

# **Chapter 9**

## **Orthogonal Frequency –Division Multiplexing**

**9.1 Multicarrier Modulation**

**9.2 OFDM and DFT**

**9.3 Block Transmission with Guard Intervals**

**9.4 Circular Convolution and DFT**

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**9.6 OFDM Implementation**

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**9.9 Multicarrier CDMA**

**Appendix B: Multi-Carrier CDMA (MC-CDMA)**

**SC-FMA**

- **Orthogonal frequency division multiplexing(OFDM) is a multicarrier modulation technique that has recently found widespread applications in high-data-rate communication systems ,such as digital subscriber lines ( DSL) , wireless LAN ( IEEE 802.11a/g/n), digital video broadcasting, and now WiMAX and other emerging wireless broadband systems ( such as 3G LTE, 4G cellular systems )**
- **OFDM's popularity for high data rate applications stems from its efficient and flexible management of ISI in highly dispersive channels.**

## 9.1 Multicarrier Modulation

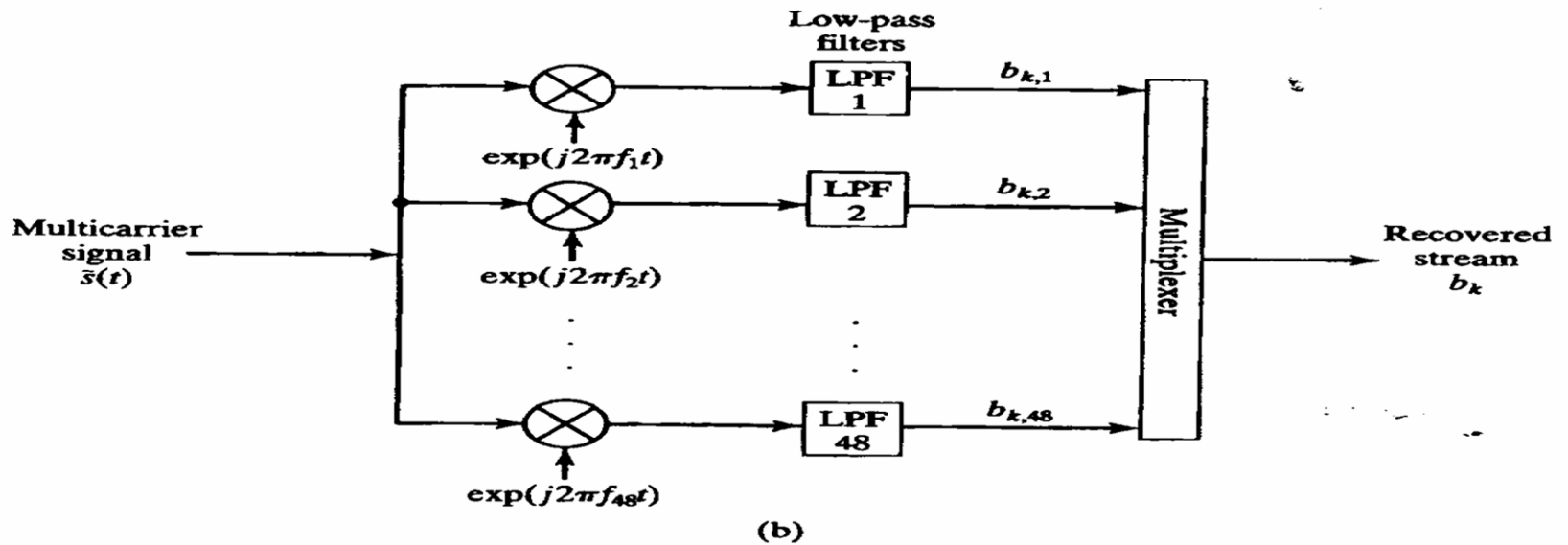
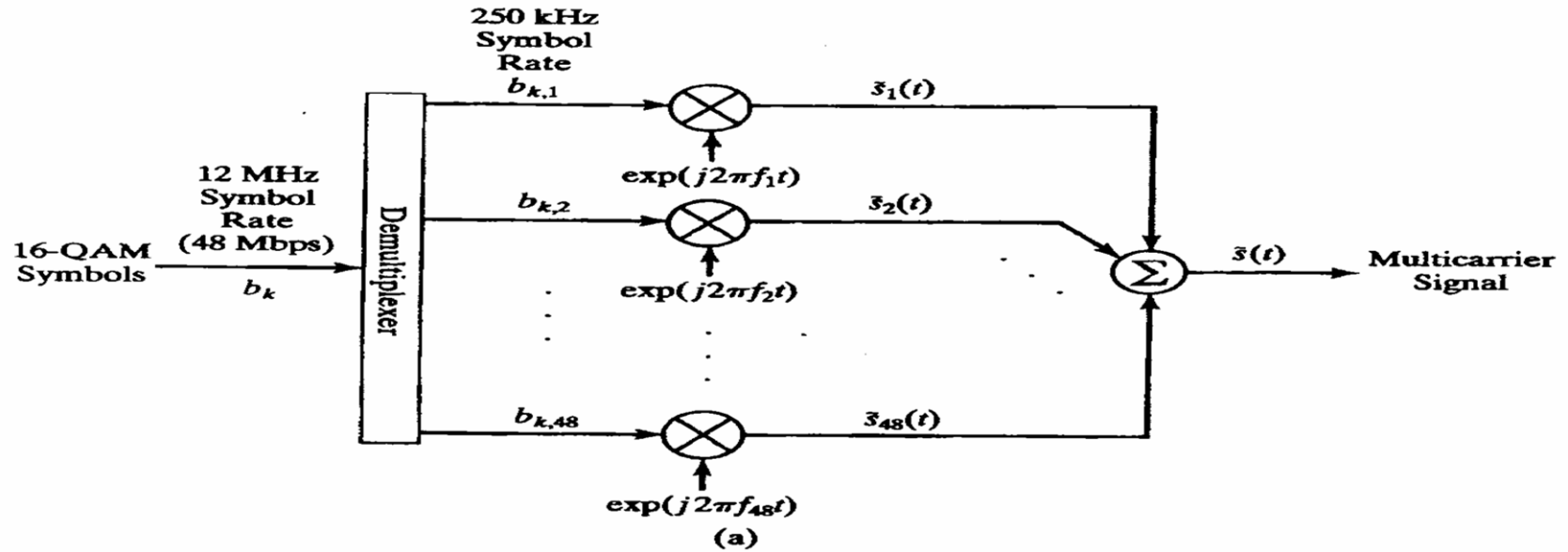
- The basic idea of multicarrier modulation follows naturally from the competing desires for high data rate and ISI-free channels.
- To avoid ISI, the symbol time  $T_s$  has to be significantly larger than the channel's delay spread  $\tau$ . For wideband channels that provide the high data rates needed by today's applications, the desired symbol time is usually much smaller than the delay spread, so the ISI is severe.
- To overcome this problem, multicarrier modulation divides the high-rate transmit stream into  $L$  low-rate substreams, each of which has  $T_s / L \gg \tau$  and is effectively ISI-free.

- These individual substreams are then sent over  $L$  parallel subchannels , maintaining the total desired data rate.
- Typically , the subchannels are **orthogonal** under ideal propagation conditions , in which case multicarrier modulation is often referred to as orthogonal frequency division multiplexing (OFDM).
- The number of substreams is chosen to ensure that each subchannel has a bandwidth less than the **coherence bandwidth** of the channel ,so the subchannels experience relatively **flat fading** . Thus the ISI on each subchannel is small.

Moreover , in the digital implementation of OFDM , the ISI can be completely eliminated through the use of a **cyclic prefix**.

- A simple illustration of a multicarrier transmitter and receiver is given in Fig.10.1 .

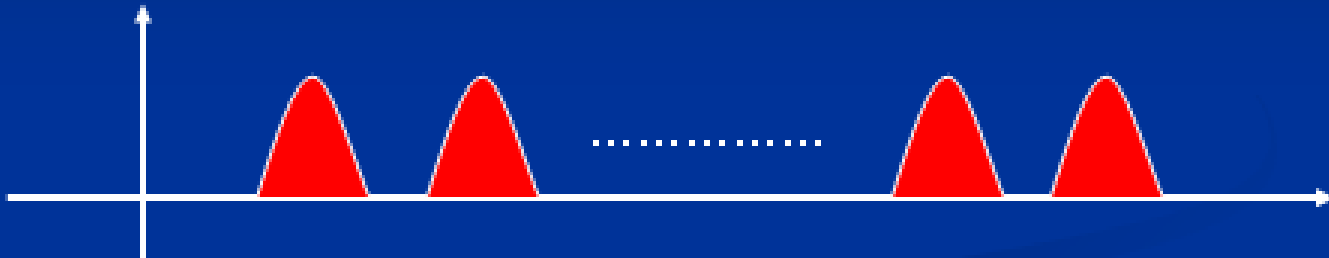
Fig.9.1 Multicarrier system (a) transmitter (b) receiver



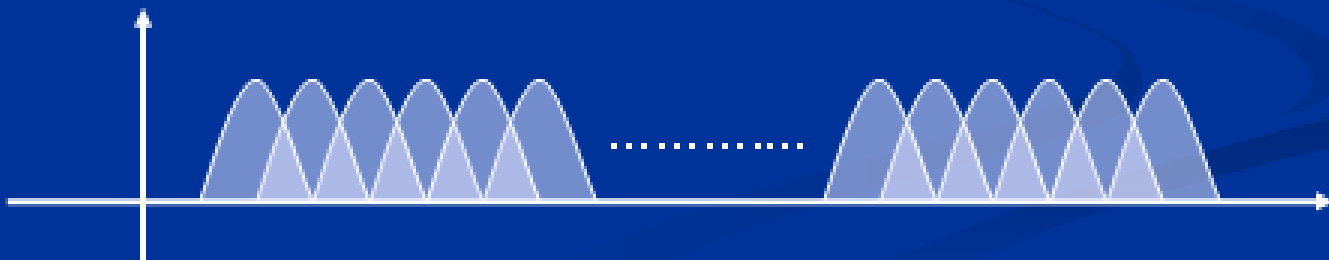
# Conventional FDM signal vs. OFDM signal in frequency domain

## OFDM

### ■ Conventional FDM



### ■ OFDM



## 9.2 OFDM and DFT

- **The use of discrete Fourier transform (DFT) to implement an OFDM system was proposed by Weinstein and Ebert in 1971. The fast Fourier transform (FFT) algorithm can be employed to efficiently to compute DFT. The FFT and its inverse, the IFFT, can create a multitude of orthogonal subcarriers using a single carrier.**
- **DFT and IDFT**

**The DFT transform a set of samples in the time domain into an equivalent set of samples in the frequency domain. The inverse discrete Fourier transform (IDFT ) performs the reverse operation.**

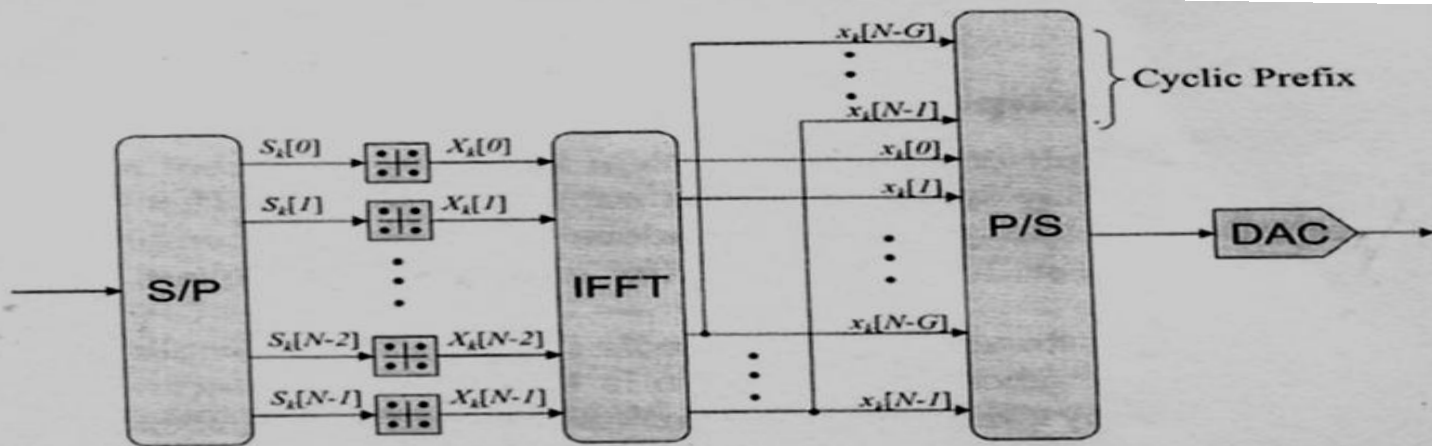
- The transform pair is described as

$$\text{DFT : } B_m = (1/\sqrt{L}) \sum_{n=0}^{L-1} b_n \exp(-j2\pi mn/L), \\ m = 0, 1, 2, \dots, L$$

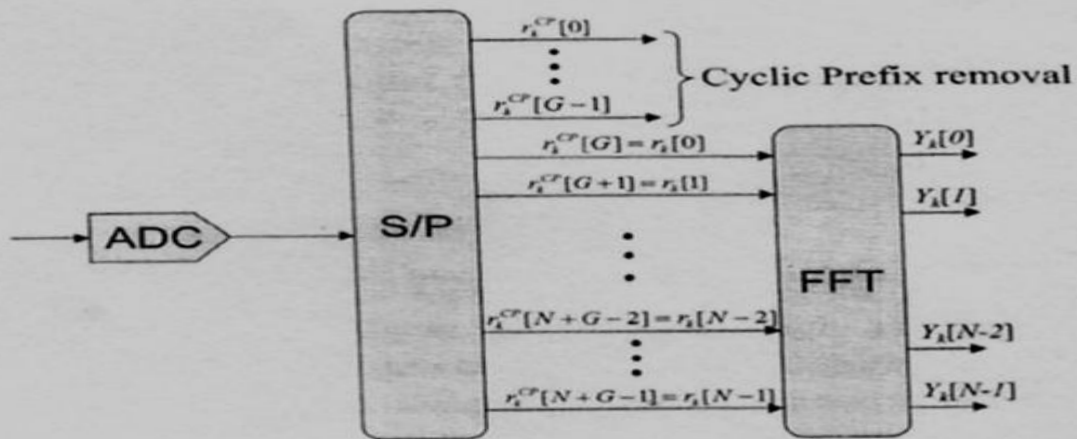
$$\text{IDFT : } b_n = (1/\sqrt{L}) \sum_{m=0}^{L-1} B_m \exp(j2\pi mn/L), \\ n = 0, 1, 2, \dots, L \\ (10.1)$$

- Fig.10.xx shows a block diagram of OFDM transmitter and receiver.





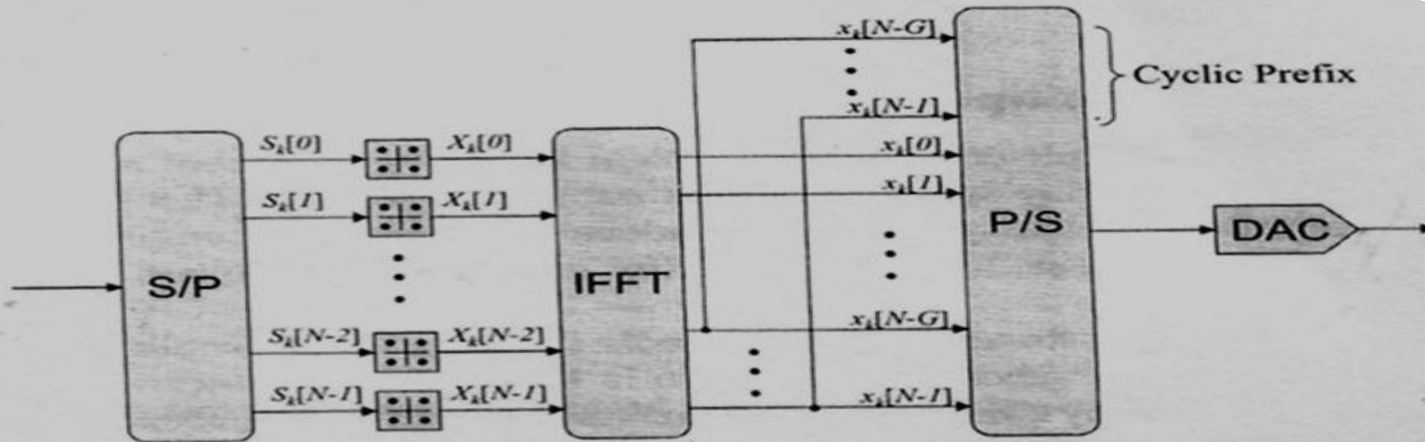
**OFDM Transmitter**



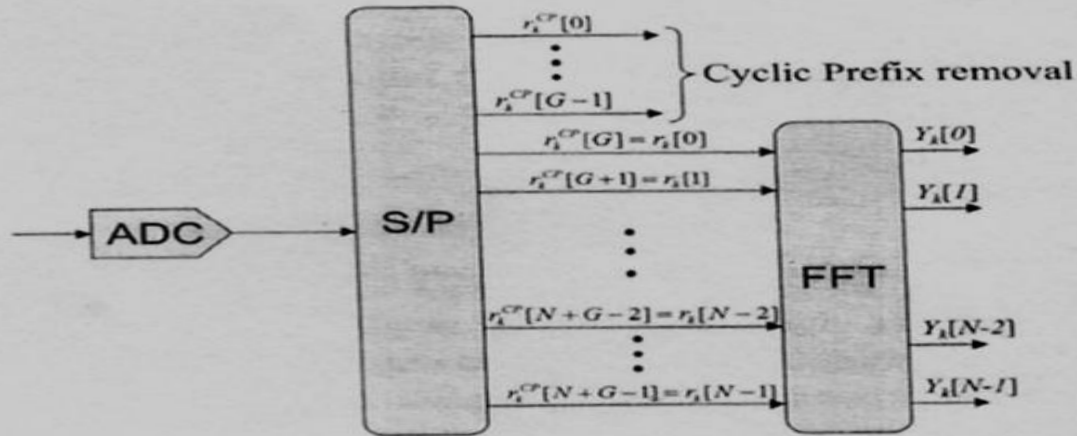
**OFDM Receiver**

Figure 5.4 OFDM system model: (a) transmitter; (b) receiver.

Fig.9.2 OFDM system model (a) transmitter (b) receiver



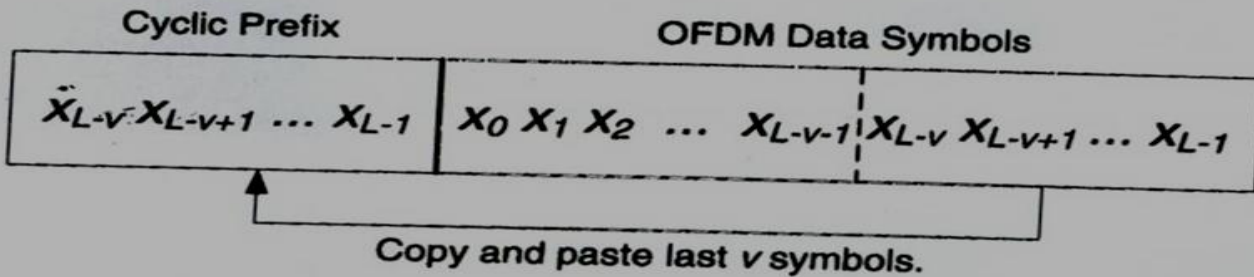
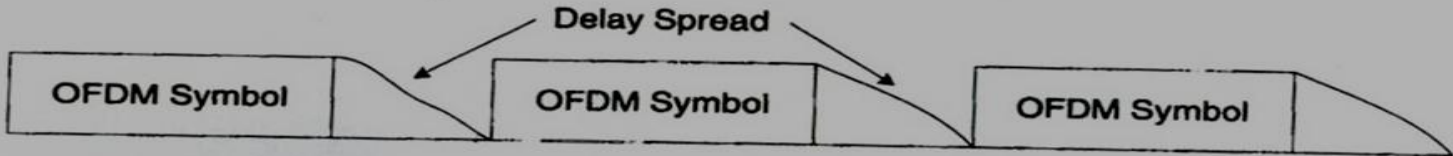
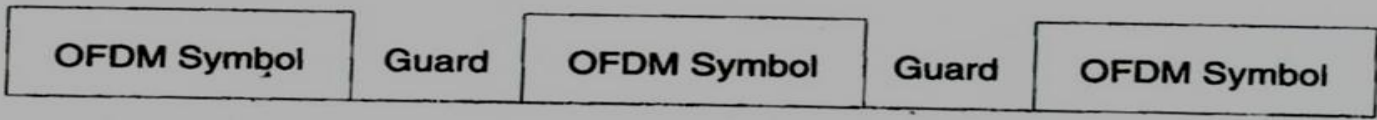
OFDM Transmitter



OFDM Receiver

## 9.3 Block Transmission with Guard Intervals

- An **OFDM symbol** is obtained by grouping  $L$  data symbols. Each OFDM symbol lasts for a duration of  $T$  seconds, where  $T = L T_s$ .
- To keep each OFDM symbol independent of other after passing through a wireless channel, a guard interval between OFDM symbols is introduced.
- After receiving a series of OFDM symbols, as long as  $T_g$  is larger than  $\tau$ , each OFDM symbol will interfere only with itself.



The OFDM cyclic prefix

## 9.4 Circular Convolution and DFT

- The  $n$ -point **circular convolution** of  $x[n]$  and  $h[n]$  is defined as

$$y[n] = x[n] \star h[n] = h[n] \star x[n] = \sum_k h[k] x[n-k]_N \quad (10.2)$$

where  $x[n-k]_N$  denotes  $[n-k]$  modulo  $N$ . In other words,  $x[n-k]_N$  is a periodic version of  $x[n-k]$ .

It can be verified that if

$$\text{DFT} \{y[n]\} = \text{DFT} \{h[n] \star x[n]\} \quad (10.3)$$

$$\text{then } Y_m = H_m X_m, \quad 0 \leq m \leq N-1 \quad (10.4)$$

$$\text{and } X_m = Y_m / H_m \quad (10.5)$$

- When an input data stream  $x[n]$  is sent through a linear time-invariant Finite Impulse Response (FIR) channel  $h[n]$  , the output is the **linear convolution** of the input and the channel :

$$y[n] = x[n] * h[n] = h[n] * x[n] = \sum_k h[k] x[n-k] \quad (10.6)$$

- Thus, the channel output is not a circular convolution . However, the linear convolution can be turned into a circular convolution by adding a special prefix to the input called a **cyclic prefix**.

## 9.5 Cyclic Prefix

- Consider a discrete-time channel with finite impulse response  $h[n] = h_0, h_1, h_2, \dots, h_\nu$ , of length  $\mu + 1$
- The input sequence of length  $N$  is

$$x[n] = x_0, x_1, \dots, x_{N-1}$$

- The cyclic prefix of  $x[n]$  is defined as  $x_{N-\nu}, \dots, x_{N-1}$ .

It consists of the last  $\mu$  values of the  $x[n]$  sequence.

For each input sequence of length  $N$ , these last  $\mu$  samples are appended to the beginning of the sequence.

This yields a new sequence,  $x_{cp}[n]$ , of length  $N + \nu$ ,

$$0 \leq \mu \leq N - 1,$$

where  $x_{cp}[n] = x_{N-\nu}, \dots, x_{N-1}, x_0, \dots, x_{N-1}$

- It is noted that  $\mu + 1 = T_m / T_s$ , where  $T_m$  is the channel delay spread and  $T_s$  is the sampling interval associated with the sequence.

- Suppose  $x[n]$  is input to a discrete-time channel with impulse response  $h[n]$ . The channel output  $y[n]$ ,  $0 \leq n \leq N-1$ , is then

$$y[n] = x_{cp}[n] * h[n] = \sum_{k=0}^{N-1} h[k] x_{cp}[n-k]$$

$$= \sum_{k=0}^{N-1} h[k] x[n-k]_N$$

$$= x[n] \star h[n] \quad (10.7)$$

Thus, by appending a cyclic prefix to the channel input, the linear convolution associated with the channel impulse response becomes a circular convolution.

- Taking the DFT of the channel output in the absence of noise then yields

$$Y_i = X_i H_i \quad 0 \leq i \leq N-1$$



- **The input sequence  $x[n]$  can be recovered from the channel output  $y[n]$ , for the known  $h[n]$ , by**

$$x[n] = \text{IDFT} \{Y_i / H_i\} \quad (10.8)$$

## 9.6 OFDM Implementation

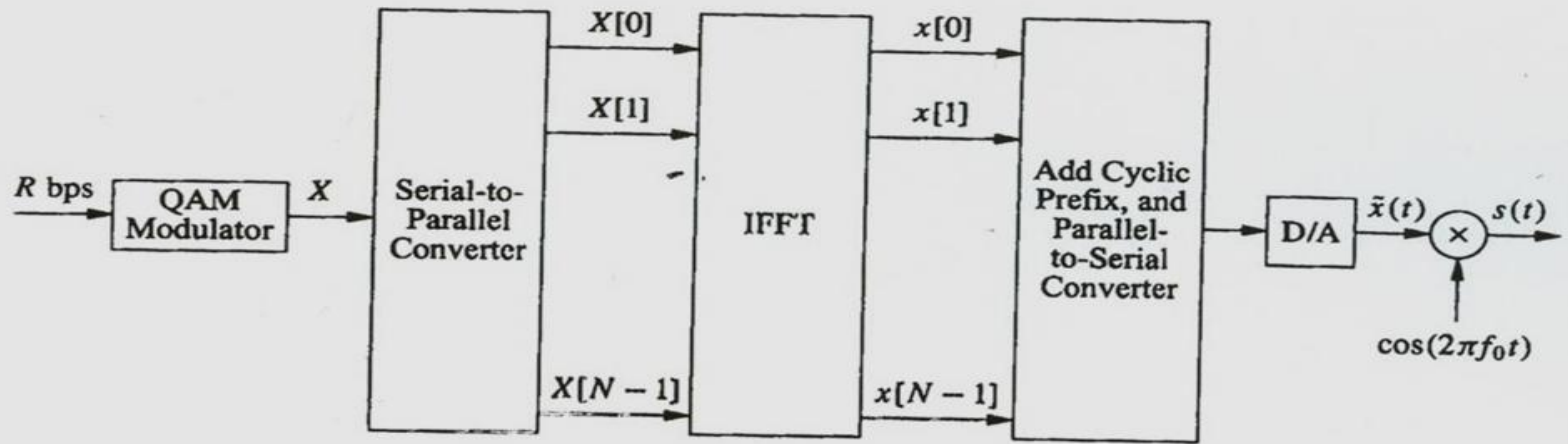
- **The OFDM implementation of multicarrier modulation is shown in the following figure.**
- **The input data stream is modulated by a QAM modulator , resulting in a complex symbol stream  $X_0, X_1, \dots, X_{N-1}$ . This symbol stream is passes through a serial-to-parallel converter, whose output is a set of N parallel QAM symbols , which are frequency component of the OFDM modulator output.**

- In order to generate  $s(t)$ , the frequency components are converted into time –domain samples by performing an inverse DFT on these  $N$  symbols, using the IFFT algorithm.
- The IFFT yields the OFDM symbols consisting of the sequence  $x[n] = x_0, \dots, x_{N-1}$ , where

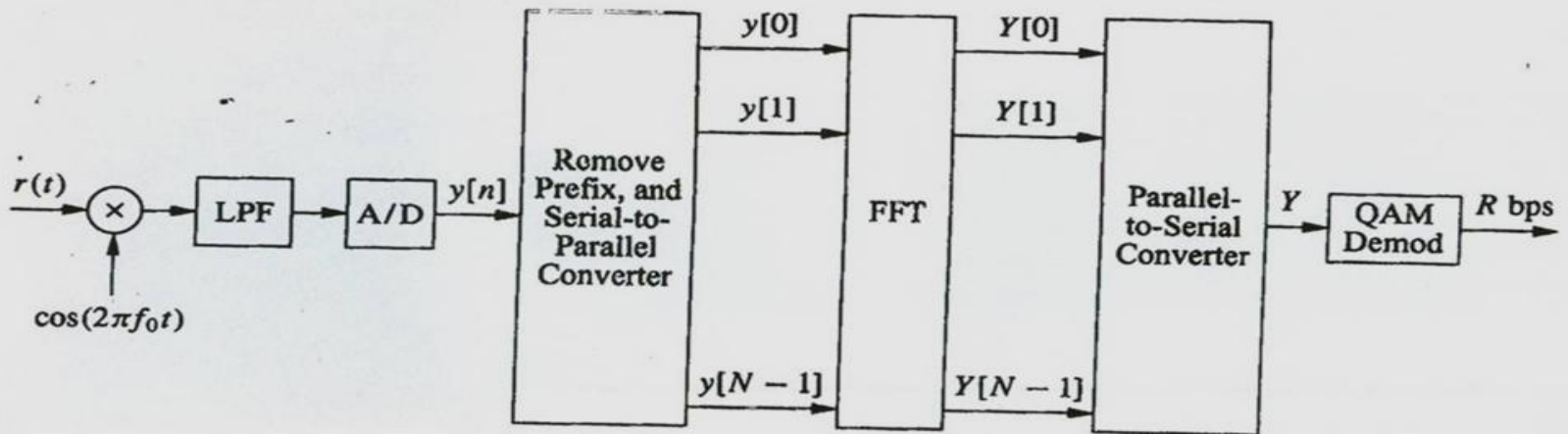
$$x_n = (1/\sqrt{N}) \sum_{i=0}^{N-1} X_i \exp(j2\pi ni / N) \quad , 0 \leq i \leq N-1$$

(10.9)

- The cyclic prefix is then added to the OFDM symbol and the resulting time samples  $x_{cp}[n]$  are passed through a D/A converter, resulting in the baseband OFDM signal  $x(t)$ , which is then upconverted to frequency  $f_c$ .

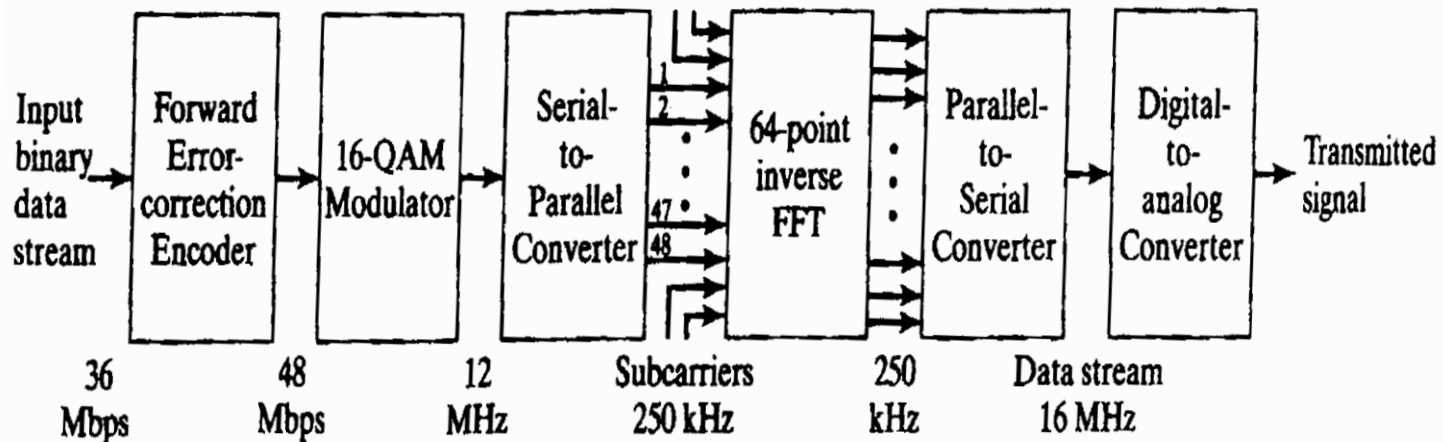


**Transmitter**

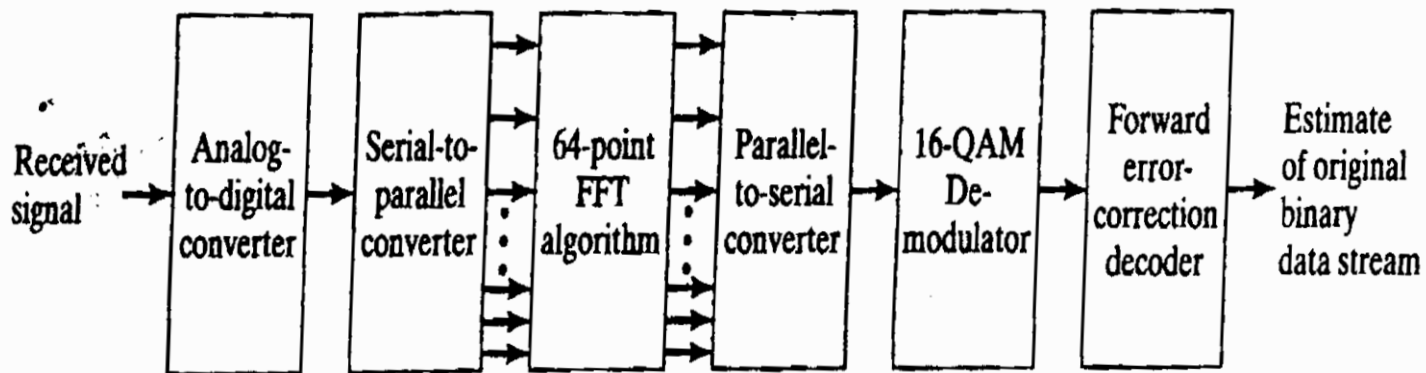


**Receiver**

OFDM with IFFT/FFT implementation.



(a)



(b)

FIGURE 3.34 Block diagram of (a) OFDM transmitter, and (b) OFDM receiver.

## Example : IEEE 802.11a Wireless LAN Standard

- In 802.11a ,  $N = 64$  subcarriers :  
48 are actually used for data transmission , the outer 12 are zeroed in order to reduce adjacent channel interference , and 4 are used as pilot symbols for channel estimation.
- The cyclic prefix consists of  $\nu = 16$  samples , so the total number associated with each OFDM symbol is 80.
- Coded bits are packetized , and the transmitter gets periodic feedback from the receiver about the packet error rate , which it uses to pick an appropriate error correction code and modulation technique. The same code and modulation must be used for all the subcarriers at any given time.
- The error correction code is a convolutional code with one of three possible code rates :  $r = \frac{1}{2}$  ,  $\frac{2}{3}$  , or  $\frac{3}{4}$  .
- The modulation types that can be used on the subchannels are BPSK, QPSK, 16-QAM , or 64-QAM .

- The total system bandwidth of 300 MHz is divided into 20-MHz channels that can be assigned to different users. Since the channel bandwidth  $B$  ( and sample rate  $1 / T_s$  ) is 20 MHz and since there are 64 subcarriers evenly spaced over that bandwidth , the subcarrier bandwidth is :

$$B_N = 20 \text{ MHz} / 64 = 312.5 \text{ kHz}$$

- Since  $\nu = 16$  and  $1 / T_s = 20 \text{ MHz}$  , the maximum delay spread for which ISI is removed is

$$T_m < \nu T_s = 16 / (20 \text{ MHz} ) = 0.8 \text{ } \mu\text{sec}$$

which corresponds to delay spread in an indoor environment.

The symbol time per subchannel is equal to the OFDM symbol time , that is

$$T_N = 80 T_s = 80 / (20 \cdot 10^6) = 4 \text{ } \mu\text{sec}$$

The data rate per subchannel is  $(\log_2 M) / T_N$

## **9.7 Challenges in OFDM Transmission**

- 1. Peak-to-Average Power Ratio (PAR)**
- 2. Frequency and Timing Offset .... Synchronization**
- 3. Frequency-Selective Fading ... Channel Estimation**



## 9.8 OFDMA

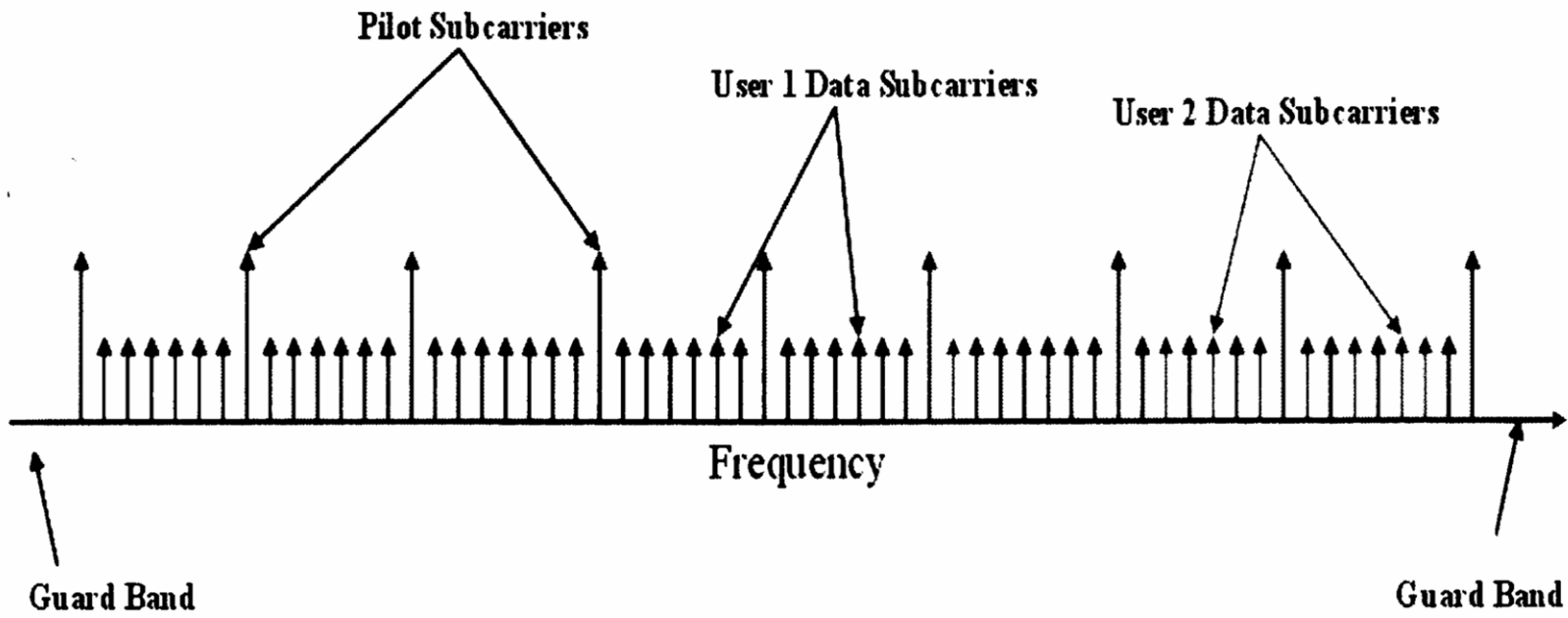
- **OFDMA is a multi-user version of the OFDM digital modulation scheme. Multiple access is achieved in OFDMA by assigning subsets of subcarriers to individual users as shown above.**

**This allows simultaneous low data rate transmission from several users.**

- **Based on feedback information about the channel conditions, adaptive user-to-subcarrier assignment can be achieved.**

**If the assignment is done sufficiently fast , this further improves the OFDM robustness to fast fading and narrow-band cochannel interference.**

- **OFDMA can also be seen as an alternative to combining OFDM with time-division multiple access .**



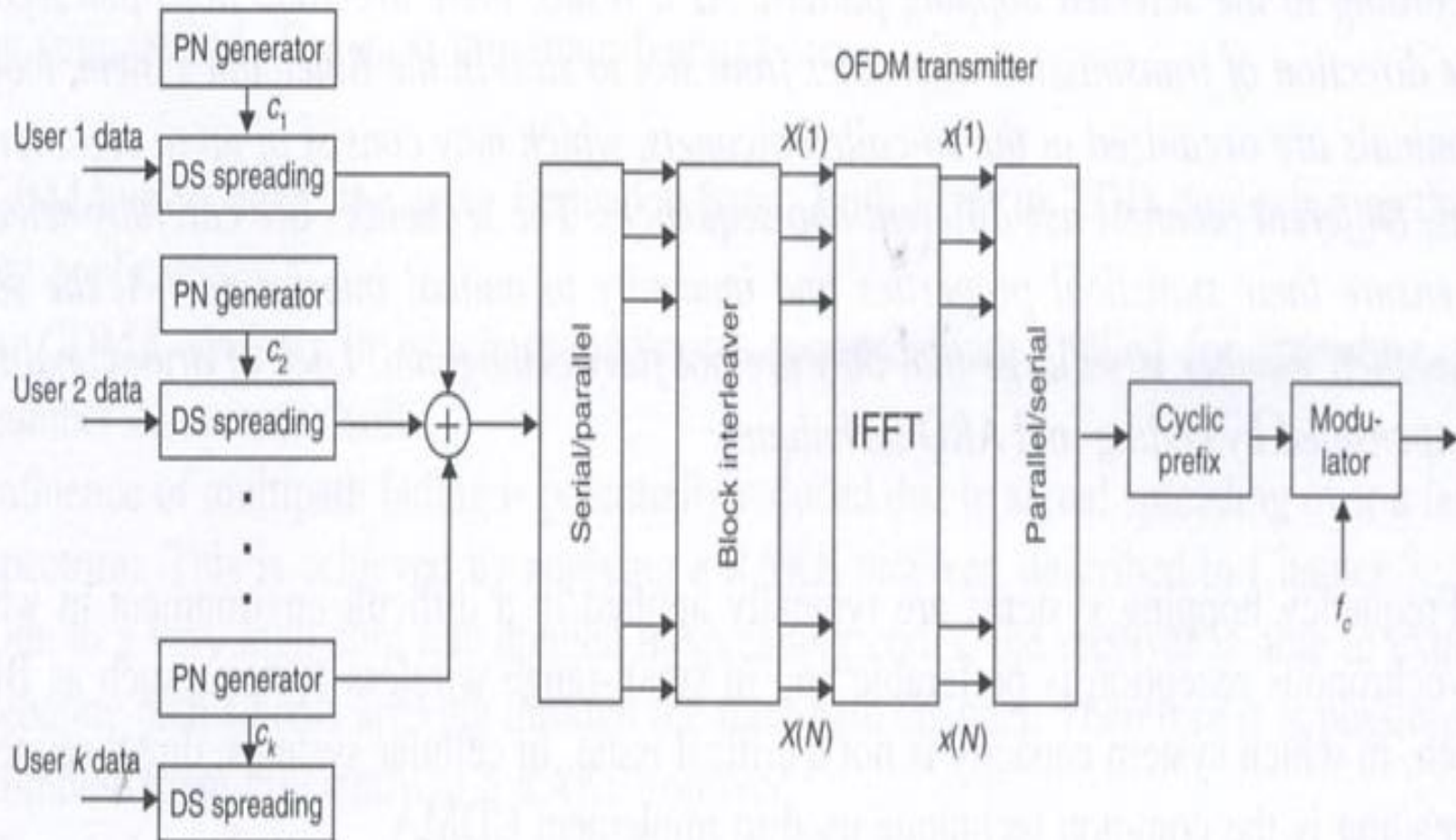
## 9.9 MC-CDMA

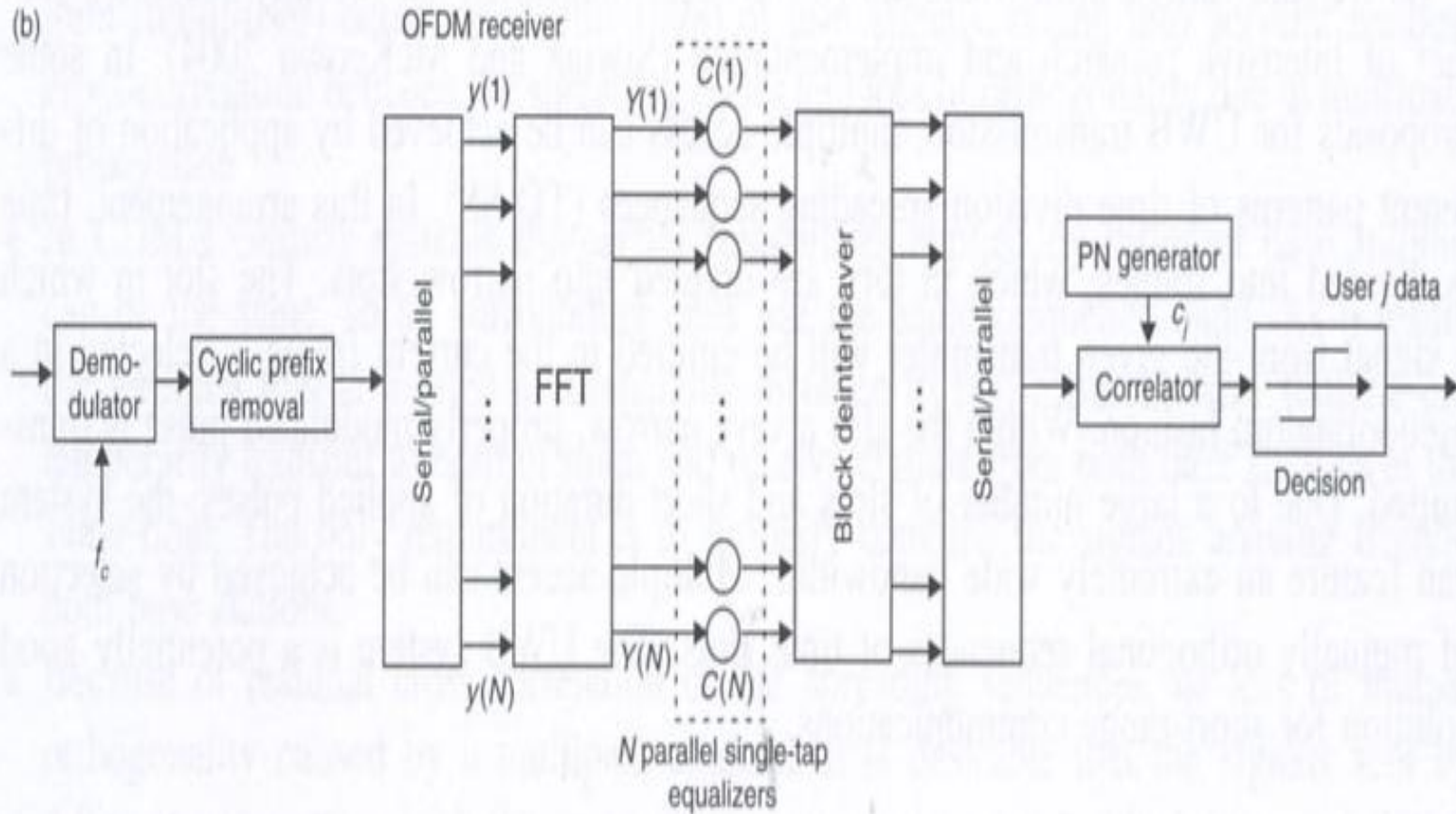
- **Multicarrier CDMA (MC-CDMA) is a technique that combines the advantages of OFDM and CDMA . It is very effective both at combating ISI and as a mechanism for allowing multiple users to share the same channel.**
- **Multi-carrier CDMA schemes can be divided into two types.**
- **In Type 1 MC-CDMA the original data stream is spread using a given spreading code and then a different subcarrier is modulated with each chip. The spreading operation is performed in the **frequency domain** , thus the MC-CDMA can be treated as a serial concatenation of DS spreading and multi-carrier modulation . Fig.xx shows the scheme of the MC-CDMA transmitter used in the downlink transmission.**

- **In Type 2 MC-CDMA scheme , the data stream is initially serial-to-parallel converted into several low-rate substreams . The resulting substreams are subsequently spread using a given spreading code and then each spread substream modulates its own subcarrier . The operation is performed in **time domain** , as shown in Fig.xx ..**

- **Type 1 MC-CDMA**

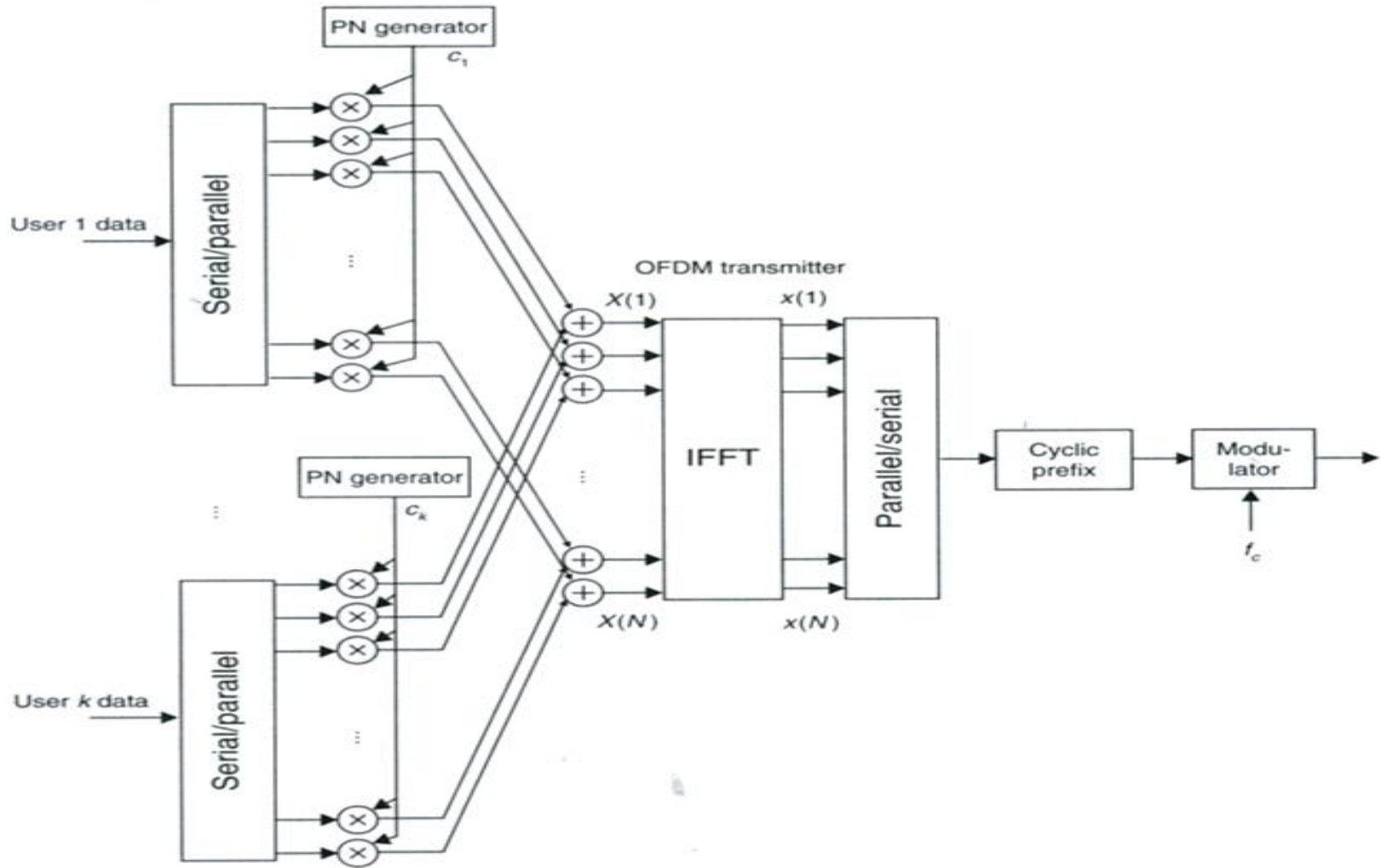
(a)

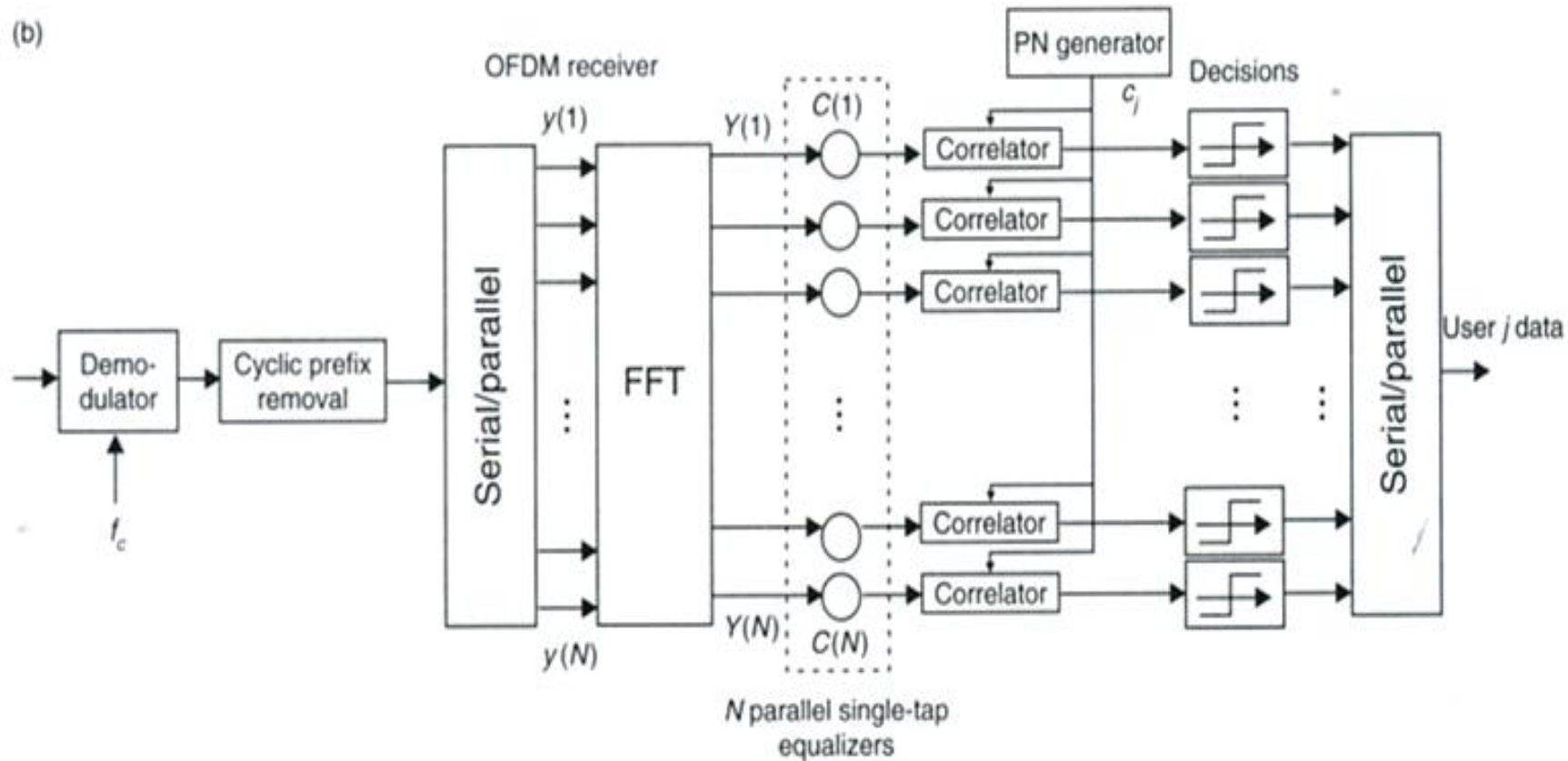




**Figure 9.8** Transmitter (a) and receiver (b) of the first type of multi-carrier CDMA scheme in downlink transmission

- **Type 2 MC-CDMA**





**Figure 9.9** Transmitter (a) and receiver (b) of the second type of multi-carrier CDMA scheme in



## Appendix : A Brief History of OFDM

**1966** R.W Chang shows that multicarrier modulation can solve the multipath problem without reducing data rate. This is generally considered the first official publication on multicarrier modulation.

*R.W.Chang"Synthesis of bandlimited orthogonal signals for multichannel data transmission" Bell Systems Technical Journal, 45:1775-1796, December 1966.*

**1971** Weinstein and Ebert show that multicarrier modulation can be accomplished using DFT.  
*S.Weinstein and P. Ebert," Data transmission by frequency-division multiplexing using the discrete Fourier transform", IEEE. Trans. Communications,19(5): 628- 634, October 1971.*

**1985** Cimini at Bell Labs identifies many of the key issues in OFDM transmission and does a proof- of-concept design.

*L.J.Cimini,"Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing", IEEE Trans. Communications,33(7):665-675, July, 1985*

**1993** DSL adopts OFDM, also called discrete multitone, following successful field trials/competitions at Bellcore versus equalizer-based systems.

**1999** The IEEE 802.11 committee on wireless LANs releases the 802.11a standard for OFDM operation in 5GHz UNI band.

- 2003** The IEEE 802.16 committee releases an OFDM- based standard for wireless broadband access for metropolitan area networks under revision 802.16a. 802.16a also specifies additional MAC-layer option, including support for OFDMA.
- 2003** The IEEE 802.11 committee releases the 802.11g standard for operation in the 2.4 GHz band.
- 2003** The multiband OFDM standard for ultrawideband is developed, showing OFDM's usefulness in low-SNR system.
- 2004** IEEE Std 802.16-2004 replaces IEEE Standards 802.16- 2001, 802.16c-2002, and 802.16a-2003. It addresses only fixed systems
- 2005** The IEEE 802.16 committee publishes formally 802.16e-2005. It specifies scalable OFDM for the physical layer and makes further modifications to the MAC layer to accommodate high-speed mobility.
- 2009** 3GPP release 8 LTE/LTE-A specifications completed and released.
- 2010** IEEE 802.11n standard is ratified, which performs MIMO-OFDM for wireless LANs for peak data rates of 600 Mbps .

## WiMAX and 802.16

- **WiMAX** is defined as **Worldwide Interoperability for Microwave Access** by the **WiMAX Forum**, formed in June 2001 to promote conformance and interoperability of the **IEEE 802.16** standard, officially known as **WirelessMAN**.
- **WiMAX** aims to provide wireless data over long distances, in a variety of different ways, from point to point links to full mobile cellular type access. In practical terms this enables a user, for example, to browse the **Internet** on a **laptop** computer without physically connecting the laptop to a wall jack. The Forum describes **WiMAX** as "a standards-based technology enabling the delivery of **last mile** wireless broadband access as an alternative to cable and DSL."

## **Fixed WiMAX**

- **This is a phrase frequently used to refer to systems built using 802.16-2004 ('802.16d') as the air interface technology.**

## **Mobile WiMAX**

- **A phrase frequently used to refer to systems built using 802.16e-2005 as the air interface technology.**

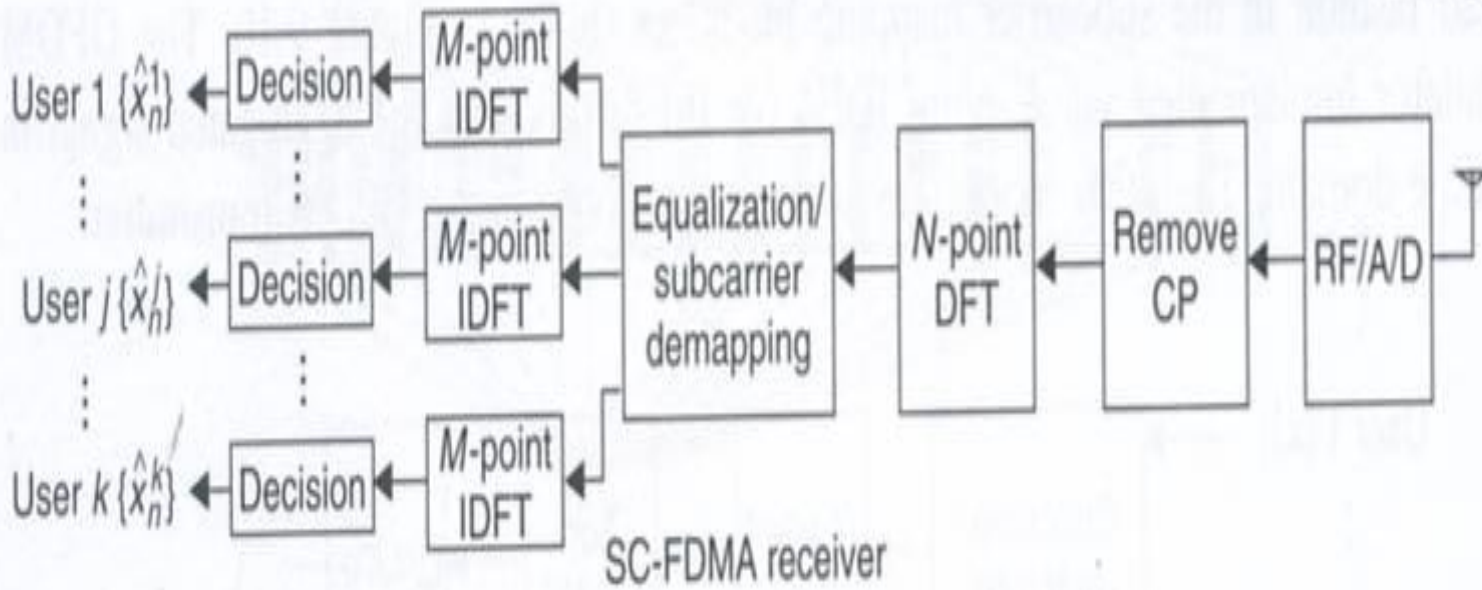
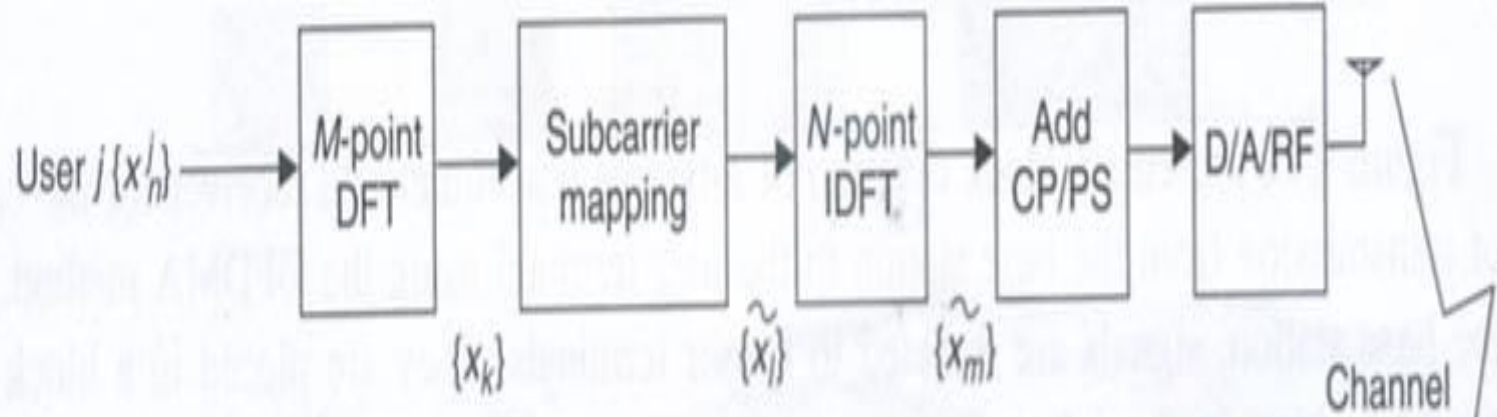
## **Appendix-10.A SC-FDMA**

- **Single-carrier FDMA (SC-FDMA) combines the desirable characteristics of OFDM with the low PAPR of single-carrier transmission schemes. Just like in OFDM, SC-FDMA divides the transmission bandwidth into multiple parallel subcarriers, with the orthogonality between the subcarriers being maintained in frequency-selective fading channel by the use of cyclic prefix or guard period.**
- **The main objective of SC-FDMA is to introduce transmission with lower PAPR than OFDMA. Since OFDMA shows envelope fluctuations, and signal with high PAPR requires highly linear power amplifiers to reduce the distortion. The design of mobile terminals are complex and they become power hungry.**
- **Another objective is to address frequency offset drawback of OFDMA.**

- In OFDM, FFT is applied on the receiver side on each block of symbols, and IFFT on the transmitter side.  
In SC-FDMA, both FFT and IFFT are applied on the transmitter side, and also on the receiver side.
- In OFDM, equalization is achieved on the receiver side after the FFT calculation, by multiplying each Fourier coefficient by a complex number. Thus, frequency-selective fading and phase distortion can be combated by utilizing a simple equalizer per subcarrier after FFT. But, SC-FDMA utilizes a complex equalizer before sending the resultant to IFFT. IFFT removes the effect of the FFT at the transmitter.
- In SC-FDMA, multiple access is made possible by inserting silent Fourier-coefficients on the transmitter side before the IFFT, and removing them on the receiver side before the IFFT. Different users are assigned to different Fourier-coefficients (sub-carriers).

- **SC-FDMA** first runs an FFT over the groups of input bits to spread them over all sub-carriers and then uses the result for the IDFT which creates the time signal. This is why SC-FDMA is sometimes also referred to as **FFT spread OFDM**.
- While SC-FDMA adds additional complexity at both the transmitter and receiver side, the 3GPP standardization body has nevertheless decided for it because of its useful properties : **low PAPR** and **low sensitivity to carrier frequency offset** .

### SC-FDMA transmitter



### SC-FDMA receiver



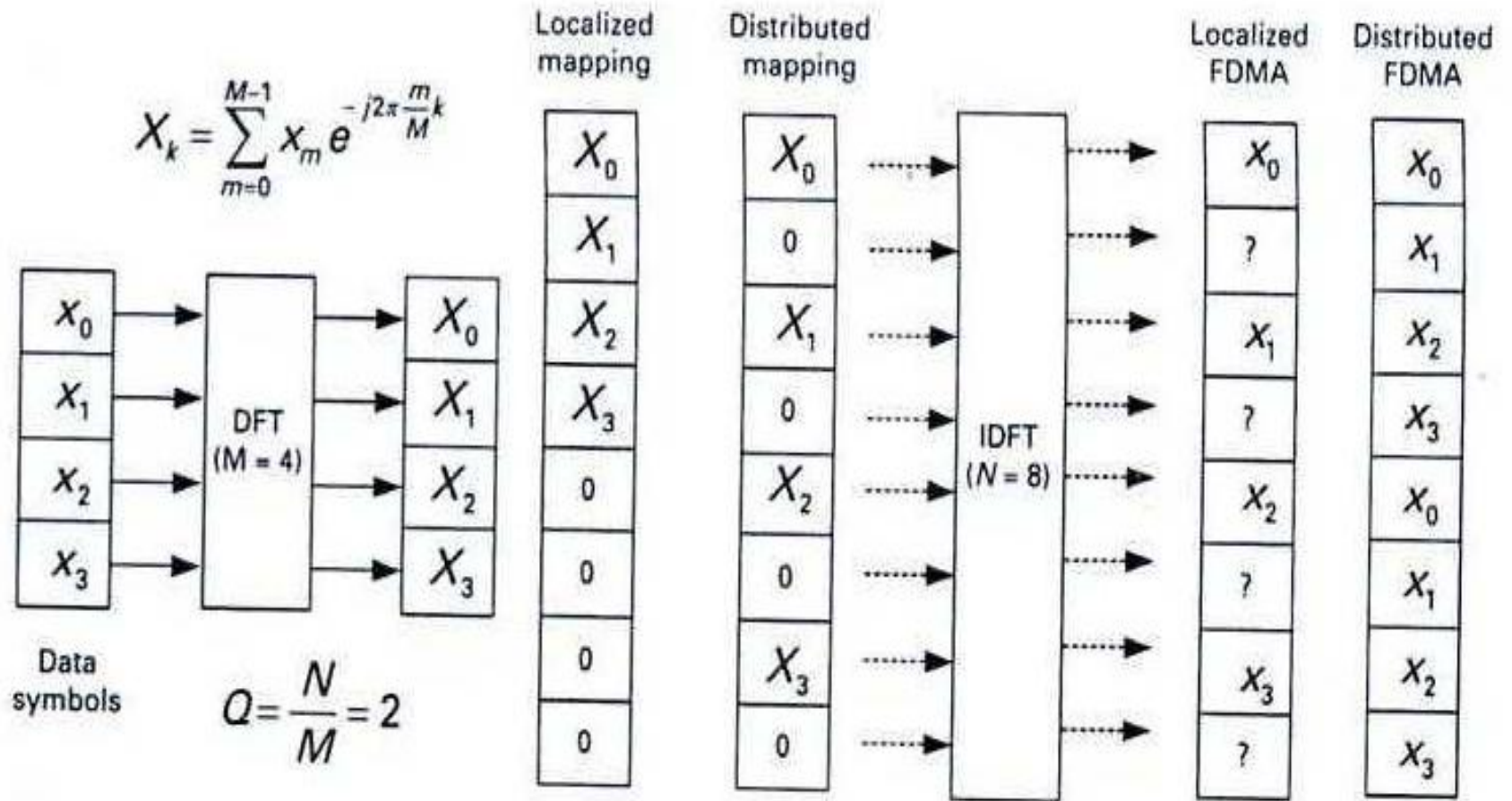
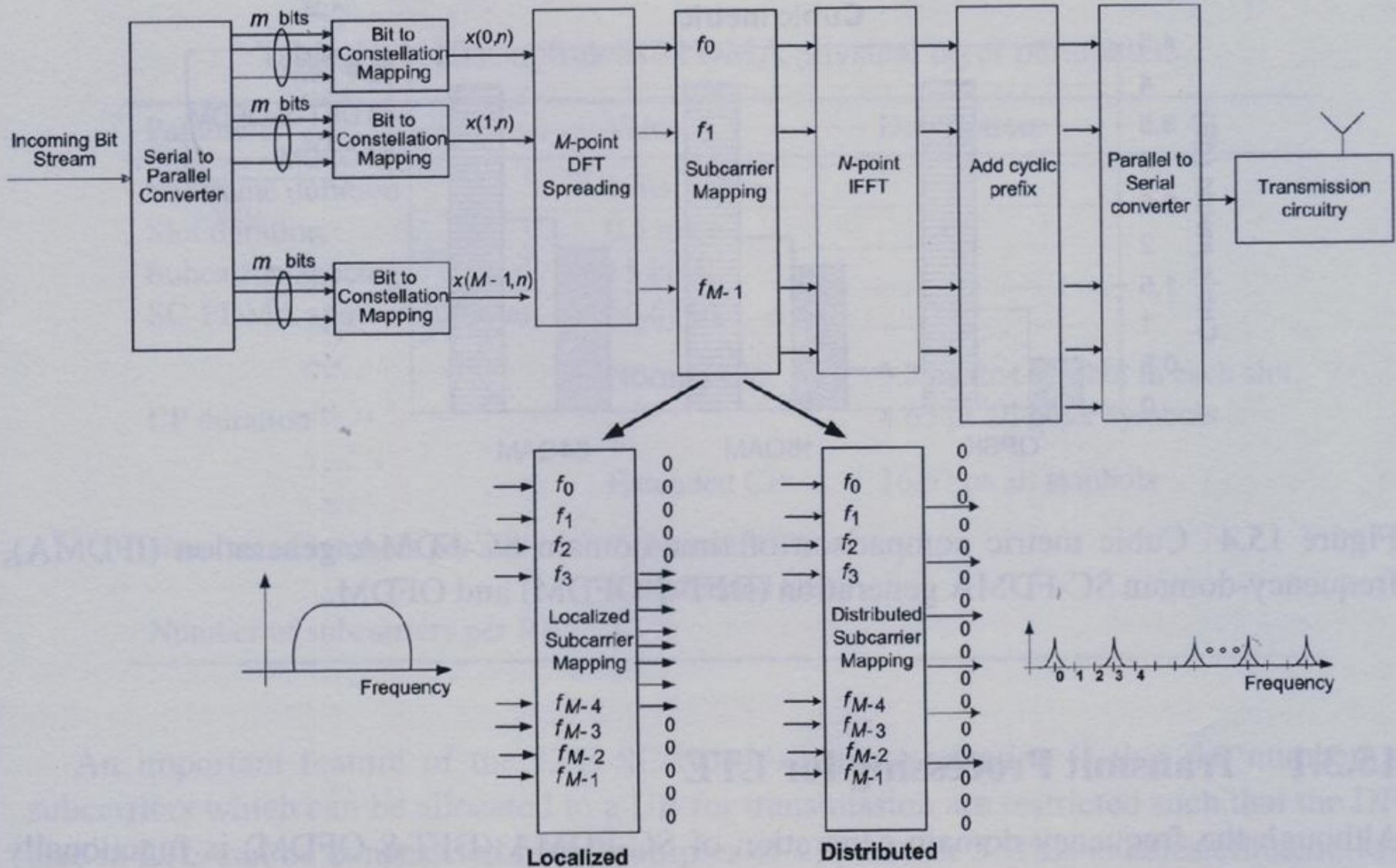
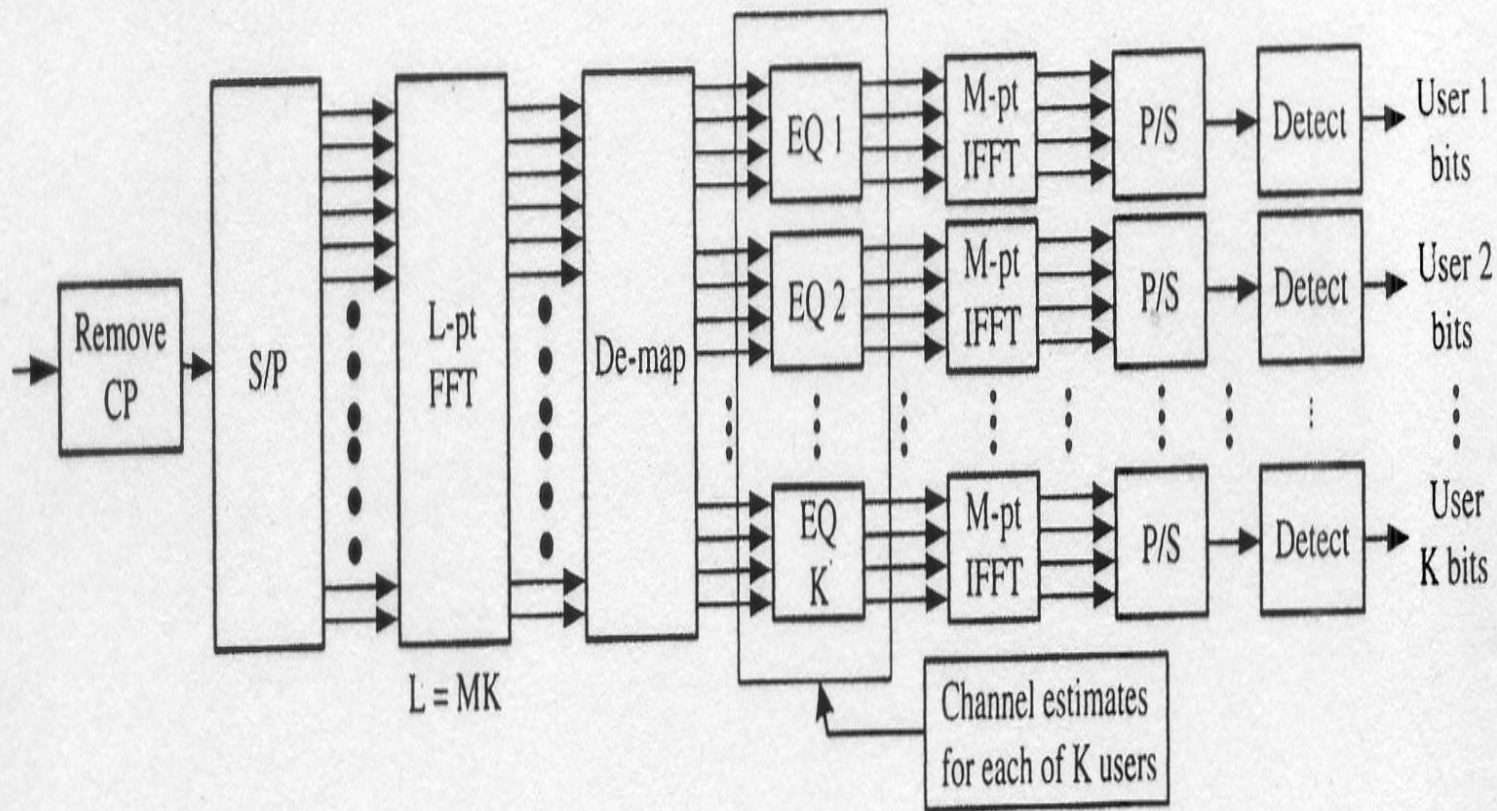


Figure 4.1. Subcarrier mapping for Localized and Distributed FDMA.

# SC-FDMA frequency domain transmit processing





**Figure 4.9** SC-FDMA uplink receiver. Much like the OFDMA receiver, here we explicitly assume that each user occupies a fraction  $M/L$  of the spectrum.

- **The  $N$  subcarriers of the user into  $M$  subcarrier is mapped in either distributed or localized manner , as illustrated in Fig. xx.**

**Distributed mapping ( . also denoted as interleaved FDMA or IFDMA )**

**Localized mapping maps subcarriers allocated to user adjacent to each other.**

## References

- **Hyung G. Myung, Junsung Lim, and David J. Goodman, “Single Carrier FDMA for Uplink Wireless Transmission”, IEEE Vehicular Technology Magazine, vol. 1, no. 3, Sep. 2006, pp. 30-38 .**
- **H. Ekström, A. Furuskär, J. Karlsson, M. Meyer, S. Parkvall, J. Torsner, and M. Wahlqvist, “Technical Solutions for the 3G Long-Term Evolution,” IEEE Commun. Mag., vol. 44, no. 3, March 2006, pp. 38–45 .**
- **H. Sari, G. Karam, and I. Jeanclaude, “Transmission Techniques for Digital Terrestrial TV Broadcasting,” IEEE Commun. Mag., vol. 33, no. 2, Feb. 1995, pp. 100–109 .**