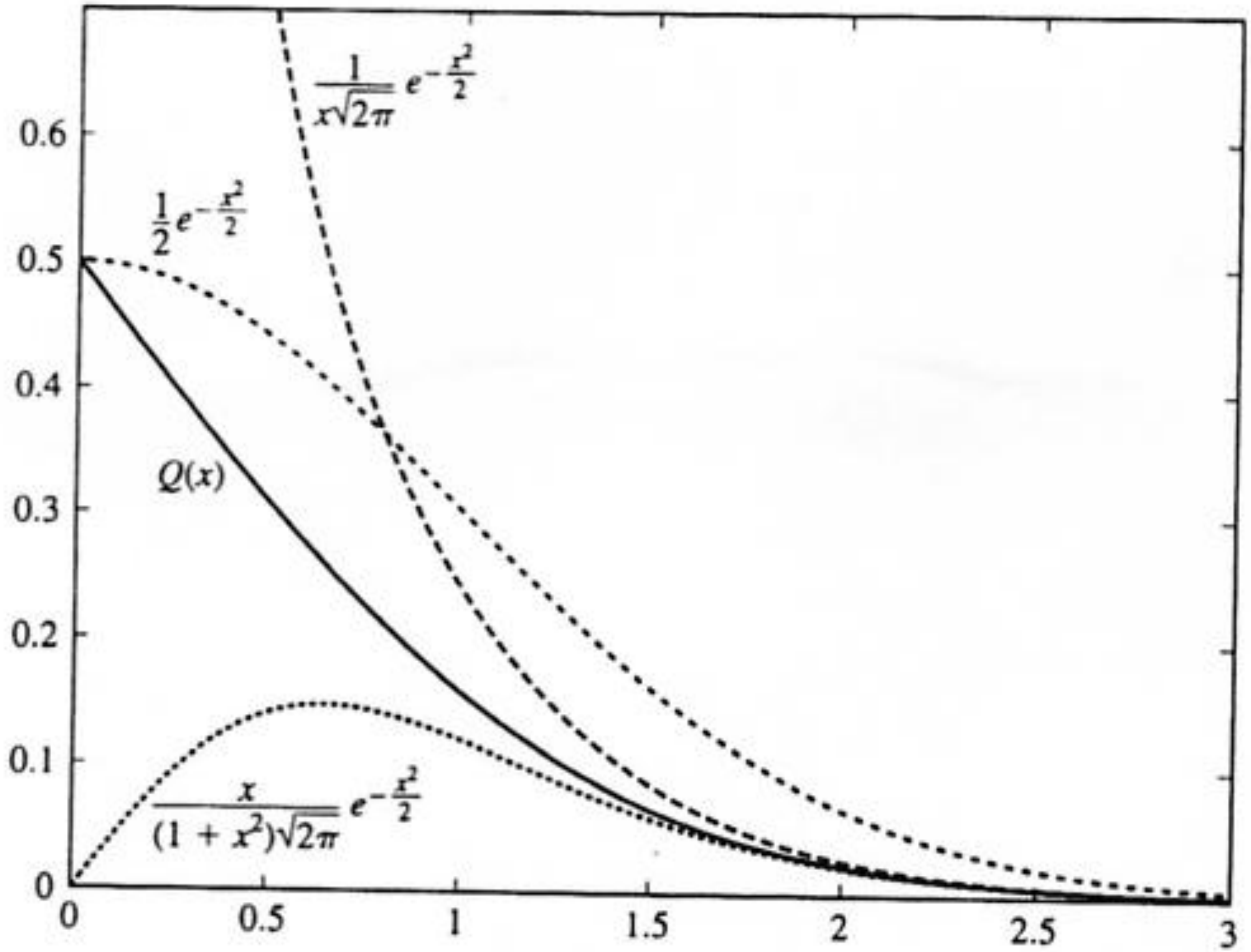


FIGURE 2.3–2
Plot of $Q(x)$ and its upper and lower bounds.

4.5.4 Optimum Detector

- **Suppose that in each time slot of duration *T* ,one of the *M* possible signals {$s_1(t)$, $s_2(t)$,… , $s_M(t)$ } is transmitted .**

  For signal-space representation ,a received signal y (t) is applied

  to a bank of correlators . The outputs of the correlators define the observation vector

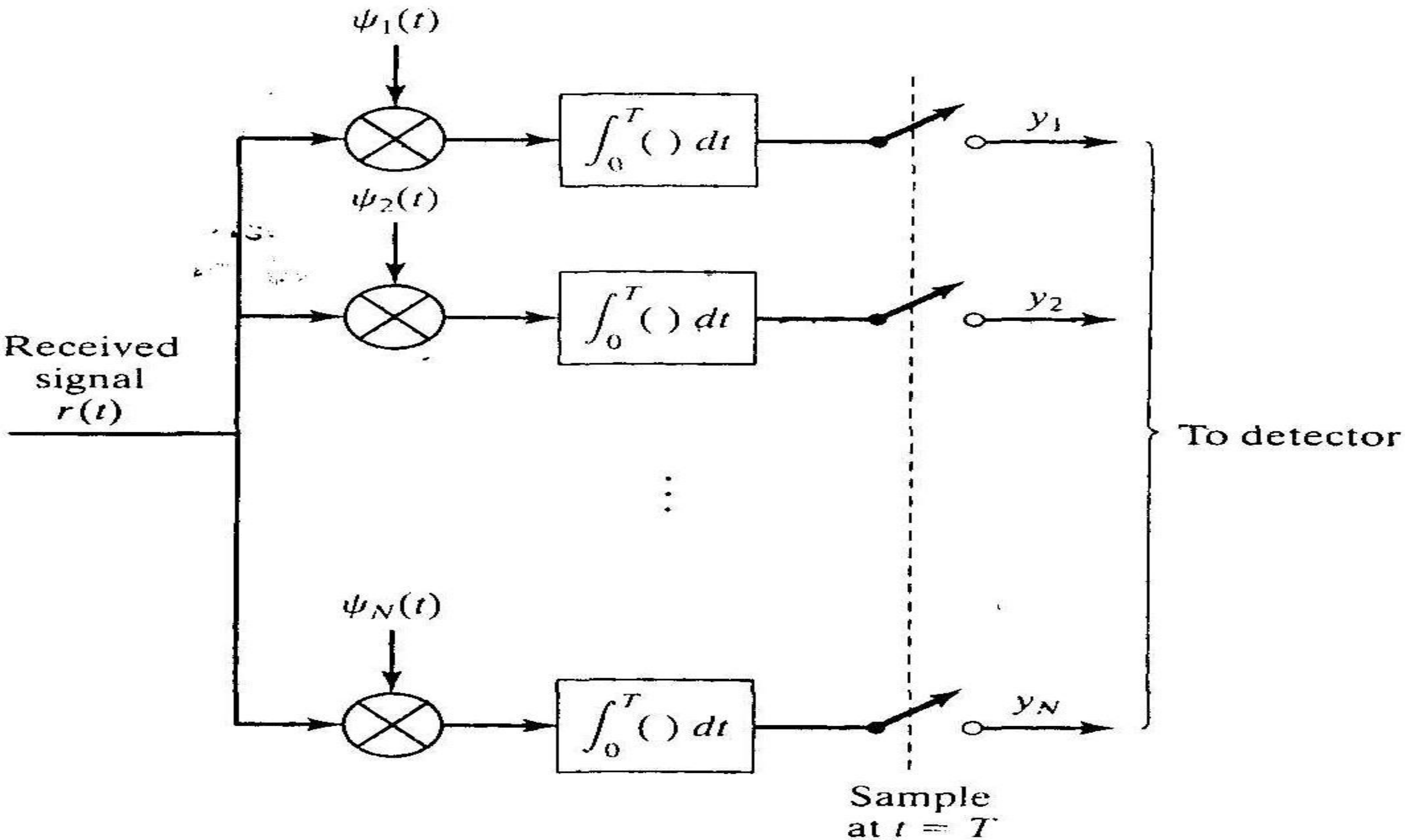
  $$\mathbf{y} = \{ y_1 , y_2 ,\dots , y_M \}$$

  The vector y is the sum of two vectors : the signal vector $\mathbf{s}_k$ and the noise vector $\mathbf{w}$. That is,

  $$\mathbf{y} = \mathbf{s} + \mathbf{w}$$

- **Optimum detection :**

  Given the observation vector *y* , perform a mapping from *y* to an estimate *x*^ of the transmitted symbol *x* , in a way that would minimize the probability of error in the decision-making process.



Correlation-type demodulator.

Suppose that , given the observation vector $\mathbf{y}$, we make the decision

$x\wedge = s_j$. The probability of error in this decision , denoted by $p_e(s_j$ not sent $\mid \mathbf{y})$, is simply

$$p_e(s_j \text{ not sent } \mid \mathbf{y}) = 1- p_e(s_j \text{ sent } \mid \mathbf{y})$$

The optimum decision criterion can be stated as follows.

Choose $x\wedge = s_j$

if $p_e(s_j \mid \mathbf{y}) \geqq p_e(s_k \mid \mathbf{y})$ for all $k \neq j$

where $k = 1,2,\ldots,M$.

This decision criterion is referred to as the maximum *a posterior* probability (MAP) rule .

Using Bayes's rule , we may express the posterior probabilities as

$$p(s_j \mid \mathbf{y}) = p(\mathbf{y} \mid s_j) \, p(s_j) \, / \, p(\mathbf{y})$$

where $p(\mathbf{y} \mid s_j)$ is the conditional pdf of the observation vector given s_j , $p(s_j)$ is the a priori probability of the signal s_j being transmitted , and
$p(\mathbf{y})$ is the unconditional pdf of $\mathbf{y}$, independent of the transmitted symbol.
Accordingly, we may restate the MAP detection criterion as follows.

Choose $\hat{x} = s_j$
if $p(s_k) \, p(\mathbf{y} \mid s_k)$ is maximum for $k = j$.

- **Maximum likelihood detection ;**

  In the case where the messages are equiprobable *a priori* , i.e.,

  when $p(s_k) = 1/M$ for all k , the optimum detection criterion reduces to

  Choose $\hat{x} = s_j$

  if $p(\mathbf{y} \mid s_k)$ is maximum for $k = j$.

  This detection criterion is referred to as Maximum Likelihood (ML) criterion .

4.5.5 ML Detection of Signals in AWGN

- **Suppose that in each time slot of duration T, one of the M possible signals $\{s_1(t), s_2(t), \ldots, s_M(t)\}$ is transmitted over AWGN channel. The messages are equiprobable a priori, i.e., $p(s_k) = 1/M$ for all k.**
- **The received signal $y(t)$ is applied to a bank of correlators. The outputs of the correlators define the observation vector**

$$\mathbf{y} = \{ y_1, y_2, \ldots, y_M \}$$

The conditional pdfs of the random variables

$\{ y_1, y_2, \ldots, y_M \}$ are given by

$$p(\mathbf{y} \mid s_k) = \Pi_{k=1}^{N} \; p(y_k \mid s_{mk}) \quad , m = 1,2,\ldots,M ,$$

where

$$p(y_k \mid s_{mk}) = [1/\sqrt{(\pi N_0)}] \; \exp\{ -(y_k - s_{mk})^2 / N_0 \}$$

$$k = 1,2,\ldots, N$$

Using the vector form $\|\mathbf{y}-\mathbf{s}_m\|^2=\Sigma_{k=1}^{N}(y_k-s_{mk})^2$

we obtain

$$p(\mathbf{y}\mid s_m)=[1/(\pi N_0)^{N/2}]\exp\{-\|\mathbf{y}-\mathbf{s}_m\|^2/N_0\}\quad,$$

$$m=1,2,\ldots,M\,.$$

The log-likelihood function is then given by

$$\ln p(\mathbf{y}\mid \mathbf{s}_m)=-(N/2)\ln(\pi N_0)-\Sigma_{k=1}^{N}(y_k-s_{mk})^2/N_0$$

We can see that the maximum of ln $p(\mathbf{y}\mid \mathbf{s}_m)$ over $\mathbf{s}_m$ is equivalent to finding the signals $\mathbf{s}_m$ that minimize the Euclidean distance

$$D(\mathbf{y},\mathbf{s}_m)=\Sigma_{k=1}^{N}(y_k-s_{mk})^2/N_0\quad,m=1,2,\ldots,M\,.$$

D(y , s_m) is denoted as distance metrics .

Since $\Sigma_{k=1}^{N}(y_k-s_{mk})^2$ can be expanded as

$$\Sigma_{k=1}^{N}(y_k-s_{mk})^2=\Sigma_{k=1}^{N}y_k^2-2\,\Sigma_{k=1}^{N}(y_k s_{mk})+\Sigma_{k=1}^{N}s_{mk}^2$$

$$=\|\mathbf{y}\|^2-2\,\mathbf{y}\cdot\mathbf{s}_m+\|\mathbf{s}_m\|^2$$

The term $\| \mathbf{y} \|^2$ is common to all decision metrics; hence it may be ignored in the computations of metrics. Thus we have a modified distance metrics

$$D'(\mathbf{y}, \mathbf{s}_m) = -2\, \mathbf{y} \cdot \mathbf{s}_m + \| \mathbf{s}_m \|^2$$

Therefore , selecting the signal s_m that minimizes $D'(\mathbf{y}, \mathbf{s}_m)$ is equivalent to sel3cting the signal that maximize the correlation metrics $C(\mathrm{y}, \mathrm{s}_m)$ defined by

$$C(\mathbf{y}, \mathbf{s}_m) = -D'(\mathbf{y}, \mathbf{s}_m)$$
$$= 2\, \mathbf{y} \cdot \mathbf{s}_m - \| \mathbf{s}_m \|^2$$

Furthermore, if all signals s_m have the same energy , $\| s_m \|^2$ may also be ignored in the computation of distance metrics.

4.5.6 ML Detection of M-ary PAM Signals in AWGN

- **With an optimum receiver for *M*-ary PAM signals transmitted over AWGN channel , the probability of error of ML detection can be estimated as follows.**
- **Assume that the message symbols are equiorobable, i.e.,**

$p(d_i) = 1/M$, where $d_i = \pm 1 , \pm 1 ,\ldots, \pm (M-1)$ and $M = 2^k$.

The received signal is expressed by

$$y(t) = d_i\, s(t) + w(t)$$

The energy of the basic pulse s(t) is denoted by E_s .

The sample U at the correlator output is given by

$$U = \int_0^T [d_i\, s(t) + w(t)]\, s(t)\, dt = d_i E_s + Z_w$$

where Z_w is a Gaussian random variable that results from the correlation of noise W(t) and signal s(t) .This random variable has zero mean and variance $\sigma_Z^2 = E_s N_0 / 2$. Therefore U is also a Gaussian random variable with mean equal to $d_i E_s$ and variance σ_Z^2 .

- **In general , symbol error probability can be calculated from the formula**

$$p_M(\varepsilon) = \Sigma_{i=1}^{M} \; p(d_i)\, p(\varepsilon \mid d_i \text{ transmitted})$$

The placing of the thresholds as shown in Fig.xx helps evaluate the probability of error . On the basis that all amplitude levels are equally *a priori* , the average probability of a symbol error is simply the probability that the noise variable w(t) exceeds in magnitude one-half of the distance between levels . However , when either one of the two outside levels , ± (*M* – 1) , is transmitted , an error occurs in one direction only . Thus , we have

$$p_M(\varepsilon) = [(M\text{-}1) / M]\; p(\mid U - E_s \mid > E_s)$$

$$= [(M\text{-}1) / M]\,(2/ \sigma_Z)\,(1/ \sqrt{(2\pi)})$$

$$\int_{Es}^{\infty} \exp(-u^2/ 2\, \sigma_Z^2)\, du$$

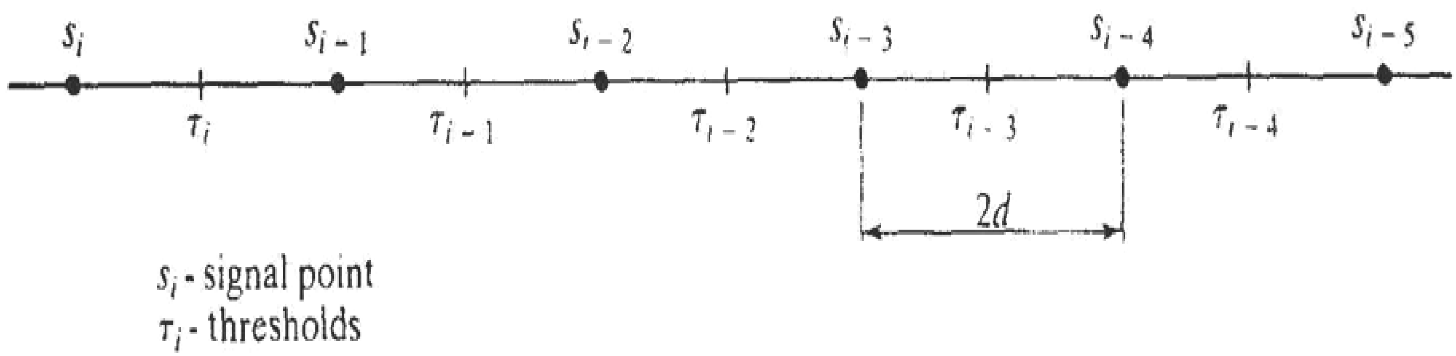
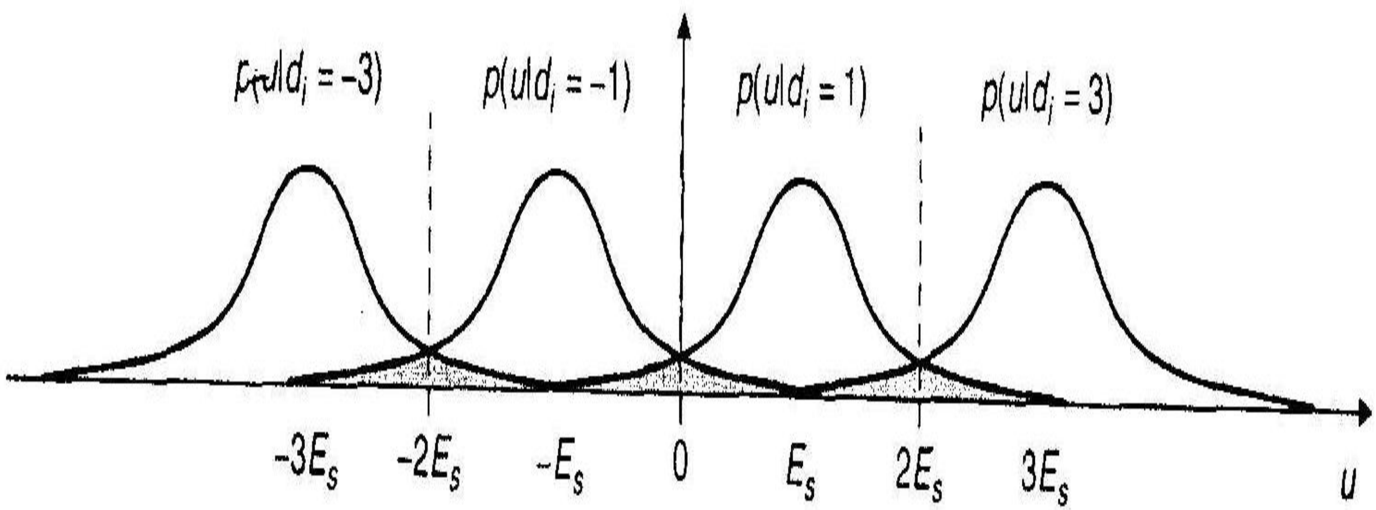


Figure 8.44 Placement of thresholds at midpoints of successive amplitude levels.



With some mathematical manipulations , we obtain

$$p_{eM} = \{ 2(M\text{-}1)/M \} \, Q(2E_s / N_0)$$

The average energy of the M-ry PAM signal can be calculate as

$$E_{av} = (2 / M) \, [\, 1+9+25+\ldots+(M\text{-}1)^2 \,] \, E_s$$

$$= (M^2 - 1) \, E_s / 3$$

An M-ary symbol carries an information of $\log_2 M$ bits . Hence, the average bit energy is

$$E_b = (M^2 - 1) \, E_s / 3 \log_2 M$$

Therefore the bit error rate is given by

$$p_{eM} = \{ 2(M\text{-}1)/M \} \, Q[\sqrt{\{(6 \log_2 M)(E_B / N_0)/ M^2 \}}$$

$$\fallingdotseq 2 \, Q[\sqrt{\{(6 \log_2 M)(E_B / N_0)/ M^2 \}}$$

M>> 1

Tabulation of $Q(z)$

z	$Q(z)$	z	$Q(z)$
0.0	0.50000	2.0	0.02275
0.1	0.46017	2.1	0.01786
0.2	0.42074	2.2	0.01390
0.3	0.38209	2.3	0.01072
0.4	0.34458	2.4	0.00820
0.5	0.30854	2.5	0.00621
0.6	0.27425	2.6	0.00466
0.7	0.24196	2.7	0.00347
0.8	0.21186	2.8	0.00256
0.9	0.18406	2.9	0.00187
1.0	0.15866	3.0	0.00135
1.1	0.13567	3.1	0.00097
1.2	0.11507	3.2	0.00069
1.3	0.09680	3.3	0.00048
1.4	0.08076	3.4	0.00034
1.5	0.06681	3.5	0.00023
1.6	0.05480	3.6	0.00016
1.7	0.04457	3.7	0.00011
1.8	0.03593	3.8	0.00007
1.9	0.02872	3.9	0.00005
		4.0	0.00003