Chapter 5

Modulation and Passband Transmission

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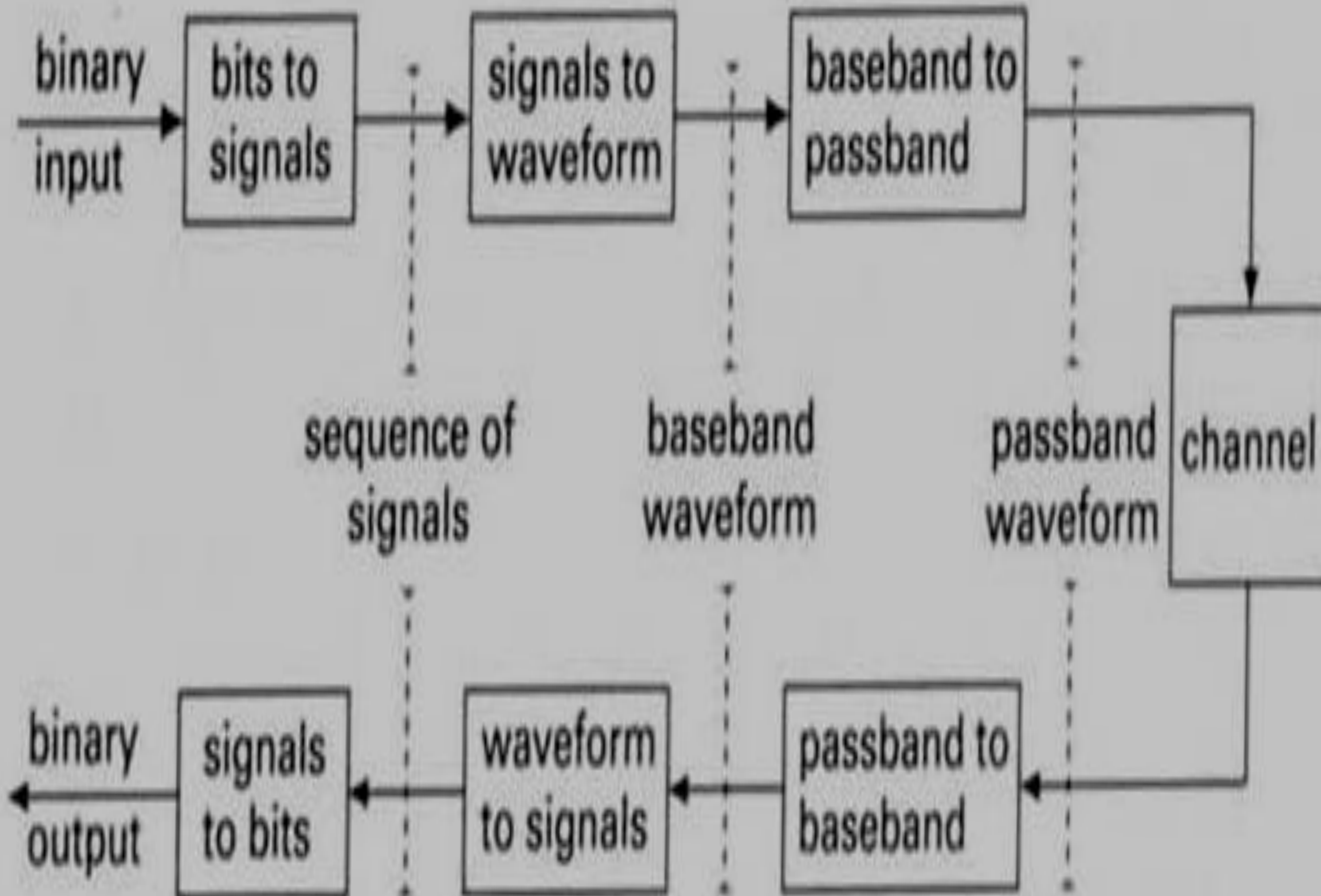


Layers of a modulator (channel encoder) and demodulator (channel decoder).

5.1 Modulation Techniques

- **Modulation is formally defined as the process by which some characteristics of a carrier wave is varied in accordance with an information-bearing signal .**

  The information-bearing signal is referred to as modulating signal. The output of the modulation process is referred to as the modulated signal .
- **The device that performs the modulation process in the transmitter is referred to as modulator , and the device used to recover the information-bearing signal in the receiver is referred to as a demolulator.**

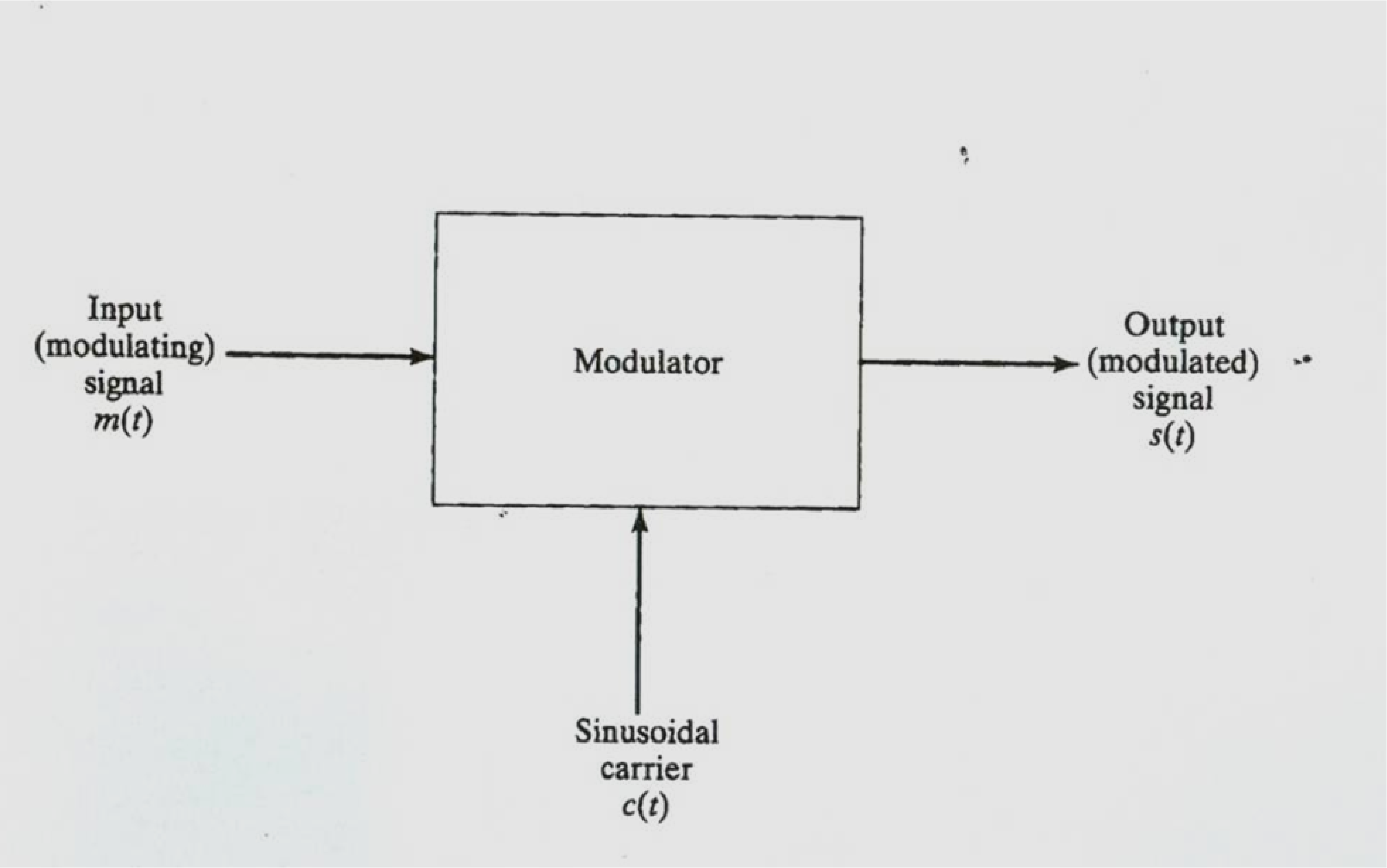
- **Purposes of modulation ?**

There are several benefits that result from the use of modulation in digital communication system :

1. **Modulation is used to shift the spectral content of a message signal (information-bearing) so that it lies inside the operating frequency band of the wireless communication channel.**
2. **Modulation (e.g., FM) provides a mechanism for putting the information content of a message signal into a form that may be less vulnerable to noise or interference.**
3. **Modulation permits the use of multiple access techniques.**

- **Fig. 5.1 shows the block diagram of a modulator supplied with a sinusoidal carrier c (t)**

Fig.5.1 Block diagram of modulator



- **Popular spectral- efficient digital modulation techniques in wireless communications :**
  - **- GSM system adopted GMSK**
  - **- US IS-95 and CDMA2000 adopted QPSK/OQPSK**
  - **- WCDMA adopted QPSK**
  - **- IEEE 802.11b adopted BPSK and DQPSK**
  - **- IEEE 802.11a adopted BPSK/QPSK , 16-QAM ,and 64-QAM**
  - **- IEEE 802.16 adopted BPSK, QPSK , 16-QAM ,and 64-QAM**

5.1.1 Analog Modulation

- **In the analog case, the message signal *m(t)* is a continuous function of time t. Consequently, the modulated signal *s(t)* , likewise , a continuous function of time.**

  Therefore, the analog modulation is also called *continuous wave* (CW) *modulation* .

- **The modulation process is on the basis of which parameter of the sinusoidal carrier *c (t)* is varied in accordance with the message signal *m(t)* .**

  ***c(t)* is typically sinusoidal and is written as**

$$c(t) = A_c \cos (2\pi f_c t + \theta) \qquad (5.1)$$

- **There are two kinds of modulation : amplitude modulation and angle modulation.**

- **In amplitude modulation, the amplitude of the carrier ,A_c , is varied linearly with the message signal *m*(*t*) .**

- **By definition, amplitude modulation (AM), produced by an analog message signal *m(t)* is described by**

$$s(t) = A_c\,(1 + k_a\,m(t))\cos(2\pi f_c t) \qquad (5.2)$$

where k_a is the modulation index.

- **Fig. 5.2 shows the spectrum of the AM signal .**

The retention of the carrier in the composition of the AM signal represents a loss of transmitted signal power.

- **The double sideband –suppressed carrier (DSB-SC) modulation is defined simply as the product of the message signal and the carrier ; that is,**

$$s(t) = c(t)\,m(t) = A_c\,m(t)\cos(2\pi f_c t) \qquad (5.3)$$

Fig. 5.3 shows the corresponding spectra.

- **The AM signal and the DSB-SC modulated signal share a common feature : They both require the use of a transmission bandwidth equal to twice the message bandwidth, namely 2*W*.**

Fig. 5.2 (a) Message spectrum

(b) Spectrum of the corresponding AM signal

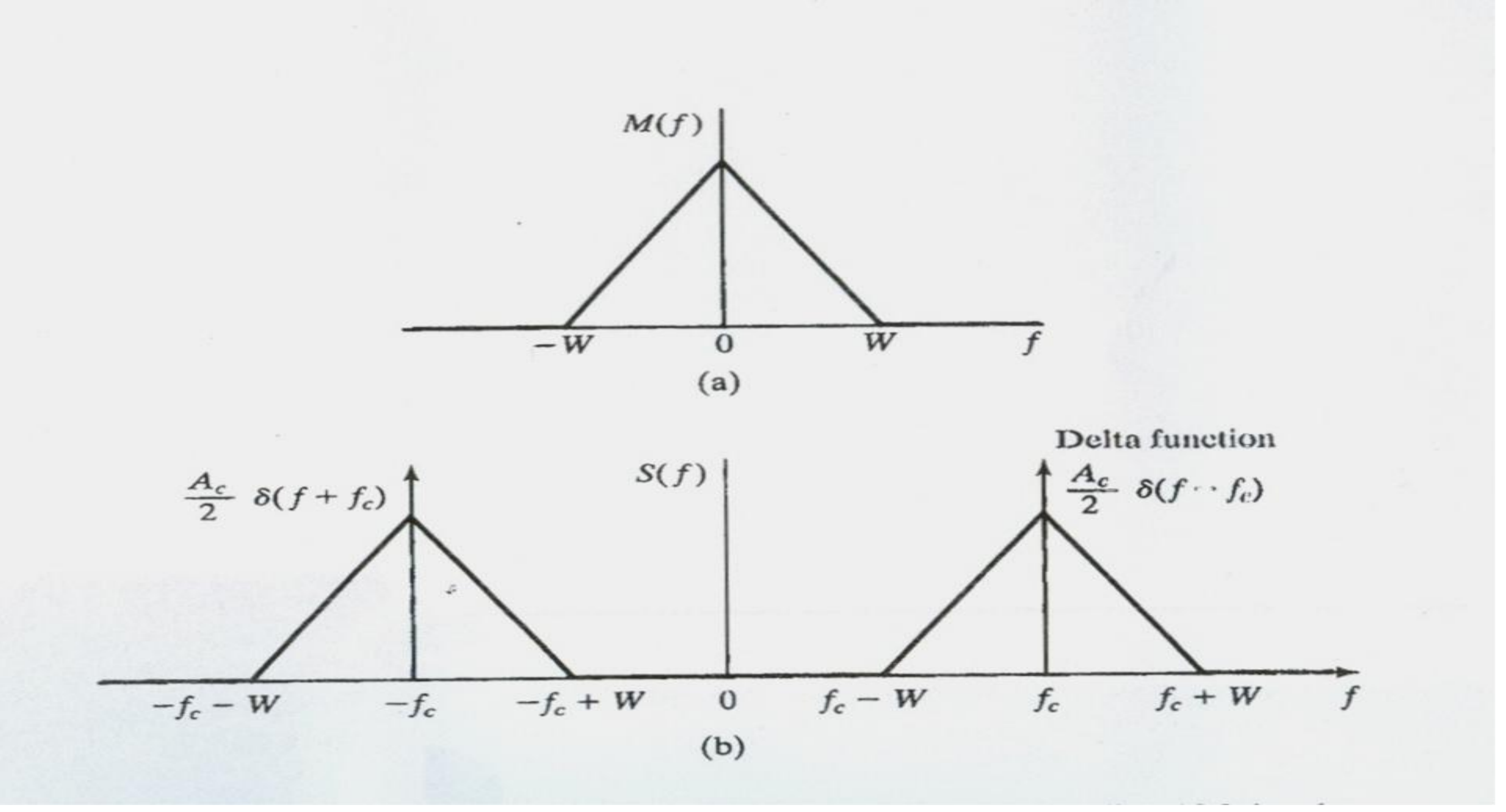
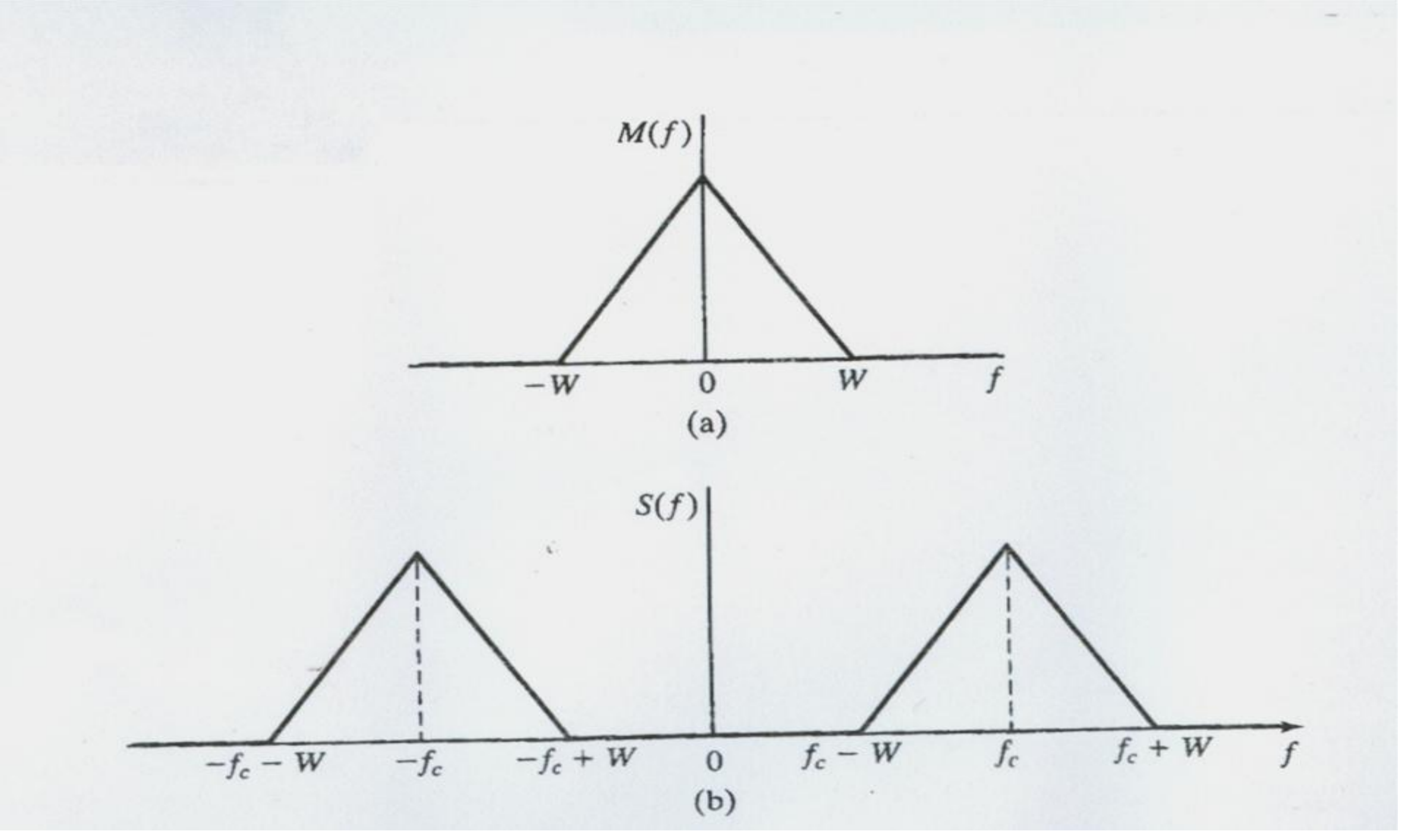


Fig. 5.3 (a) Message spectrum

(b) Spectrum of the corresponding DSB-SC signal



- **Angle modulation itself can be classified into two kinds :** **frequency modulation and phase modulation.** **In these modulations, the frequency or the phase of the carrier is varied linearly with the message signal.**
- **In frequency modulation (FM), the instantaneous frequency of the sinusoidal carrier , denoted by $\Psi(t)$, is varied in accordance with the information-bearing signal $m(t)$.**
- **The frequency- modulated signal can be described in the time-domain by**

$$s(t) = A_c \cos \left(2\pi f_c t + 2\pi k_f \int m(\tau) d\tau \right) \qquad (5.4)$$

where k_f is the selectivity (or index) of frequency modulation.

5.1.2 Digital Modulation

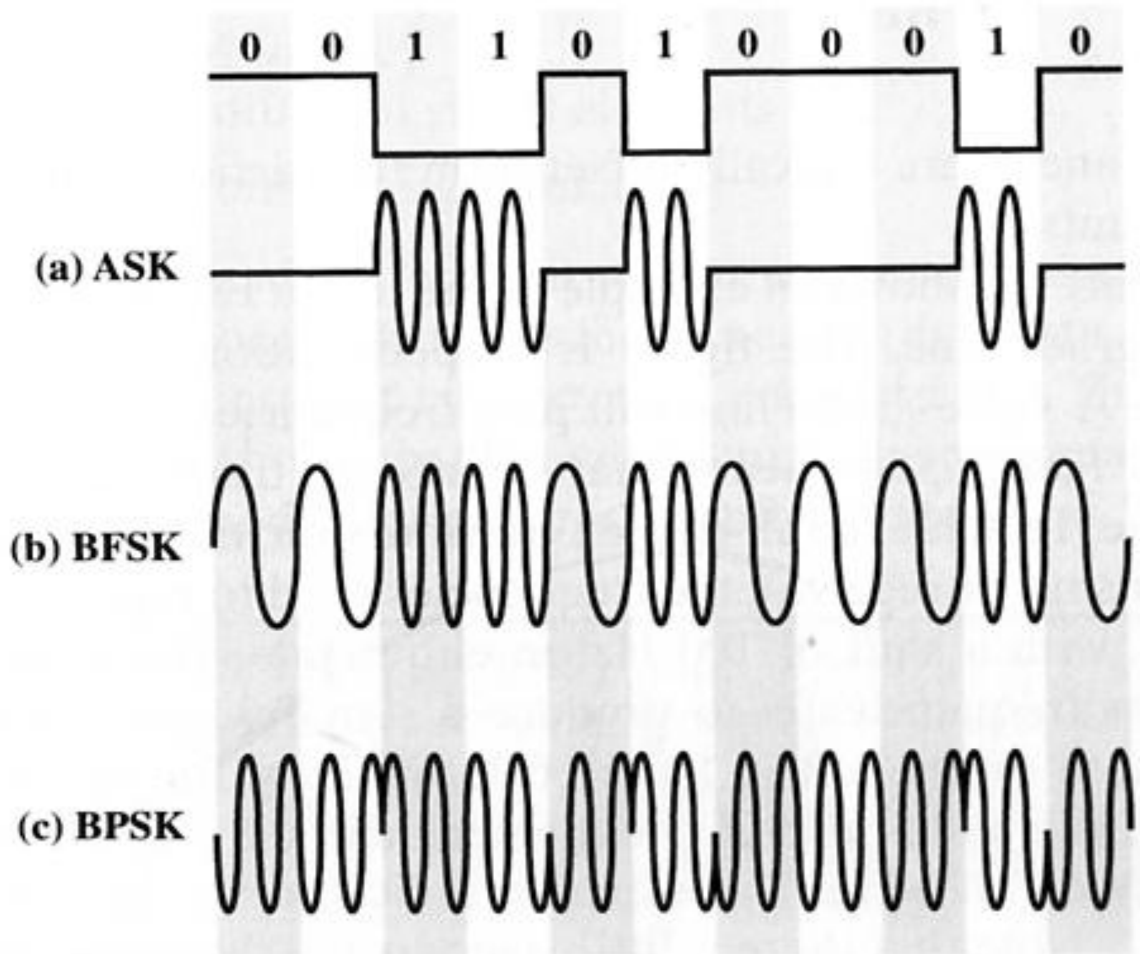
- **To transmit digital signal waveforms through a bandpass channel, the baseband signal waveforms $m_k(t)$, $k=1,2,\ldots,K$ are used to modulate a sinusoidal carrier by varying its amplitude, frequency, or phase. The corresponding modulation techniques are referred to as amplitude-shift keying (ASK), frequency-shift keying (FSK), or phase-shift keying (PSK), respectively.**

  Another important class of digital modulation technique is quadrature amplitude modulation (QAM) which is a hybrid of amplitude and phase modulations.

  Binary ASK , also known as on-off keying , was one of the earliest forms of digital modulation used in radio telegraphy at the beginning of 20th century. Simple ASK is no longer widely used in digital communication system.

- **Fig.5.4 illustrates the waveforms of binary ASK, FSK, and PSK.**

Fig.5.4 Waveforms of ASK, BFSK, and BPSK



5.2 Complex Representation of Bandpass Signals and Systems

5.2.1 Bandpass Signals

- **A general representation of bandpass signal (such as linearly modulated signal) is given by**

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad (5.5)$$

where $s_I(t)$ is referred to as in-phase component of $s(t)$ and $s_Q(t)$ is referred to as the quadrature phase component.

- **It can be shown that $s(t)$ can be expressed in another form as**

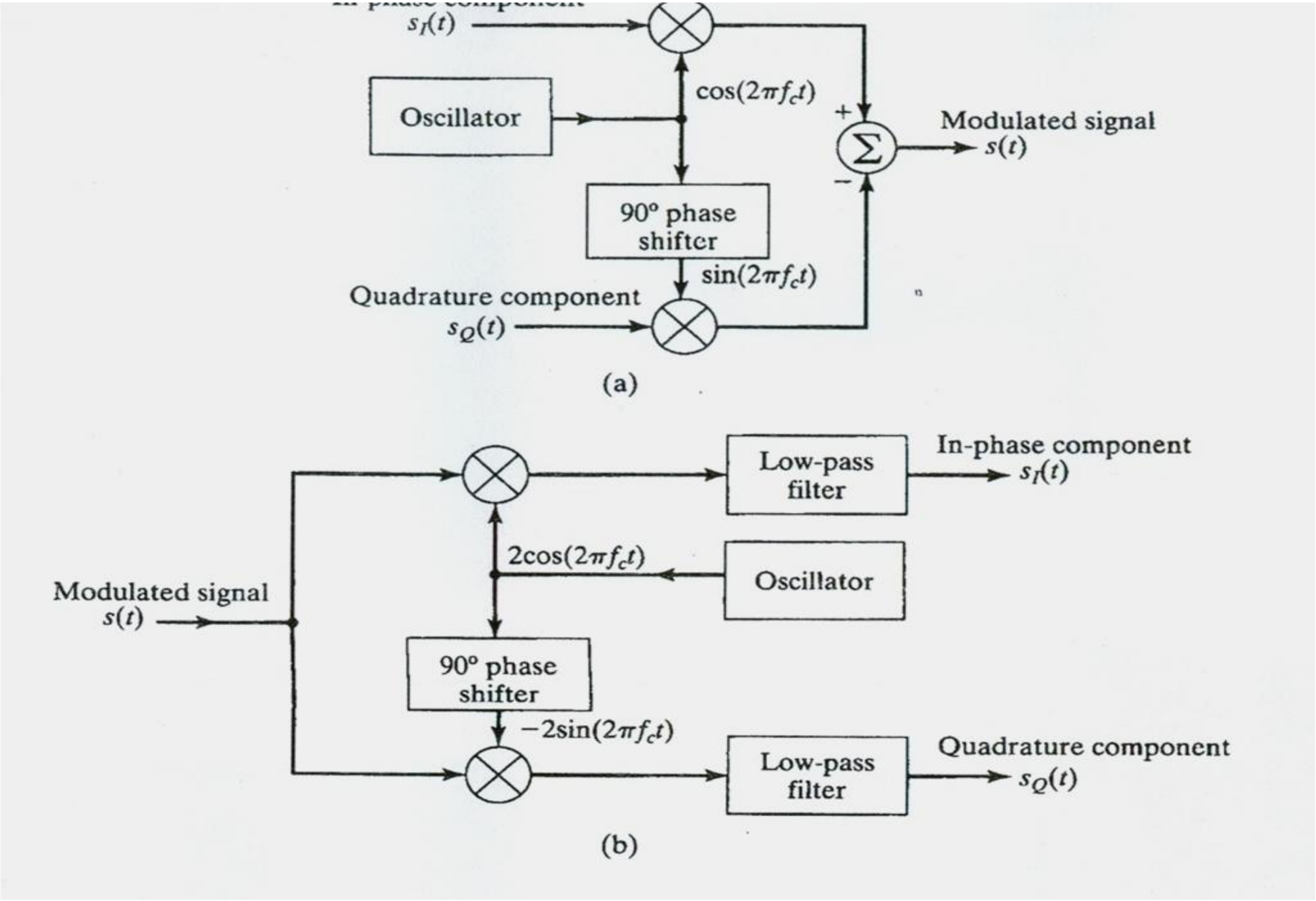
$$\tilde{s}(t) = Re\{s(t)\exp(j2\pi f_c t)\} \quad (5.6)$$

and $\tilde{s}(t) = s_I(t) + j s_Q(t) \quad , \ j = \sqrt{-1}$ **(5.7)**

- **Given the two components $s_I(t)$ *and* $s_Q(t)$, we may thus use the scheme shown in Fig.5.5(a) to synthesize the modulated signal $s(t)$. The in-phase component $s_I(t)$ and the quadrature phase component $s_Q(t)$ are both real-valued functions of time that are uniquely defined in terms of baseband signal $m(t)$.**
- **The in-phase and quadrature components are orthogonal to each other, occupying exactly the same bandwidth as the baseband signal $m(t)$.**
- **The complex envelope $s(t)$ given in Eq.(5.6) completely preserves the information content of the modulated signal except for the carrier frequency f_c .**

Fig.5.5 (a) Synthesizer to construct a modulated signal

(b) Analyzer for a modulated signal



5.2.2 Linear Bandpass Systems

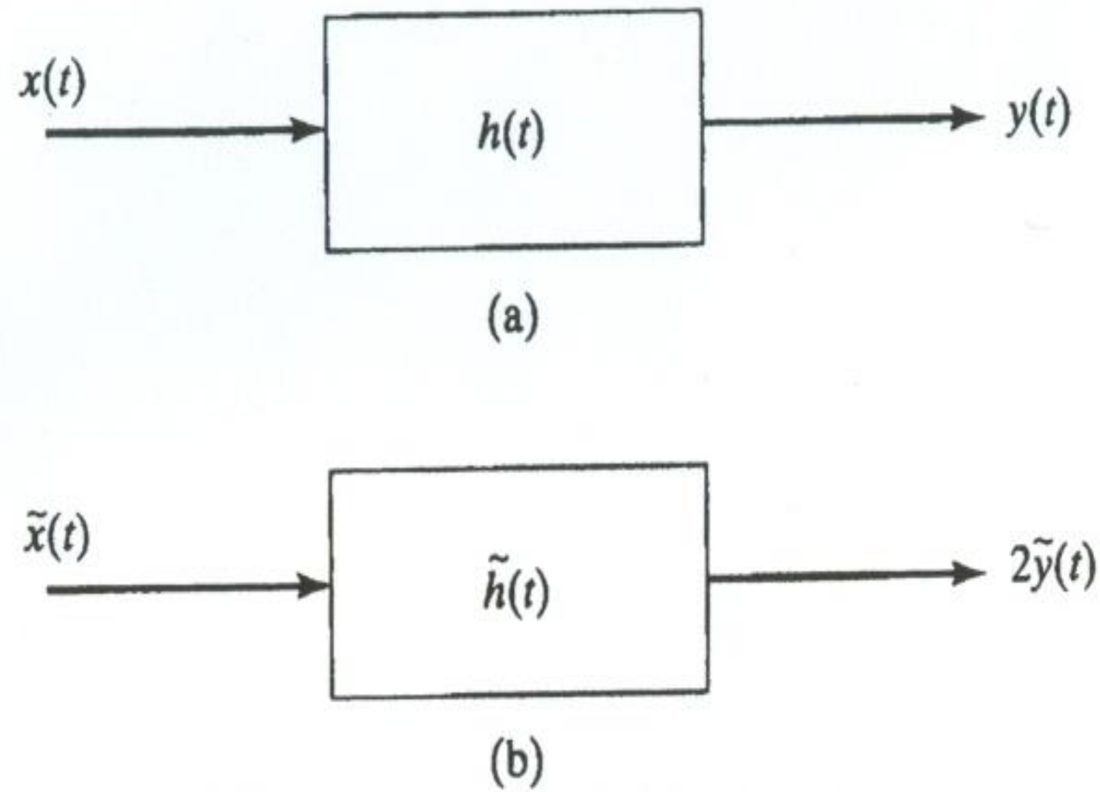
- **Consider a linear band-pass system with impulse response *h(t)* and fed by an input signal *x(t)* to produce an output signal *y(t)*, as illustrated in Fig.5.6.**
- **Two assumptions are made :**
  1. The system is narrowband
  2. The input signal is a modulated signal whose carrier frequency , f_c , is the same as the midband frequency of the system.
- **The analysis of the system can be simplified by the equivalent complex baseband model shown in Fig.5.6 (b).**

The impulse response of the equivalent may be expressed as

$$\tilde{h}(t) = h_I(t) + j\, h_Q(t) \qquad (5.8)$$

where $h_I(t)$ is the in-phase component and $h_Q(t)$ is the quadrature component.

Fig.5.6 (a) Block diagram of linear bandpass system
(b) Equivalent complex baseband model



- **The impulse response of the original band-pass system, *h(t)* , is related to the complex impulse , $\tilde{h}(t)$ as**

$$h(t) = \text{Re}\ \{\ \tilde{h}(t)\ \exp\left(j2\pi f_c t\right)\}\qquad (5.9)$$

$$x(t) = \text{Re}\ [\tilde{x}(t)\ \exp(j2\pi f_c t)]$$

$$y(t) = \text{Re}\ [\tilde{y}(t)\ \exp(j2\pi f_c t)]$$

- ***x(t)* and *y(t)* are related by the complex convolutional integral**

$$\tilde{y}(t) = (\tfrac{1}{2}) \int_{-\infty}^{\infty} \tilde{x}(\lambda)\ h(t-\lambda)\ d\lambda$$

$$= (\tfrac{1}{2}) \int_{-\infty}^{\infty} h(\lambda)\ \tilde{x}(t-\lambda)\ d\lambda \qquad (5.10)$$

- **Note that the carrier frequency f_c has been eliminated from the equivalent model.**

5.3 Signal Space Representation of Digitally Modulated signals

- **The digitally modulated signal *s(t)* assumes one of a discrete set of possible forms. The corresponding mapping of the complex envelope *s(t)* onto a signal space is referred to as a signal constellation.**
- **We are focusing on two-dimensional signal constellation whose structure naturally depends on the specific form of modulation under study.**
- **The traditional approach is to use energy-normalized versions of the in-phase component $s_I(t)$ and quadrature component $s_Q(t)$ of the modulated signal $s(t)$ as the horizontal axis and vertical axis of the two-dimensional signal space, respectively.**

Note : V.A. Kotel'nikov (1947) was the first to introduce the use of signal space into

communication system characterization in his Ph.D. dissertation

- **The normalized coordinates of unit energy are denoted by $\psi_1(t)$ and $\psi_2(t)$, respectively, defined in turn by**

$$\psi_1(t) = \sqrt{(2/T)}\ \cos(2\pi f_c t) \qquad 0 \leqq t \leqq T \qquad (5.11)$$

and $$\psi_2(t) = \sqrt{(2/T)}\ \sin(2\pi f_c t) \qquad 0 \leqq t \leqq T \qquad (5.12)$$

where T is the symbol duration and the carrier frequency is an integer multiple of $1/T$.

- **$\psi_1(t)$ and $\psi_2(t)$ have the following properties :**
  1. **Orthogonality, i.e.,**

$$\int \psi_1(t)\,\psi_2(t)\,dt = 0 \qquad (5.13)$$

  2. **Unit energy , i.e. ,**

$$\int \psi_1^2(t)\,dt = \int \psi_2^2(t)\,dt = 1 \qquad (5.14)$$

- **Fig. 5.7 shows the signal constellations of BPSK and QPSK.**
- **The signal space characterization of QPSK signal constellation described in Fig.5.7(b) is given in Table 4.2,**

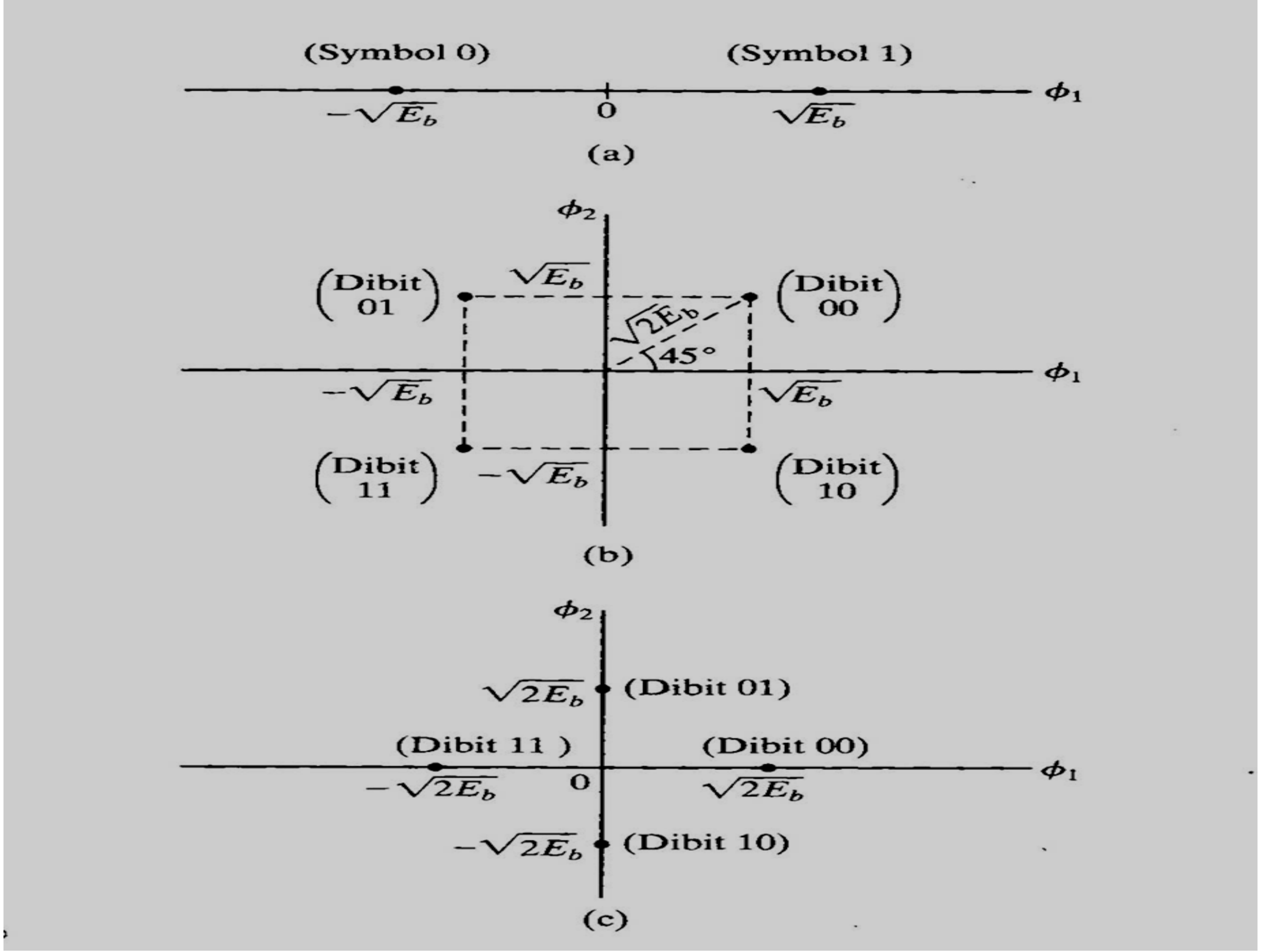
where E_b is the bit energy .

For QPSK signal, , symbol energy $E_s = 2 E_b$, $T = 2 T_b$.

- **In terms of the message points (s_{i1}, s_{i2}) defined in Fig.5.7, we express the QPSK signal as**

$$s_i(t) = s_{i1} \psi_1(t) + s_{i2} \psi_2(t) \quad i = 1,2 \text{ and } 0 \leqq t \leqq T \tag{5.15}$$

Fig.5.7 Signal Constellation for (a) BPSK (b) one version of QPSK (c) another version of QPSK



5.4 Phase-Shift Keying Techniques

- **In PSK, the phase of the carrier is discretely varied with respect to a reference phase and according to the data being transmitted.**
- **The general analytic expression for PSK is**

$$s_i(t) = \sqrt{(2E_s / T_s)}\ \cos\,(2\pi f_c t + 2\pi i /M)$$

$$0 \leqq t \leqq T_s\ ; \quad i = 1,2,\ldots, M \qquad (5.16)$$

where E_s is the symbol energy over the symbol duration T_s

5.4.1 Binary Phase-Shift Keying

- **In binary phase- shift keying (BPSK) , a binary bit stream is used to modulate a carrier's phase.**
- **Let *p(t)* denote the basic pulse used in the construction of a binary stream. Let *T* denote the bit duration (i.e., the duration of binary symbol 1 or 0). Then, the binary data stream is described by**

$$m(t) = \Sigma\, b_k\, p(t-kT) \qquad (5.17$$

where $b_k = \begin{cases} 1 & \textit{for binary symbol 1} \\ -1 & \textit{for binary symbol 0} \end{cases} \qquad (5.18)$

Example : For rectangular pulse

$$p(t) = \begin{cases} 1 & \text{for } 0 \leqq t \leqq T \\ 0 & \text{otherwise} \end{cases} \qquad (5.19)$$

- **For BPSK, the simplest form of digital phase modulation, the binary symbol 1 is represented by setting the carrier phase $\theta(t) = 0$ radian , and the binary symbol 0 is represented by setting $\theta(t) = \pi$ radian. A pair of such kind of signals are referred to as antipodal signals.**

Thus , the modulated signal is given by

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary symbol 1} \\ A_c \cos(2\pi f_c t + \pi) & \text{for binary symbol 0} \end{cases} \quad (5.20)$$

$$= \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary symbol 1} \\ -A_c \cos(2\pi f_c t) & \text{for binary symbol 0} \end{cases} \quad (5.21)$$

- **In general , we may express the BPSK signal in the compact from**

  $$s(t) = c(t)\, m(t) \qquad (5.22)$$

  As with DSB-SC modulation , BPSK (and other kinds of phase modulation) requires a transmission bandwidth twice the message bandwidth.

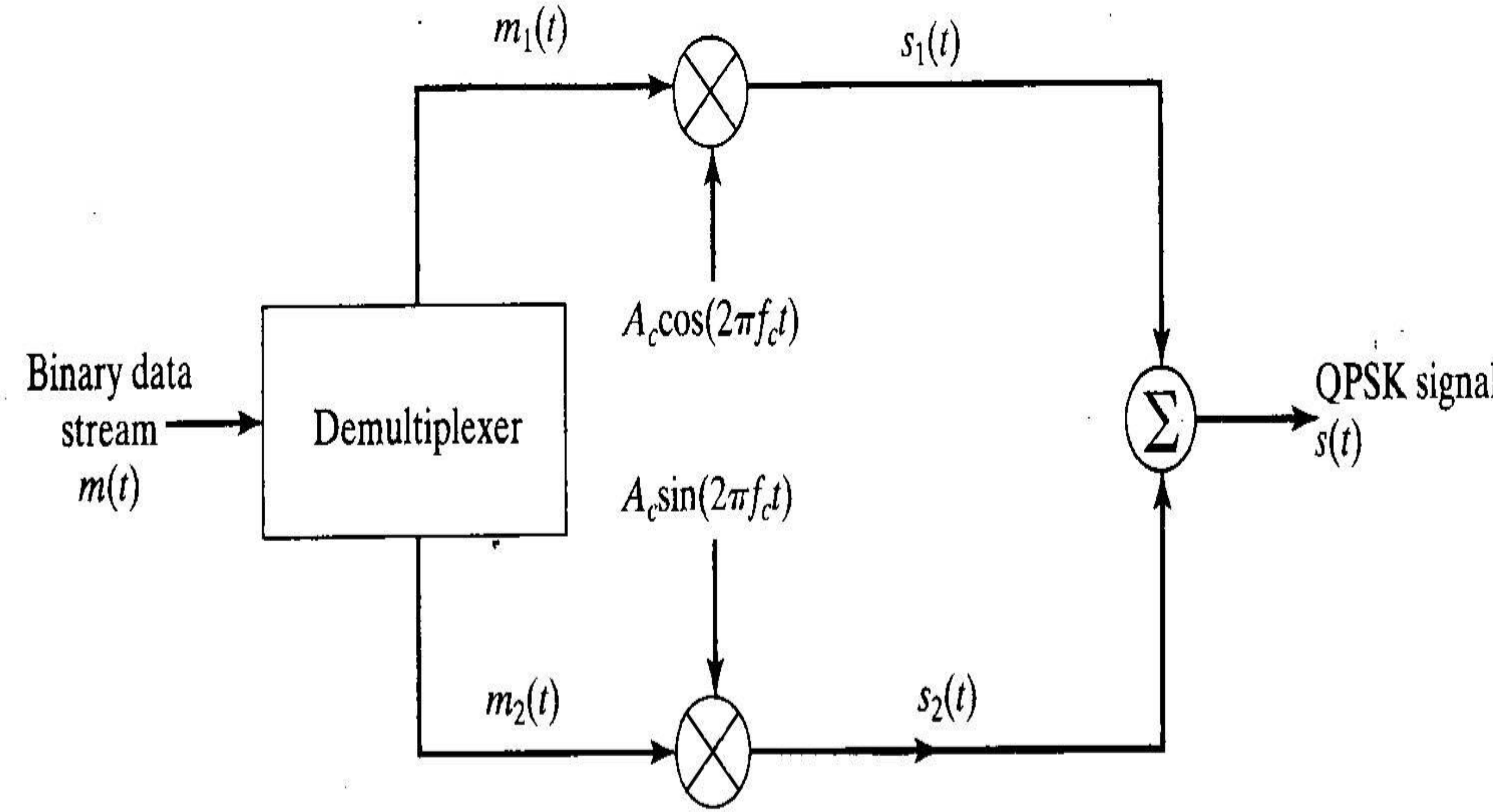
5.4.2 Quadriphase –Shift Keying (QPSK)

- **In the QPSK generator, the incoming binary data stream is first demultiplexed into two substreams , $m_1(t)$ and $m_2(t)$, as shown in Fig. 5.8.**
- **Then the phase of the carrier in QPSK assumes one of four equally-spaced values, depending on the composition of each dibit (group of two adjacent bits).**

For example, we may use 0, $\pi/2$, π, and $3\pi/2$ radians as the set of four vales available for phase-shift keying the carrier.

Specifically , the values 0 and π radians are used to phase-key one of the two substreams, $m_1(t)$, and the remaining values, $\pi/2$ and $3\pi/2$ radians , are used to phase-key the other substream, $m_2(t)$.

Fig.5.8 Block diagram of a QPSK generator



- **Then the BPSK signal produced in the upper path of Fig.4.13 is described by**

$$s_1(t) = A_c m_1(t) \cos(2\pi f_c t) \qquad (5.23)$$

The BPSK signal produced in the lower path is described by

$$s_2(t) = A_c m_2(t) \sin(2\pi f_c t) \qquad (5.24)$$

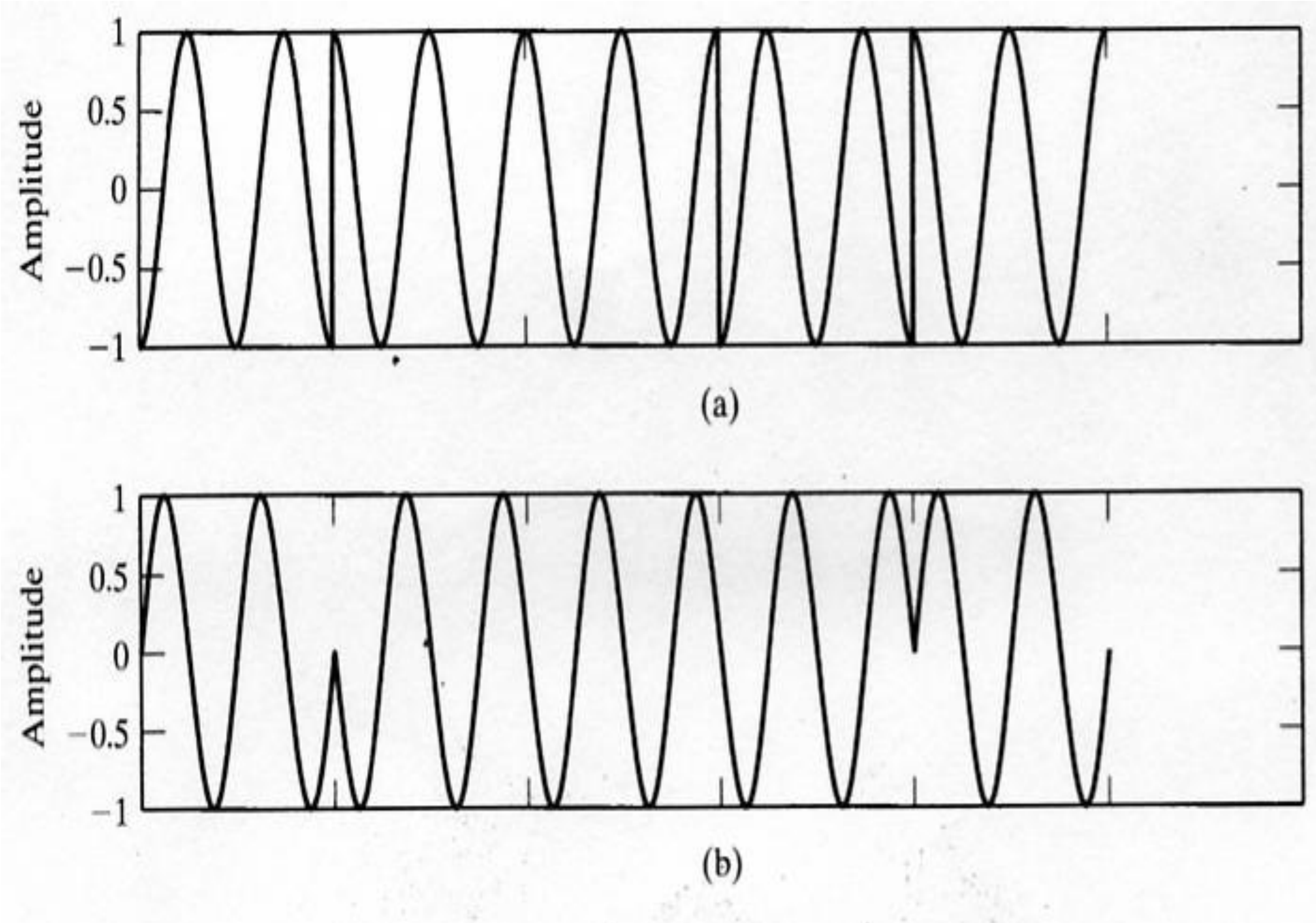
The QPSK signal is obtained by adding these two BPSK signals

$$s(t) = s_1(t) + s_2(t)$$
$$= A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t) \qquad (5.25)$$

- **Since both BPSK signals $s_1(t)$ and $s_2(t)$ are linear, the QPSK signal $s(t)$ is likewise linear .**

- **Some important properties of QPSK signal are described below.**
  - **a. The symbol duration of both $m_1(t)$ and $m_2(t)$ is twice the duration of the original binary data stream.**
  - **b. The bandwidth of both $m_1(t)$ and $m_2(t)$ is one-half that of $m(t)$.**
  - **c. The BPSK signals s_1 (t) and s_2 (t) have a common transmission bandwidth equal to twice that of $m_1(t)$ or $m_2(t)$. The QPSK signal $s(t)$ has the same transmission bandwidth as s_1 (t) or s_2 (t) .**
    Hence the transmission bandwidth of the QPSK signal is the same as that of the original binary data stream.
  - **d. The carrier amplitude is maintained constant.**
  - **e. The carrier phase undergoes jumps of 0^o, $\pm 90^o$, or $\pm 180^o$ every $2T$ seconds, where T is the bit duration.**
- **Fig.5.9 (a) shows the waveform of a typical QPSK signal .**

Fig.5.9 Waveforms of (a) conventional QPSK (b) offset QPSK



5.4.3 Offset QPSK

- **The amplitude of a QPSK signal is ideally constant. However, the carrier phase of a QPSK signal may jump by $\pm 90^{o}$ or $\pm 180^{o}$, every two bit durations. The occasional phase shift of 180^{o} can cause the signal envelope to pass through zero momentarily . This can have an undesirable effect in terms of envelope deviation if the modulated signal is filtered, which is invariably the case in a practical system.**
- **To avoid the possibility of 180^{o} phase switching , the $m_1(t)$ and $m_2(t)$ are offset in their alignment by one bit period (i.e. half symbol period). The resulting modulated signal is called offset quadriphase-shift keying (OQPSK) signal .**
- **The phase transitions likely occur in OQPSK signal are confined to 0^{o}, 90^{o} . The amplitude fluctuation in OQPSK due to filtering have a smaller amplitude than in conventional QPSK.**

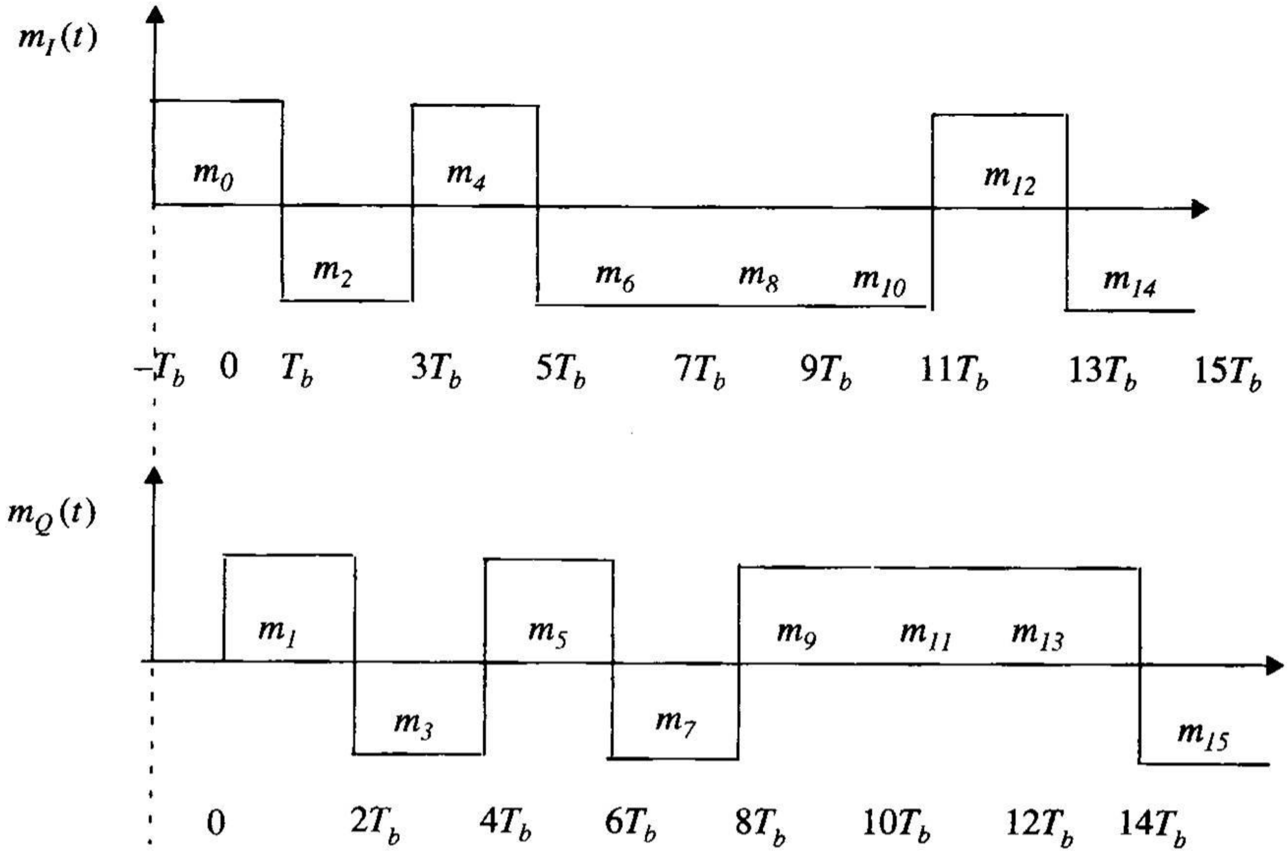
- **Ordinarily, the carrier phase of a conventional QPSK signal may reside in one of two possible discrete settings :**

  1. 0, $\pi/2$, π or $3\pi/2$ radians.

  2. $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$ radians.

  These two phase settings are shifted by $\pi/4$ radians to each other.
- **The waveforms of QPSK, OQPSK are shown in Fig.5.9.**
- **Fig.5.10 illustrates the time offset waveforms that are applied to the in-phase and quadrature phase arms of an offset QPSK modulator.**

Fig.5.10 **time offset waveforms that are applied to the in-phase and quadrature phase arms of an offset QPSK modulator.**



5.4.5 M-ary Phase Shift Keying (MPSK)

- **In *M*-ary phase-shift keying , M pulses are used to represent the message bits. The k-th pulse is expressed by**

$$s_k(t) = \sqrt{(2E_s / T_s)} \cos(2\pi f_c t + 2\pi k/M)$$

$$0 \leqq t \leqq T_b \ ; \quad k = 1, 2, \ldots, M$$

where E_s is the symbol energy over the symbol duration T_s.

The phases of successive pulses are $2\pi/M$ apart.

5.5 Frequency-Shift Keying Techniques

- **The canonical representation of a bandpass signal given in Eq.(4.21) may be expressed in the polar form**

$$s(t) = a(t)\cos(2\pi f_c t + \theta(t)) \qquad (5.26)$$

where $a(t) = \sqrt{s_I^2(t) + s_Q^2(t)} \qquad (5.27)$

is called the envelope of $s(t)$ **and**

$$\theta(t) = \tan^{-1}\left(s_Q(t) / s_I(t)\right) \qquad (5.28)$$

is called the phase of $s(t)$ **.**

5.5.1 Binary Frequency-Shift Keying

- **In binary frequency-shift keying (BFSK) , the symbols 1 and 0 are distinguished from each other by the transmission of one of two sinusoidal waves that differ in frequency by a fixed amount.**
- **A typical pair of sinusoidal waves for BFSK is described by**

$$s_i(t) = \begin{cases} \sqrt{(2E_b/T)}\cos(2\pi f_i t) & 0 \leqq t \leqq T_b \\ 0 & \text{otherwise} \end{cases} \qquad (5.29)$$

where $i = 1,2$. T is the symbol (bit) duration and E_b is the energy transmitted per bit; the frequency transmitted is

$$f_i = (n_c + i)/T \quad \text{for some fixed integer } n_c \text{ and } i = 1,2 \qquad (5.30)$$

The symbol 1 is represented by $s_1(t)$ and the symbol 0 is represented by $s_2(t)$.

- **The BFSK signal described here is a continuous- phase frequency shift keying (CPFSK).**
- **The attribute of continuous phase is desirable for signals that are to be transmitted over a bandlimited channel , as discontinuities in a signal introduces wideband frequency components.**
- **In addition , some classes of amplifiers exhibit nonlinear behavior when driving with nearly-discontinuous signals : this could have undesirable effects on the shape of transmitted signal.**

- **A useful way of representing the CPFSK signal is to express it in the conventional form of an angle-modulated signal as**

  $$s(t) = \sqrt{(2E_b/T)}\cos(2\pi f_c t + \theta(t)) \qquad (5.31)$$

  where $\theta(t) = \theta(0) + \pi h\, t / T$

  $$h = T(f_1 - f_2) \qquad (5.32)$$

  and f_1 **is the frequency of** $s_1(t)$ **and** f_2 **is the frequency of** $s_2(t)$**.**

  The dimensionless parameter h **is referred to as the deviation ratio of the CPFSK signal.**

5.5.2 Minimum Phase Shift Keying (MSK)

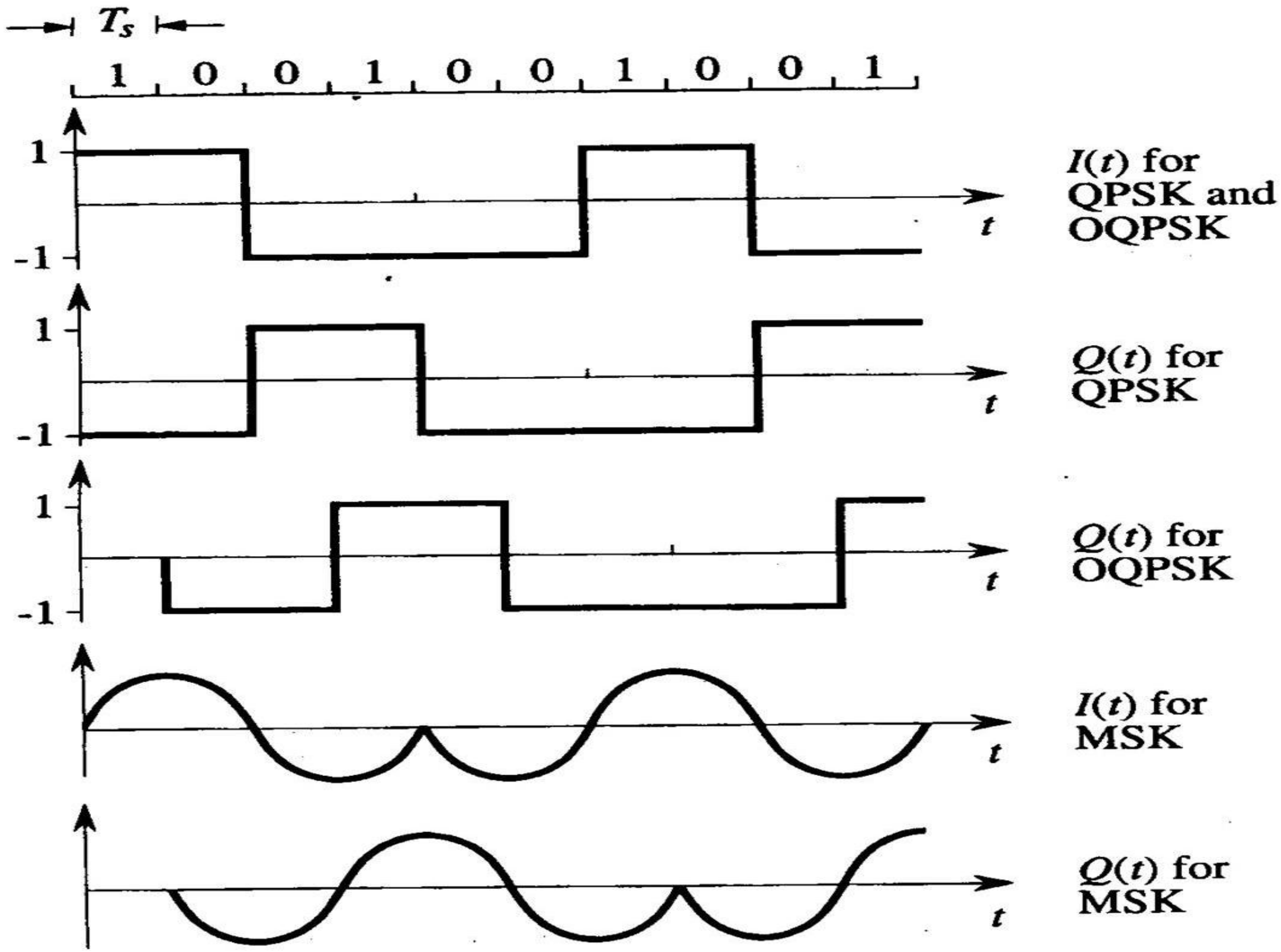
- **With h = ½ , we find from Eq.(4.49-1) that the frequency deviation (i.e. $f_1 - f_2$) equals to half the bit rate. This is the minimum frequency spacing that allows the two FSK signals representing symbols 1 and 0 , as in Eq.(4.49).**

  Thus, a CPFSK signal with a deviation ratio of ½ is commonly referred to as minimum shift keying (MSK)

- **An MSK signal can also thought as a special form of OQPSK where the baseband rectangular pulses are replaced with half-sinusoidal pulses , as shown in Fig.5.11.**
- **The baseband power spectral density of the MSK signal is given by**

$$S_B(f) = (32\, E_b / \pi^2)\, [\, \cos(2\pi Tf) / (16T^2f^2 - 1)\,]^2 \qquad (5.33)$$

Fig.5.11 In-phase and quadrature components of QPSK,OQPSK , and MSK signals



- **The MSK signal has some desirable features :**

  1. a constant envelope

  2. a relatively narrow bandwidth

  3. a coherent detection performance equivalent to that of QPSK.

- **However, the out-of-band spectral characteristics of MSK still do not satisfy the stringent requirement of wireless communications.**

5.5.3 Gaussian Minimum Shift Keying (GMSK)

- **GMSK can be viewed as a derivative of MSK.**
  In GMSK , the sidelobe level of the spectrum are further reduced by passing the modulating NRZ data waveform through a premodulation Gaussian pulse-shaping filter.
- **The transfer function *H*(*f*) and its impulse response h(t) of the shaping filter are given, respectively , by**

$$H(f) = exp\,(-\alpha^2 f^2)\,, \qquad (5.34)$$

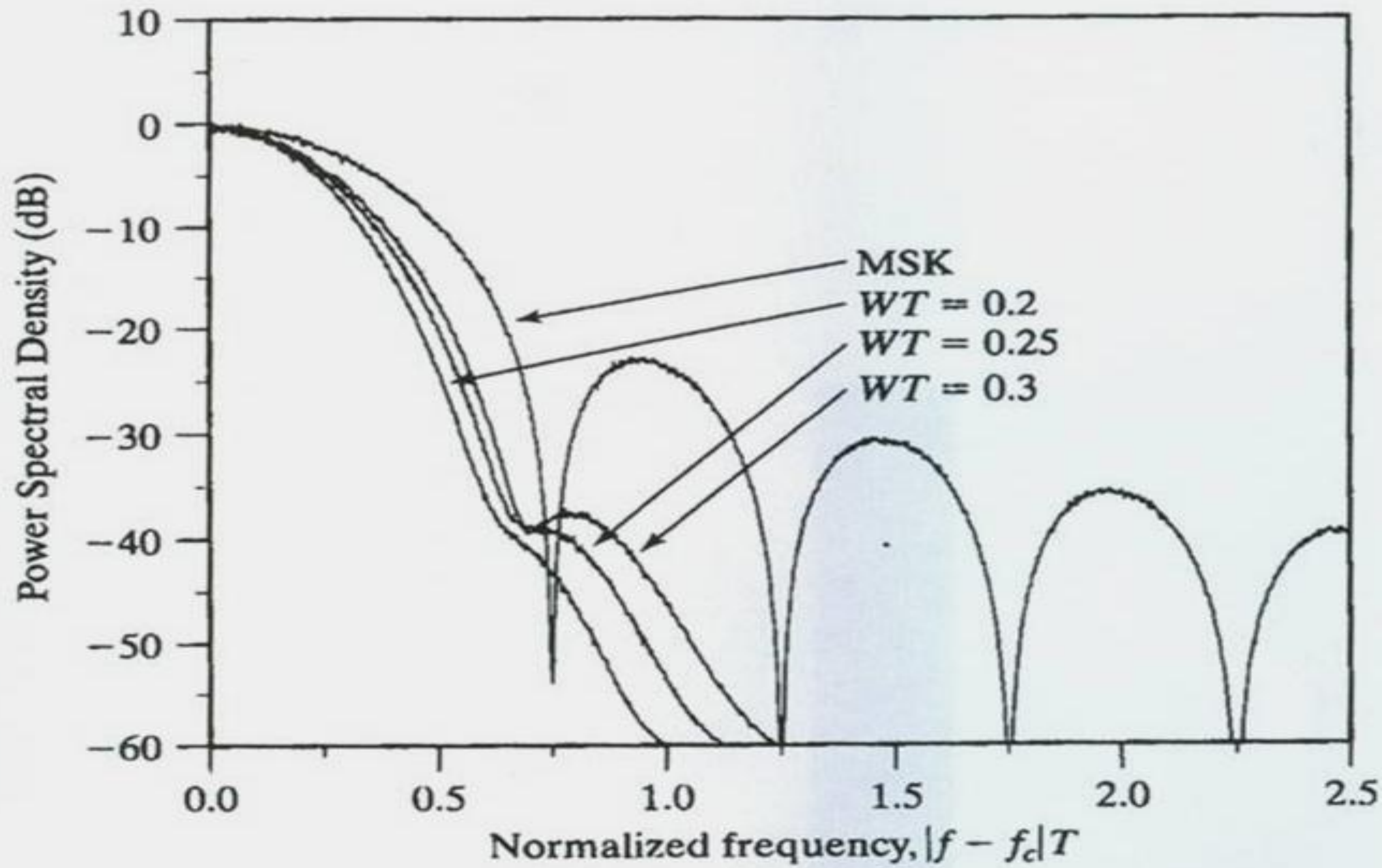
and $$h(t) = (\sqrt{\pi})/2\ \ exp\,[-(\pi/\alpha)^2 t^2] \qquad (5.35)$$

where $\alpha = 0.5887 / W$, W is the 3-dB bandwidth of the shaping filter *H*(*f*) and *T* is the symbol (bit) duration.

- **Fig.5.12 shows the power spectra of MSK and GMSK.**

- **As shown in Fig. 5.12, the choice of the time-bandwidth product *WT* offers a trade-off between spectral compactness and a reduction in out-of-band power spectrum.**
- **When *WT =0.3* , the sidelobes of the spectrum of the GMSK signal drop by an amount larger than 40 dB relative to the midband frequency , which means that the effect of adjacent channel interference is practically negligible.**
- **GMSK modulation scheme has been adopted in the second generation European mobile system GSM.**

Fig.5.12 Power spectra of MSK and GMSK signals



5.6 Quadrature Amplitude Modulation

- **In M-ary PSK modulation , the amplitude of the modulated signal was constrained to remain constant.**
- **By allowing the amplitude to also vary with phase , a new modulation scheme called quadrature amplitude modulation (QAM) is obtained.**
- **Fig.4.18 shows the signal constellation of another commonly used linear form of 16-quadrature amplitude modulation (16-QAM). The modulated signal is a hybrid of amplitude and phase modulations. Each message point of the constellation corresponds to a specific quadbit**
- **Gray Coding :**

  For signal constellation, such as M-ary PSK and QAM ,we can map the binary data symbols in such a way that the nearest neighboring symbols (in Euclidean distance) differ in only one bit position. Such a mapping is called " Gray coding".

The general for of an *M*-ary QAM signal can be defined as

$$s_i(t) = \sqrt{(2E_0/T_s)}\,[\,a_i \cos(2\pi f_c t) + b_i \sin(2\pi f_c t)\,]$$

$$0 \leqq t \leqq T,\ i = 1,2,\ldots,M$$

(5.36)

where E_0 is the energy of the signal with the lowest amplitude .

Note that *M*-ary QAM does not have constant energy per symbol.

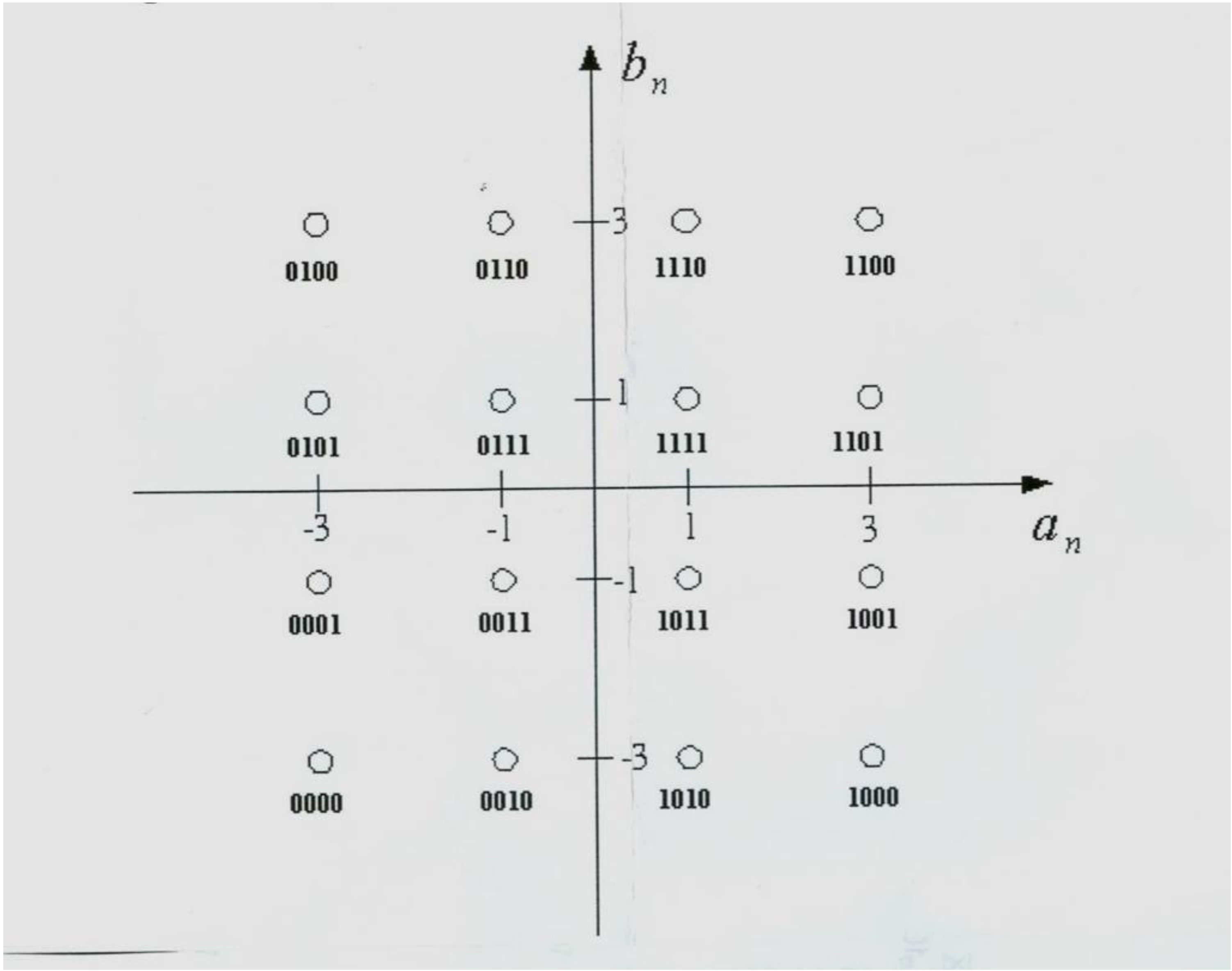
$a_i, b_i = a, 3a, \ldots, (\log_2 M - 1)$ **, where *M* is a power of 4 .**

- **The average energy of M-QAM signal is**

  $E_{av} = 2(M-1)\,E_0/3$

- **In both Fig.5.6 and Fig.5.13, the message points are identified with gray-encoded dibits and quadbits , respectively.**

- **Fig.5.13 16-QAM with Gray Coding**



5.7 Bandwidth Efficiency

- **In wireless communications, channel bandwidth and transmitted power constitute two primary " communication resources".**
- **The primary objective of spectrally efficient modulation is to maximize the bandwidth efficiency defined as the ratio of the data rate in bits per second to the effectively utilized channel bandwidth at a minimum practical expenditure of average signal-to-noise ratio.**

$$\text{Bandwidth Efficiency} = R_b / B \quad bits/s/Hz$$

- **For rectangular data pulse and null-to-null bandwidth , the bandwidth efficiencies for various modulation schemes are shown in Table 5.1.**
- **The efficiency will be limited by Shannon's theorem , i.e.,**

$$(R_b / B)_{max} = log_2 (1+ S/ N)$$

as shown in Fig.5.14.

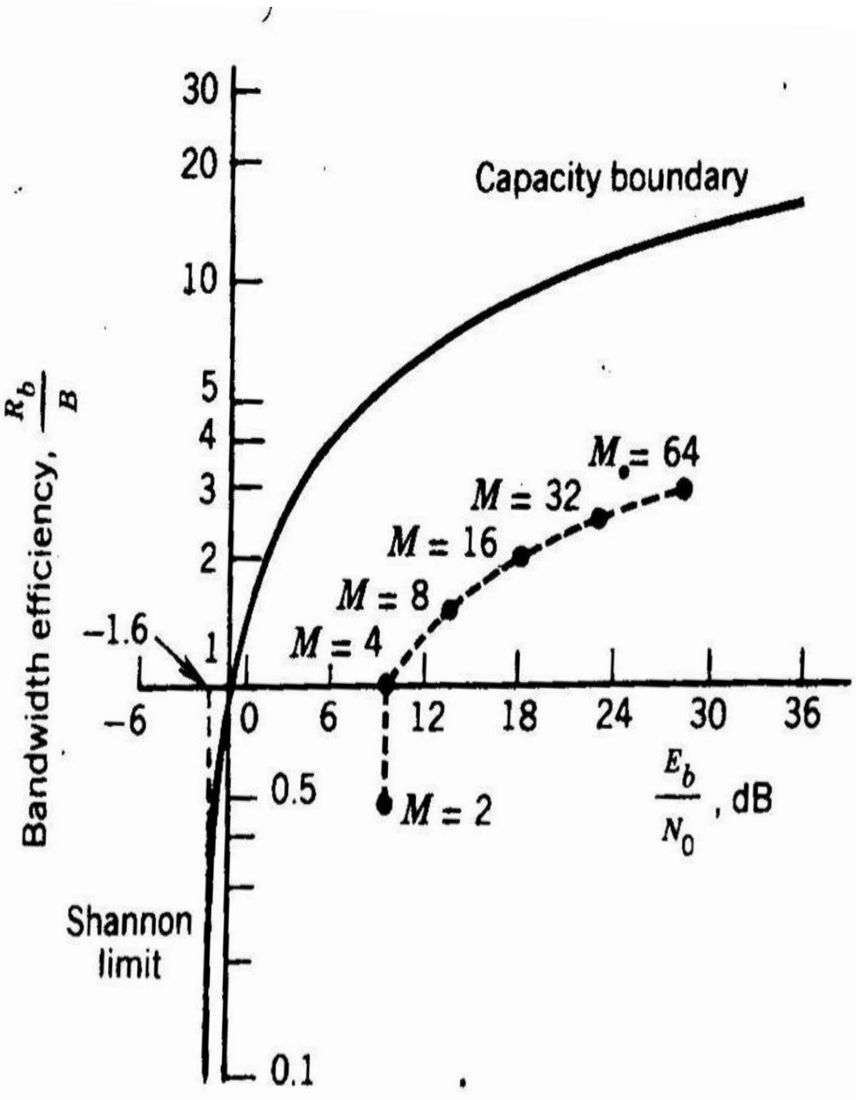
Table 5.1

Bandwidth efficiencies for various modulation schemes

With rectangular data pulse and null-to-null bandwidth

Modulation Scheme	Bandwidth Efficiency (bps/Hz)
M-ary PSK, DPSK, QAM	$0.5 \log_2 M$
MFSK; coherent	$\dfrac{2 \log_2 M}{M + 3}$
MFSK; noncoherent	$\dfrac{\log_2 M}{2M}$

Fig.5.14 Bandwidth efficiency : MPSK vs. Ideal System



(a)

- **If Nyquist (ideal rectangular) filtering is applied at baseband so that , for MPSK , the require bandwidth at an IF (or RF) is related to the symbol rate by**

  $$B = 1/T_s \;,$$

  where T_s is the symbol duration .

  Thus, the bandwidth efficiency of MPSK modulated signals using Nyquist filtering can be expressed as

  $$R_b / B = \log_2 M \quad \text{bits/s/ Hz} \qquad (5.37)$$

- **For QAM-type and MPSK signals , with raised cosine filtering , the spectral efficiency is**

  $$R_b / B = [\, 1/ \, (1+r) \,] \; \log_2 M \quad \text{bits/s/ Hz} \qquad (5.38)$$

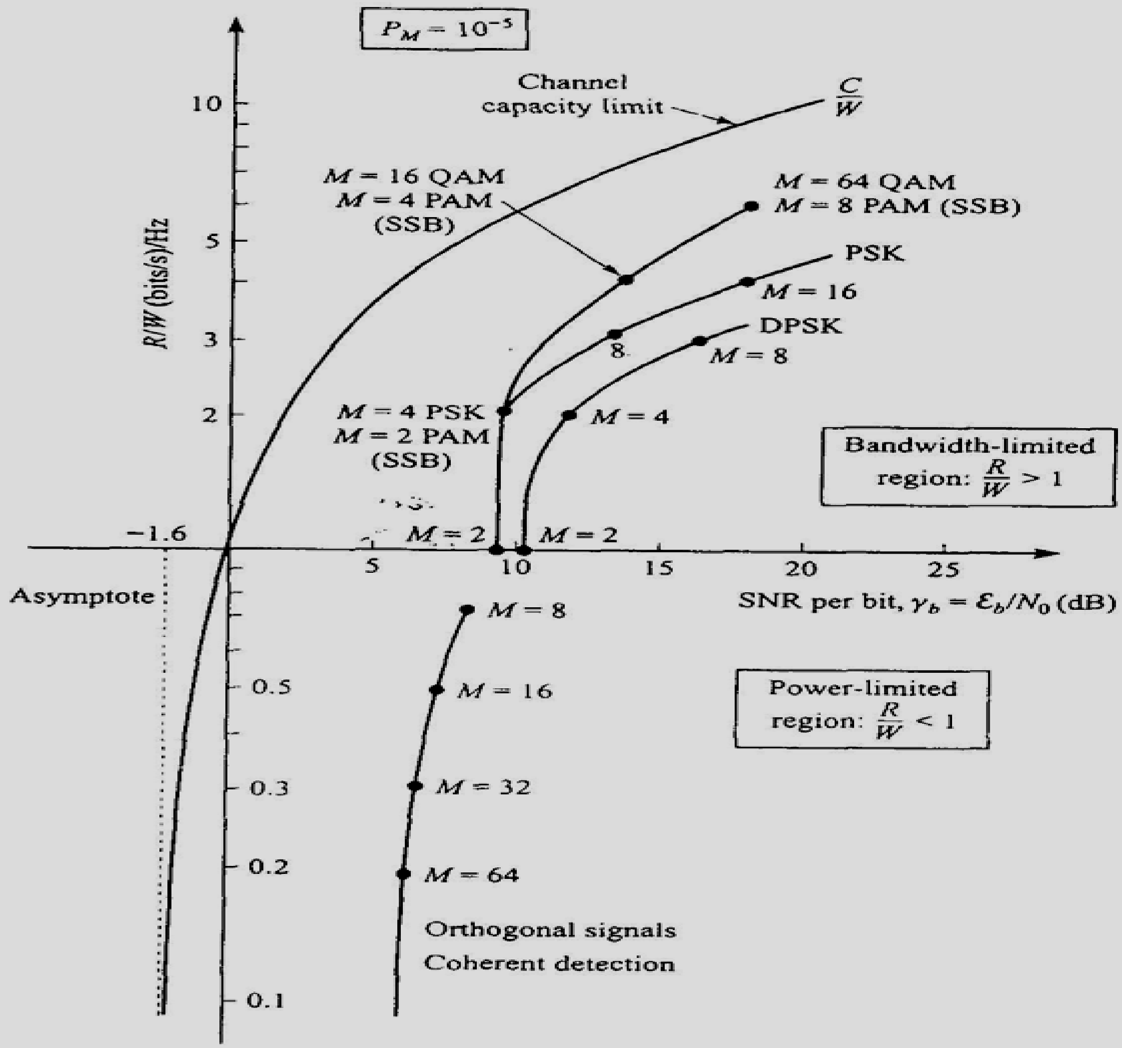
  where r is the roll-off factor of the raised-cosine filter.

  For $r = 1$,

  $$R_b / B = (\, 1/2 \,) \; \log_2 M \quad \text{bits/s/ Hz}$$

  which is the same as the case of rectangular data pulses.

Using Nyquist filtering



IGURE 4.6–1
omparison of several modulation schemes at $P_e = 10^{-5}$ symbol error probability.

- **Power-Limited Modulation System**

For the case of power-limited systems , power is scarce but system bandwidth is available , e.g., a space communication link . MFSK is a natural choice for this kind of characteristics.

For MFSK , the IF minimum bandwidth , assuming minimum tone spacing , is given by

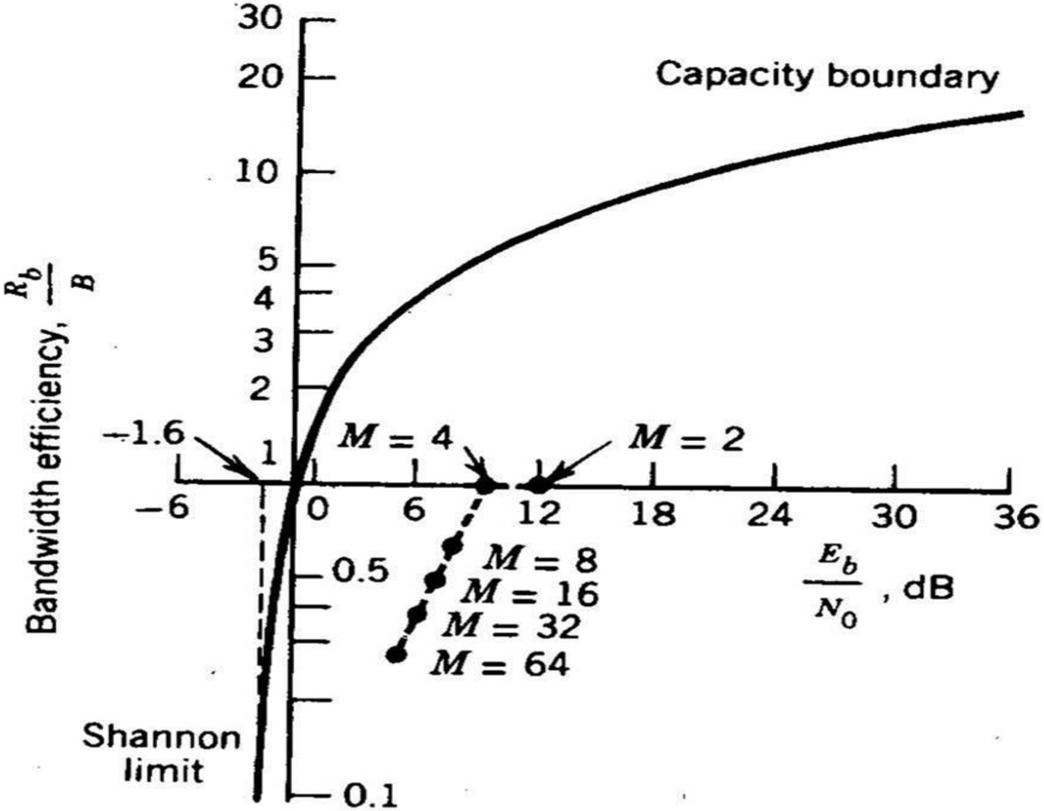
$$W = M / T_s = M R_s$$

where T_s is the symbol duration.

The spectral efficiency of the MFSK signal ,as shown in Fig.5.15, can be expressed as

$$R_b / W = (\log_2 M) / M \quad bits/s/Hz \qquad (5.39)$$

Fig.5.15 Bandwidth efficiency :MFSK vs. Ideal System



5.8 Performance of Optimum Receiver : Bit Error Rate (BER)

5.8.1 Bandpass signals corrupted by AWGN

- **Consider a bandpass signal can be expressed by**

$$s(t) = \alpha(t) \cos(2\pi f_c t + + \psi) \quad , \alpha(t) > 0 \qquad (5.40)$$

which is corrupted by the bandpss noise w(t),

where $w(t) = w_c(\mathrm{t}) \cos(2\pi f_c t) + w_s(\mathrm{t}) \sin(2\pi f_c t)$

(5.41)

Both w_c(t) and w_s(t) are uncorrelated Gaussian random variables with zero and variance $\sigma^2 = 2 N_0 B$.

The pdfs of w_c(t) and w_s(t) are given by

$$p_{wc}(x) = p_{ws}(x) = (1/\sqrt{(4\pi N_0 B)}) \exp(- x^2 / 4N_0 B)$$

(5.42)

- **The corrupted signal at the detector input is given by**

$$x(t) = s(t) + w(t)$$
$$= \alpha(t)\cos(2\pi f_c t + \Theta)$$
$$+ w_c(t)\cos(2\pi f_c t) + w_s(t)\sin(2\pi f_c t)$$
$$= A(t)\cos(2\pi f_c t + \Theta)$$

(5.43)

where $A(t) = \sqrt{[(\alpha+w_c)^2 + w_s^2]}$

$$\Theta(t) = -\tan^{-1}[w_s/(\alpha+w_c)] \qquad \textbf{(5.44)}$$

Since $A^2 = (\alpha + w_c)^2 + w_s^2]$

and then $w_c^2 + w_s^2 = \alpha^2 - 2A\alpha\cos\Theta(t) + \alpha^2 \qquad \textbf{(5.45)}$

We have

$$p_{A\Theta}(A, \Theta) = (A / 2\pi\sigma^2)$$
$$\exp[-(A^2 - 2A\alpha\cos\Theta(t) + \alpha^2)/2\sigma^2] \qquad \textbf{(5.46)}$$

and then $p_A(\alpha) = \int_{-\pi}^{\pi} p_{A\Theta}(A, \Theta)\, d\Theta \qquad \textbf{(5.47)}$

- **It can be shown that**

$$p_A(A) = (A/\sigma^2)\exp[-(A^2+\alpha^2)/2\sigma^2]\, I_0(A\alpha/\sigma^2) \quad (5.48)$$

where I_0 is the modified zero-order Bessel function of the first kind. The above equation is known as the Rician probability density function.

- For large α , α >> σ , $A \doteqdot \alpha$,

we obtain $p_A(A) = \sqrt{(A/2\pi\alpha\sigma^2)}\ exp[-(A-\alpha)^2/2\sigma^2]$

$$\doteqdot (1/\sigma\sqrt{(2\pi)}\ exp[-(A-\alpha)^2/2\sigma^2] \quad (5.49)$$

The pdf is nearly a Gaussian distribution function with mean α and variance σ^2.

- The pdf of the phase function can be obtained by

$$p_\Theta(\Theta) = \int_0^\infty p_{A\Theta}(A,\Theta)\, dE$$

$$= (1/2\pi)\ exp[-\alpha^2/2\sigma^2]$$

$$\{\ 1 + \sqrt{(2\pi)}\ (\alpha/\sigma)\ \cos\Theta\ exp[(\alpha^2\cos^2\Theta)/2\sigma^2]$$

$$[1-Q((\alpha\cos\Theta)/\sigma))]\ \} \quad (5.50)$$

5.8.2 Coherent Detection of MPSK Signals in AWGN

- **Coherent and Noncoherent Receivers :**

  In a coherent receiver , the locally generated carrier is synchronized with the carrier in the transmitter in both phase and frequency.

  In a noncoherent receiver, the phase information of carrier is ignored at the expense of degrade noise performance.

- **Table 5.2 presents a summary of the bit error rates for the following receivers :**

  coherent BPSK, QPSK ,BFSK, MSK and DPSK, noncoherent BFSK

- **Consider the *M*-ary phase shift keying signals transmitted over AWGN channel . The transmitted signal can be expressed by**

$$s(t) = \sqrt{(2E_s/T_s)} \cos(2\pi f_c t + 2\pi k/M), \quad k = 0,1,2,\ldots, M-1 .$$

$$0 \leq t \leq T$$

(3.51))

At the receiver , coherent phase reference is available. The basic function of the receiver is to detect the phase of the received pulse. This can be done by two phase detectors

With cos ($2\pi f_c t$) and sin ($2\pi f_c t$) as references , along with a logic circuit to determine the ratio of the two detected components.

- To compute the error probability p_{eM} , we note that a detection error results if the phase of ant pulse deviates by more than π / M . The pdf $p_\Theta(\theta)$ of the pase is found in Eq. (5.50) .

Hence,

$$p_{eM} = 1 - \int_{-\pi/M}^{\pi/M} p_\Theta(\Theta)\, d\Theta \qquad (5.52)$$

Note that $2 E_s / N_0 = 2 E_b \log_2 M / N_0$

Hence,

$$p_{eM} = 1 - (1/2\pi) \int_{-\pi/M}^{\pi/M} \exp[-E_b \log_2 M / N_0] \{ 1 + \sqrt{(4\pi)} (E_b \log_2 M / N_0) \cos\Theta \exp[(E_b \cos^2\Theta) \log_2 M / N_0] [1 - Q(\cos\Theta \sqrt{(2 E_b \log_2 M / N_0)})] \} d\Theta$$

(5.53)

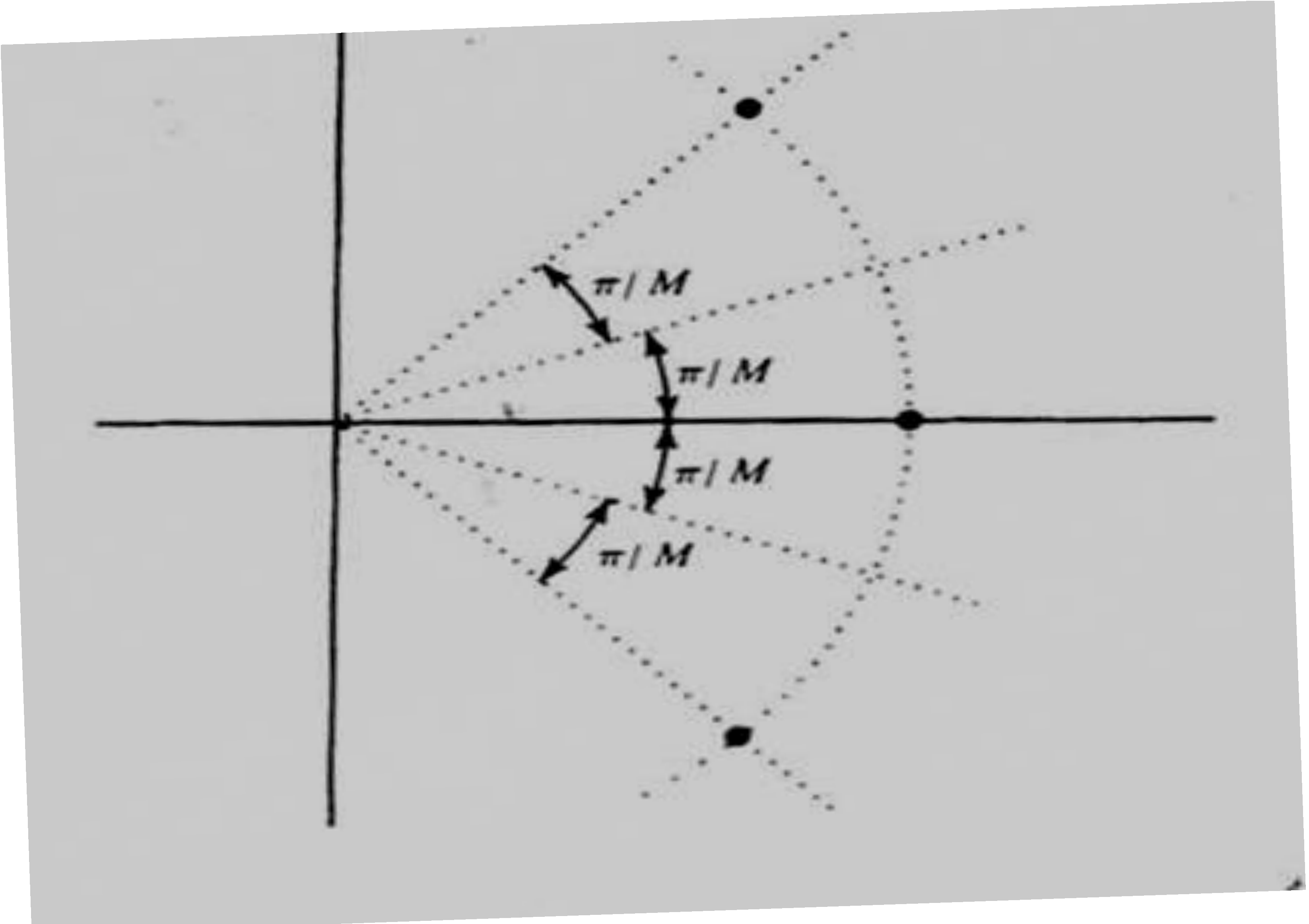
For $E_b / N_0 >> 1$ (weak noise) and M >> 2 ,

Eq. (5.53) can be approximated as

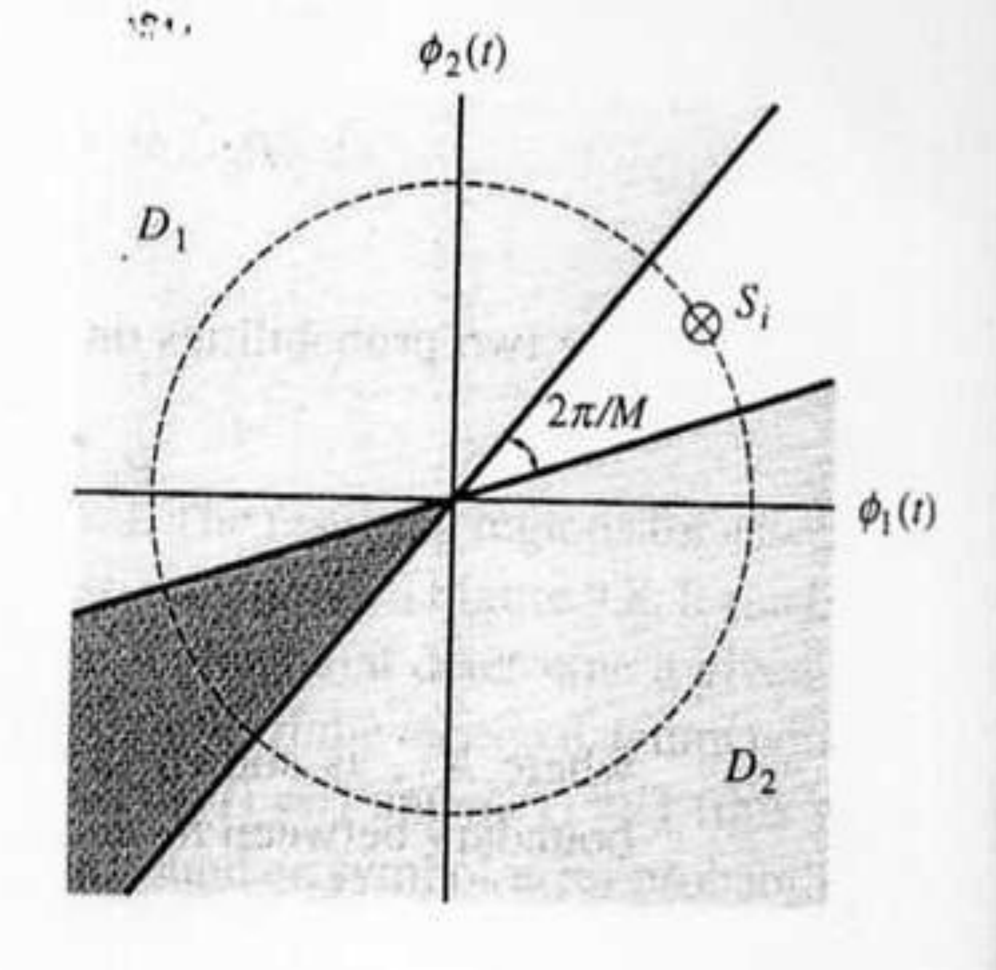
$$p_{eM} \fallingdotseq 2 Q \{ \sqrt{[(2E_b / N_0) (\log_2 M) \sin^2(\pi / M)]} \}$$

(5.54)

(Proakis , p.194)

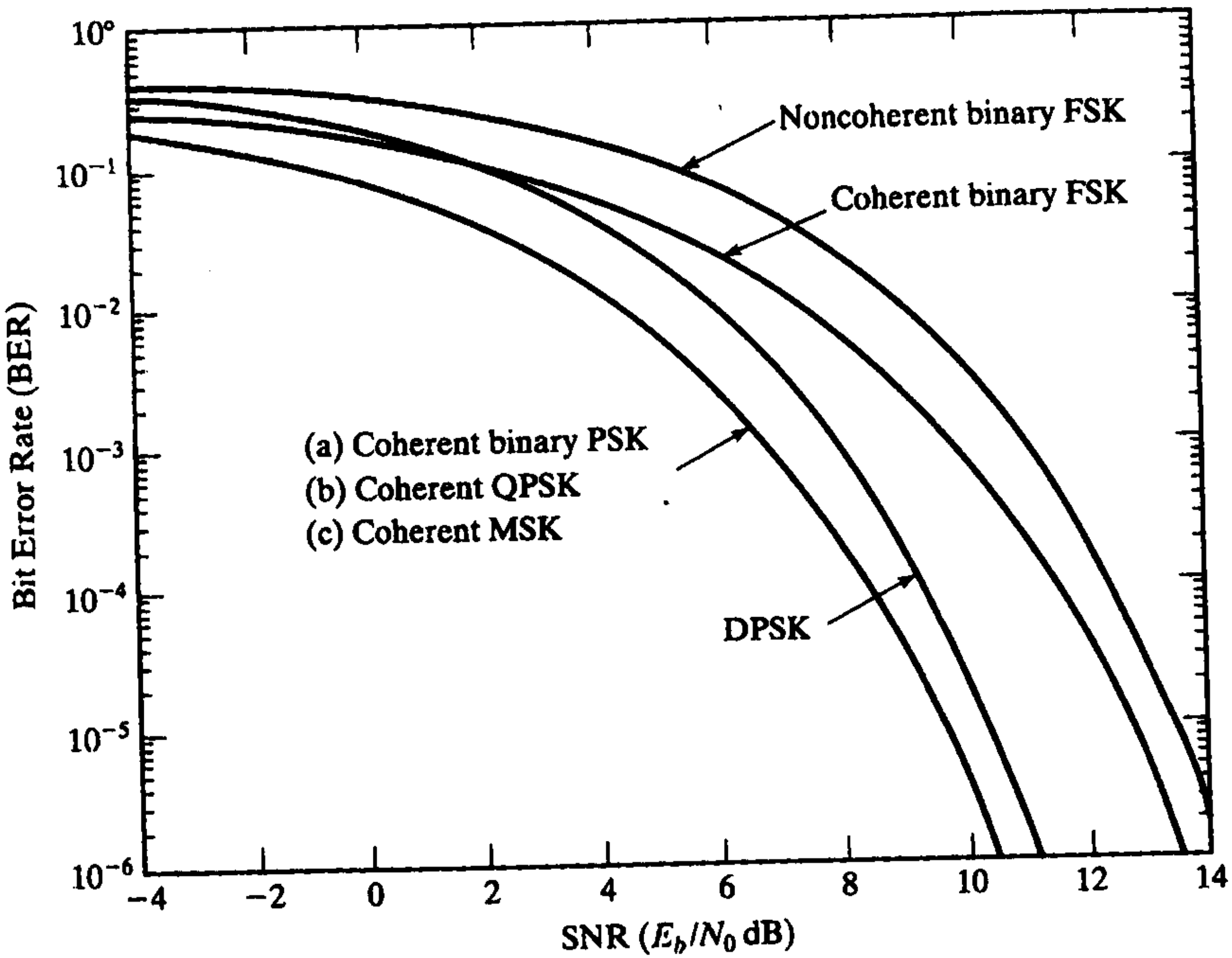


π/ M
π/ M
π/ M
π/ M



(b)

Fig.5.17 BER of different PSK and FSK systems



Note

- **For a bit error probability of 10^{-5} , BPSK modulation requires an E_b / N_0 of 9.6 dB (the optimum uncoded binary information). Therefore, for this case , Shannon's work promised the existence of a theoretical performance improvement of 11.2 dB (9.6 +1.6 = 11.2 dB) over the performance of optimum uncoded binary modulation improvement , through the use of coding techniques.**

- **Performance of *M*-ary QAM**

  For the special class of M-QAM with $M = 2^L$, an *M*-ary QAM signal can be treated as a superposition of two independent

  L-ary PAM signals placed on two orthogonal sinusoidal signals with the same carrier frequency .In these special cases, the signal constellation has a square shape.

  In detection process, the symbol decision will be correct if there is no erroneous decision in either of the two dimensions of the modulation , so we can write

$$p_C = [\, 1 - p_{L\text{-}PAM}(\varepsilon)\,]^2 \qquad (2.55)$$

Therefore

$$\begin{aligned} p_{M\text{-}QAM}(\varepsilon) &= 1 - p_C \\ &= 1 - [\, 1 - p_{L\text{-}PAM}(\varepsilon)\,]^2 \\ &= 2\, p_{L\text{-}PAM}(\varepsilon) - [\, p_{L\text{-}PAM}(\varepsilon)\,]^2 \qquad (2.56) \end{aligned}$$

- **For high value of E_b / N_0 the squared error probability $p_{L\text{-}PAM}(\varepsilon)\,]^2$ is small compared with the first component in Eq. (2.57 . Therefore the second term of (2.57) can be neglected. Consequently , we obtain**

$$p_{M\text{-}QAM}(\varepsilon) \doteqdot 2\,(\,1 - 1/\sqrt{M}\,)\,Q\,[\,\sqrt{(\,3E_{av}/\,(M-1)\,N_0\,)}\,] \quad (2.57)$$

where E_{av} is the average signal energy of M-QAM signal .

$$E_{av} = 2(M-1)\,E_0/3$$

- **The power spectrum and bandwidth efficiency of QAM modulation is identical to M-ary PSK (MPSK) modulation. In terms of power efficiency , QAM is superior to MPSK.**

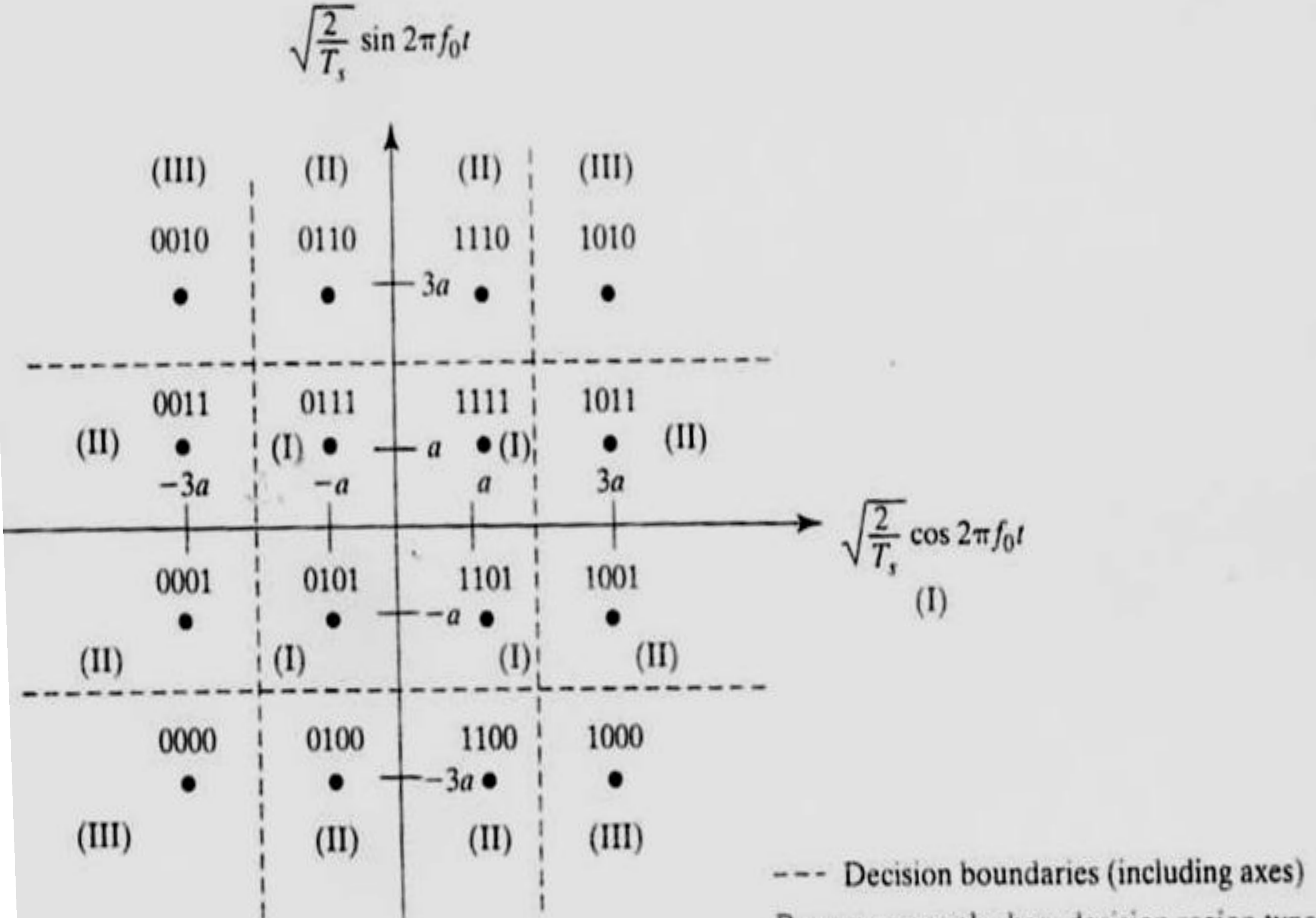
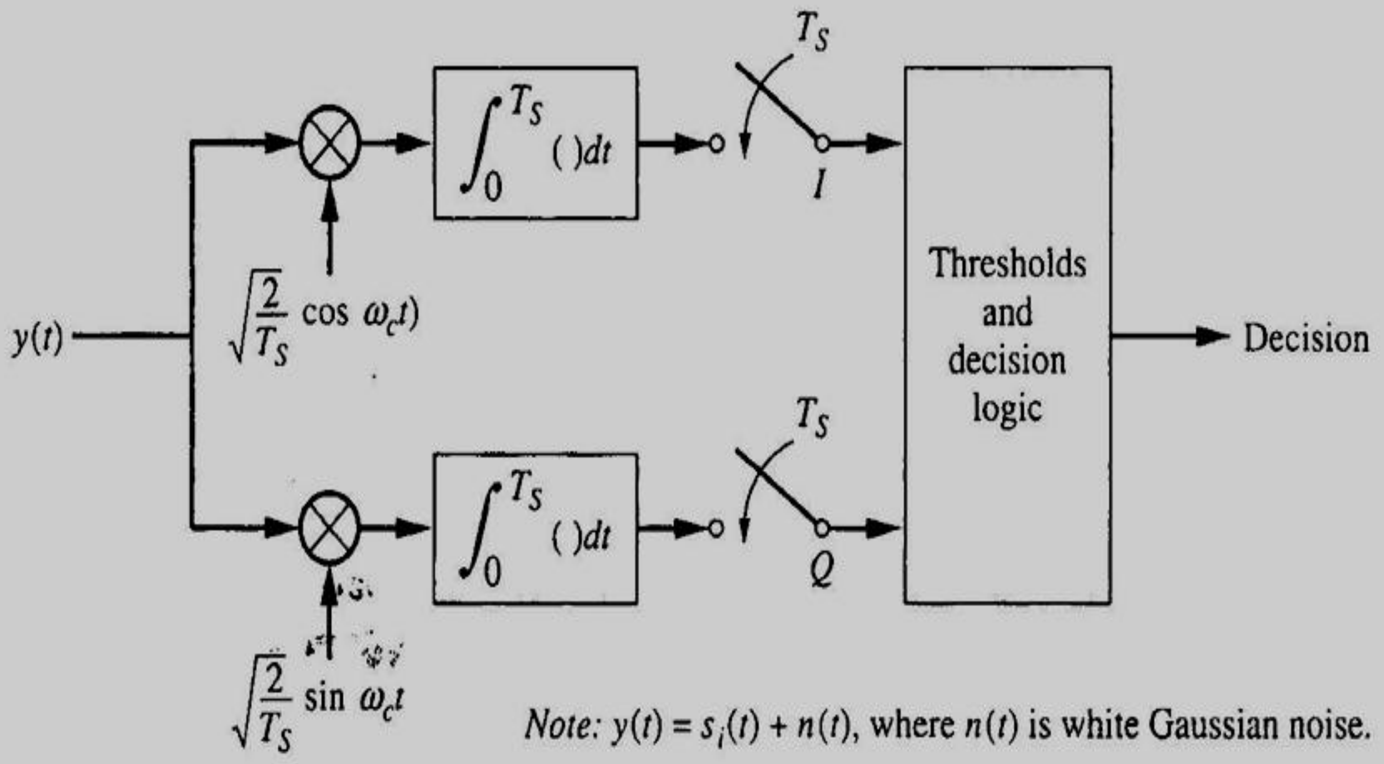


FIGURE 4–10 Signal constellation and decision regions for 16-ary QAM.

Fig.5.18



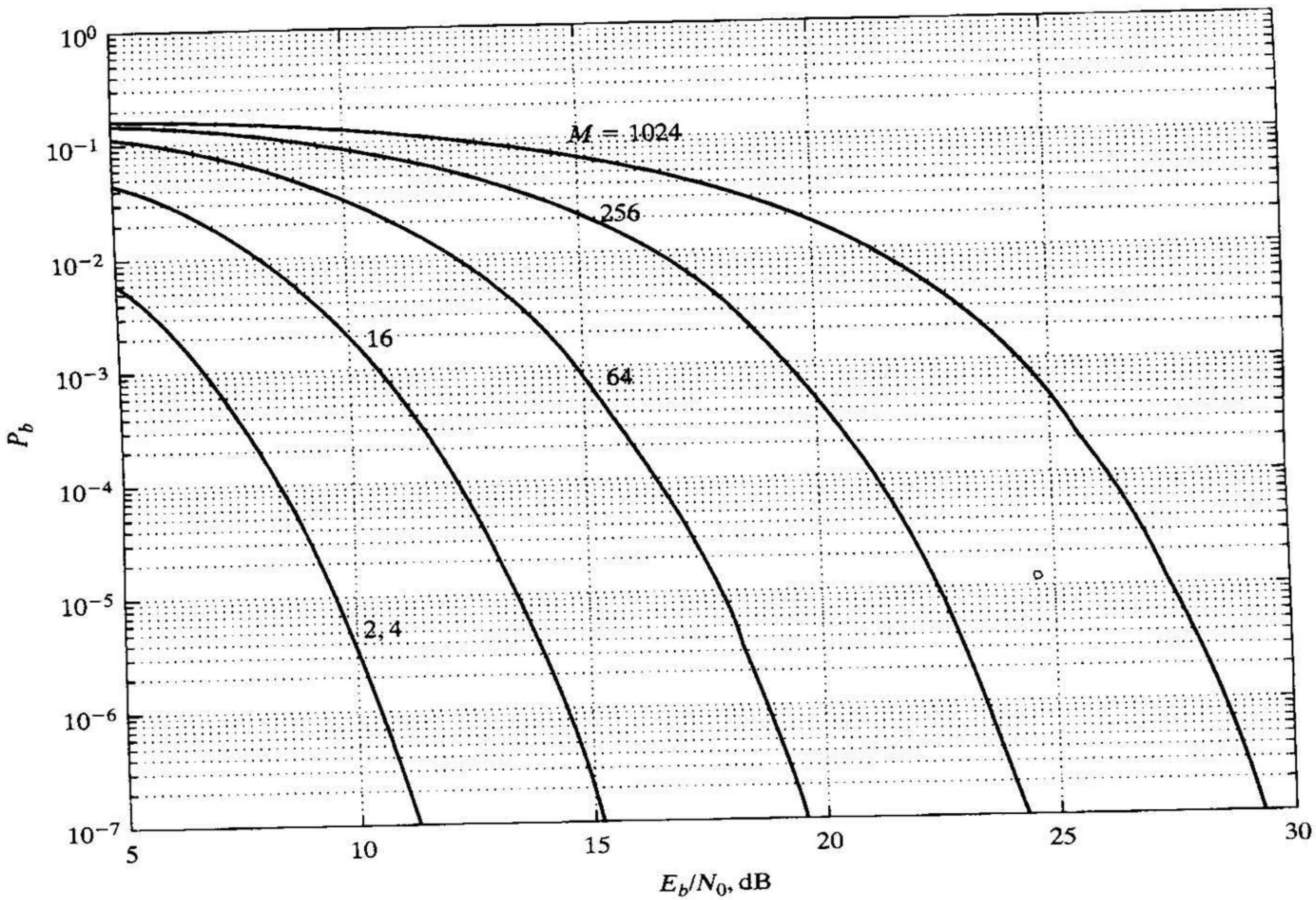
Note: $y(t) = s_i(t) + n(t)$, where $n(t)$ is white Gaussian noise.

(b)

Figure 9.11

Signal space and detector structure for 16-QAM. (a) Signal constellation and decision regions for 16-QAM. (b) Detector structure for *M*-ary QAM. (Binary representations for signal points are Gray encoded.)

Fig.5.19 Bit error probabilities of M- ary QAM in AWGN channel



5.9 Adjacent Channel Interference and Power Amplifier Nonlinearity in FDMA System

5.9.1 Adjacent Channel Interference (ACI)

- **Spectral efficiency of a wireless communication system is defined as the ratio of the permissible source rate (in bits per second) to the channel bandwidth (in Hertz). Thus, spectral efficiency is measured in bits/s/Hz .**
- **In the FDMA system, the spectral efficiency depends on how closely the individual channels (frequency bands) can be spaced. ACI is one of the most important factors that limit this spacing.**

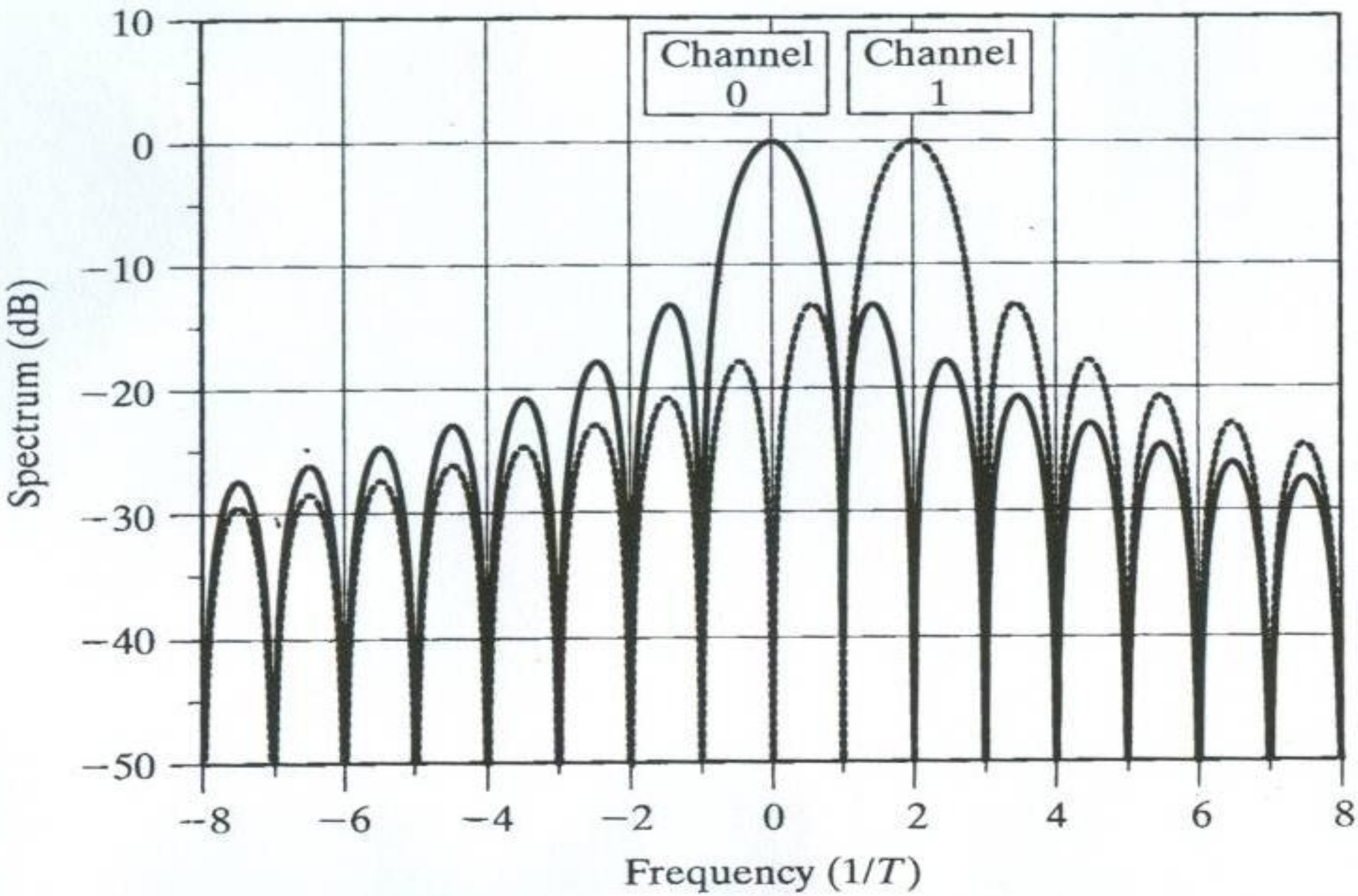
- To illustrate the ACI problem, consider a rectangular pulse p(t) of duration T and whose energy spectral density
  is defined by

$$|P(f)|^2 = |\sin(\pi f t)|^2 / |\pi f t|^2$$

$$= \text{sinc}^2(f t) \qquad (5.58)$$

- Fig.5.20 presents two plots of Eq. (5.58) : one (solid line) for channel 0 and the other (dashed line) for channel 1 .
- Pulse transmission over these two channels is accounted for by the main lobes of the respective spectra. The sidelobes account for ACI problem.
- Fig.5.20 shows that the levels of the first two sidelobes are suppressed by only 13 and 17 dB relative to the main lobes (i.e. desired signals) .

Fig.5.20 Adjacent channel interference



- **Near-far problem :**

In Fig.5.20 , the received signals are all assumed to be at the same power level. In wireless communications , signals may be transmitted at the same [power level. Because of the different propagation distance to the receivers , the transmitted signals are then received at widely differing power levels.

If, for example, the signal on channel 1 is received 30 dB stronger than the signal on channel 0 , it is possible for the ACI produced by channel 1 to completely overwhelm the desirable signal in channel 0. This phenomenon is called the near-far problem.

5.9.2 Power Amplifier Nonlinearity

- **Mobile radio terminals are designed for a certain battery life or time between recharges.**
- **A significant power consumption in mobile radios is the transmit power amplifier.**

  The power amplifiers in the transmitter have to have high efficiency. Such amplifiers, specially, Class-C amplifiers are highly nonlinear.
- **An ideal amplifier nonlinearity has the AM-to-AM characteristics illustrated in Fig.5.21.**

  That is, the amplifier acts linearly up to a given point , whereafter it sets a hard limit on the input signal. With this ideal nonlinearity , the phase distortion is assumed to be zero.

- **The operating point of the amplifier is often specified as the input back-off .**

- **The input back-off is defined as**

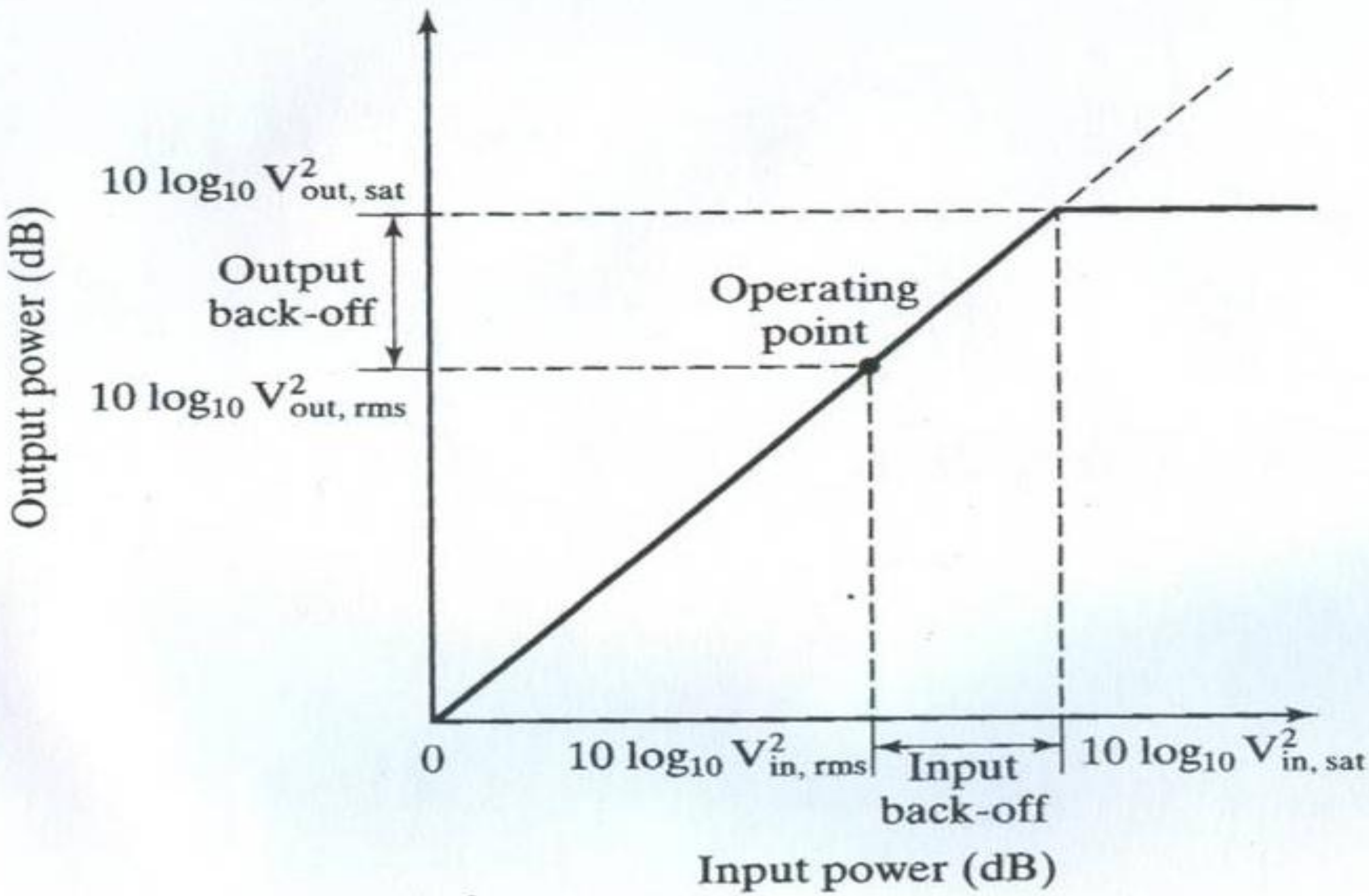
$$\textbf{Input back-off} = 10 \log_{10} (V_{in,rms} / V_{in, sat})^2$$

Alternatively, the operating point can be expressed in terms of the output back-off , defined as folows.

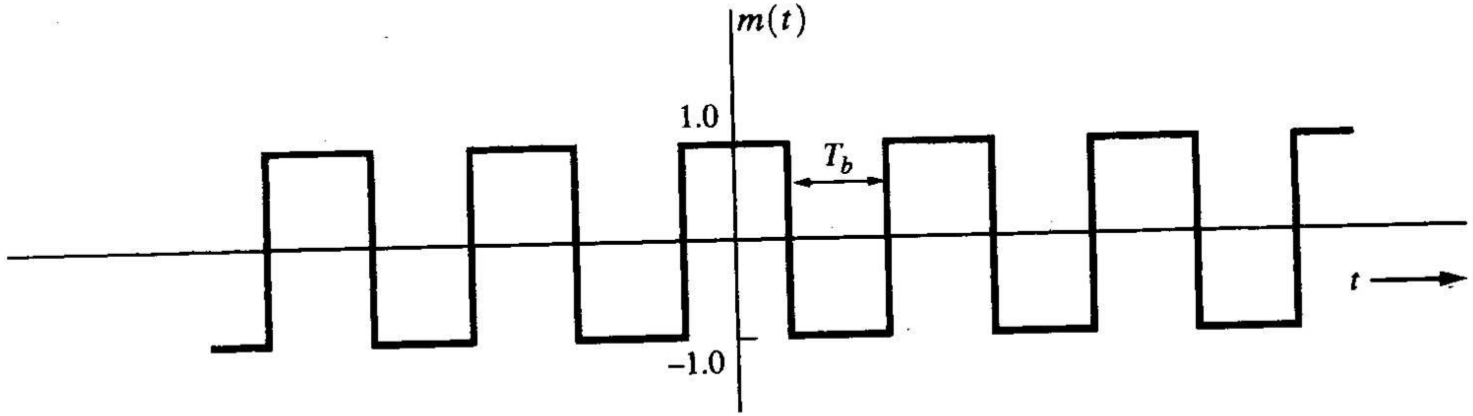
$$\textbf{Output back-off} = 10 \log_{10} (V_{out,rms} / V_{out, sat})^2$$

where $V_{in, sat}$ and $V_{out, sat}$ are the saturation input and output, respectively.

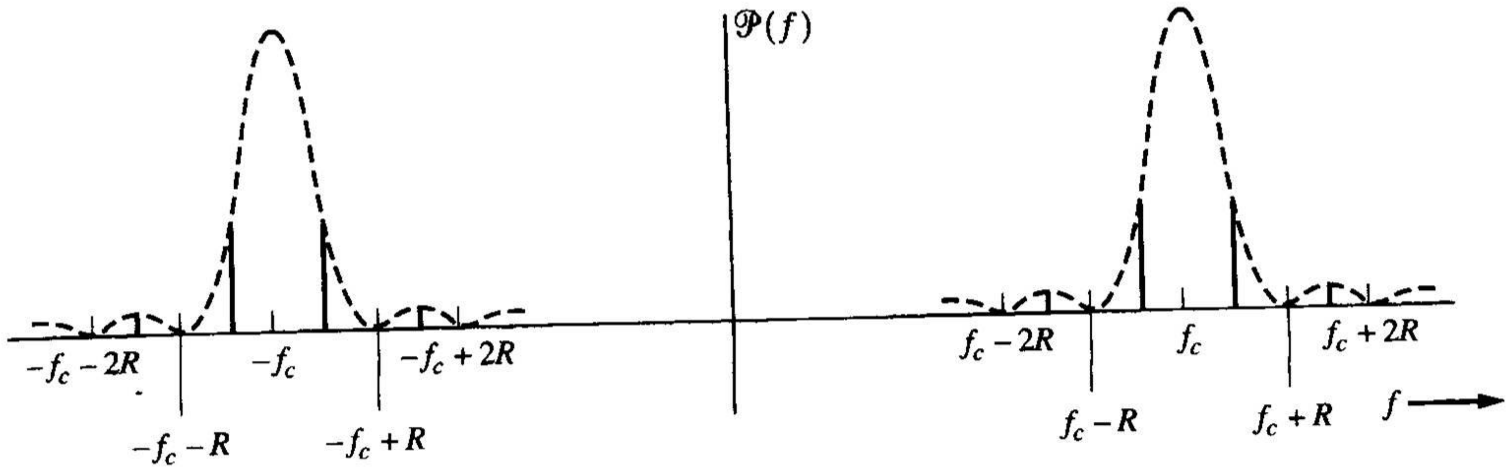
Fig.5.21 Input backoff vs. output backoff of nonlinear amplifier



Appendix : Spectrum of BPSK



(a) Digital Modulating Waveform



(b) Resulting BPSK Spectrum

Spectrum of a BPSK signal.

- **The power spectra for quadrature modulation schemes**

We consider signals having complex envelope of the form

$$z(t) = x(t) + j\ y(t)$$

where $x(t) = \Sigma\, a_k\, p(t-kT_s + \Delta t)$ **and** $y(t) = \Sigma\, b_k\, q(t-kT_s + \Delta t)$

in which $p(t)$ **and** $q(t)$ **are deterministic pulse-shape function.**

Let $P(f)$ **an** $Q(f)$ **be the Fourier transform of** $p(t)$ **and** $q(t)$**, respectively.**

$\{a_k\}$ **and** $\{b_k\}$ **are iid random processes with zero mean .**

Denote that $E[a_k^2] = A^2$ $E[b_k^2] = B^2$

The time increment Δt **is uniformly distributed in** $(0, T_s)$ **to ensure the stationarity of** $z(t)$ **.**

It can be shown that the power spectral density of $z(t)$ is

$$G_z(f) = (A^2 + B^2) \, | P(f) |^2 / T_s$$

It follows that the power spectrum of the real signal

$$s(t) = \mathrm{Re}\, [\, z(t) \exp (j\, 2\pi f_0 t + \Theta) \,]$$

is $S(f) = \frac{1}{4} \{ G_z(f - f_0) + G_z(f + f_0) \}$

where Θ *is a random variable in* $(0, 2\pi]$.

This result can be applied to compute the power spectra of BPSK, QPSK, OQPSK and MSK.

- **BPSK**

For BPSK , the quadrature component of the envelope is identically zero and the pulse-shape function is

$$p(t) = \Pi(t/T_b) \text{ and } P(f) = T_b \operatorname{sinc}(T_b f)$$

where T_b is the bit period.

The baseband spectrum of the BPSK is then given by

$$G_z(f) = A^2 T_b \operatorname{sinc}^2 (T_b f)$$

- **QPSK and OQPSK**

For QPSK , $T_s = 2T_b$ and $p(t) = (1/\sqrt{2})\, \Pi(t/2T_b)$

It can be shown that the baseband power spectrum for QPSK and OQPSK is

$$G_z(f) = 4 A^2 T_b \operatorname{sinc}^2 (2T_b f)$$

- **MSK**

For MSK , $p(t) = \cos(\pi t/2T_b)\, \Pi(t/2T_b)$ and then

$$G_z(f) = \{ 16 A^2 T_b/\pi^2 \, [\, 1-(4T_b f)^2]^2 \} \cos^2 (2\pi T_b f)$$

- **MPSK**

MPSK can be expressed as a special case of QAM by setting $a_i = b_i$

$$s_i(t) = \sqrt{(2E_s / T_s)}\ [\cos(2\pi f_c t) + \varphi_i(t)\ ] \qquad (4.54)$$

$$0 \leqq t \leqq T,\ i = 1,2,\ldots,M$$

where $\varphi_i(t) = 2\pi i / M$

- **Remarks :**

1. **Owing to PSK's simplicity, particularly compared with its competitor QAM , it is widely used in existing technologies. The most popular wireless LAN standards, 802.11 b and 802.11g , use a variety of different PSKs, depending on the data- rate required.**

2. **Notably absent from the various schemes is 8-PSK. This is because its error-rate performance is close to that of 16-QAM, only about 0.5 dB better, but its data rate is only ¾ that of 16-QAM. Thus 8-PSK is often omitted from standards .**

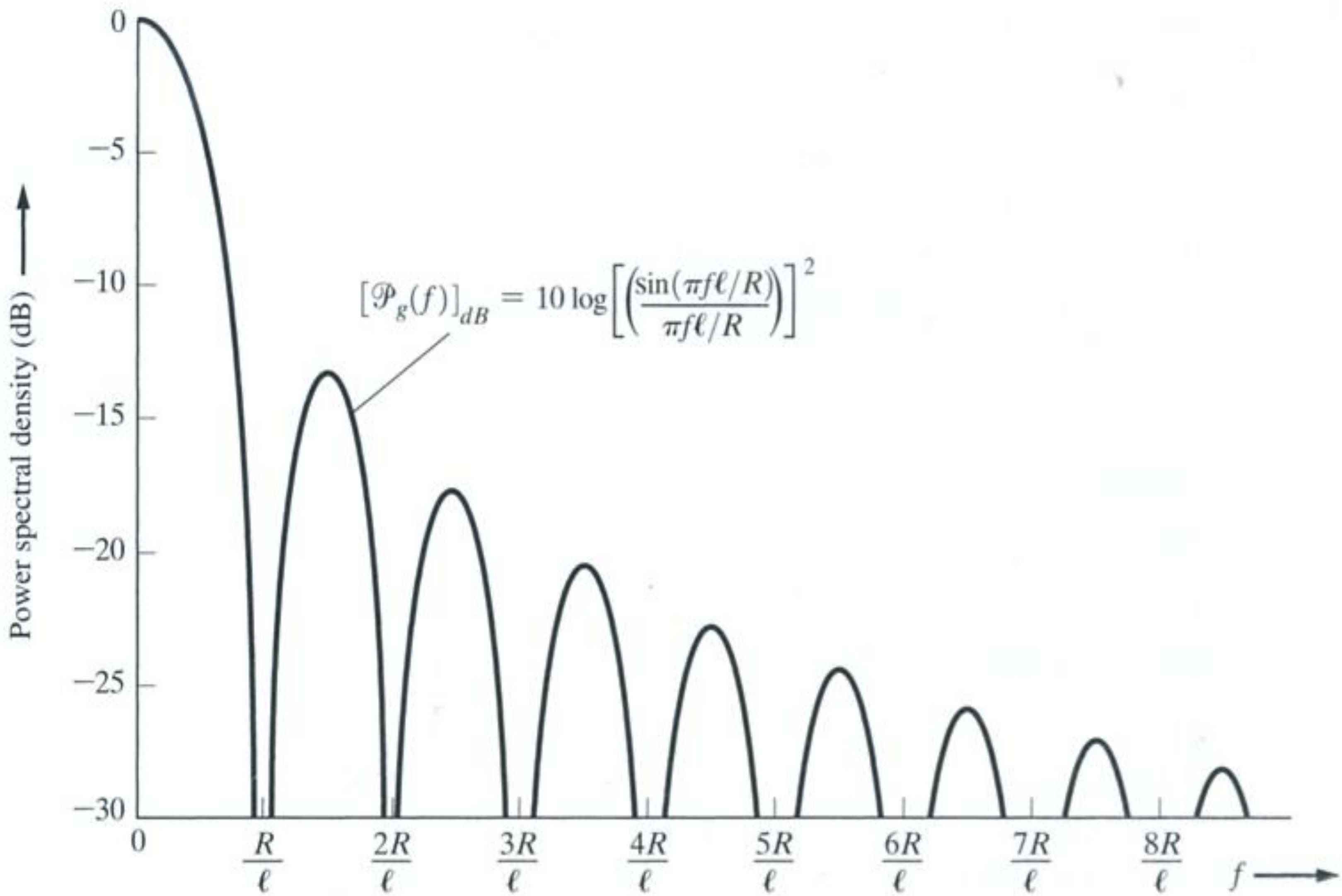


Figure 5–33 PSD for the complex envelope of MPSK and QAM with rectangular data pulses, where $M = 2^\ell$, R is the bit rate, and $R/\ell = D$ is the baud rate (positive frequencies shown). Use $\ell = 2$ for PSD of QPSK, OQPSK, and $\pi/4$ QPSK complex envelope.

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