Chapter 11

Orthogonal Frequency –Division Multiplexing

References

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5. M.-H. Hsieh and C.-H. Wei ," A Low-Complexity Frame Synchronization and Frequency Offset Compensation Scheme for OFDM Systems over Fading Channels " IEEE Trans. Vehicular technology , Sep.1999 , pp. 1596-1609.
6. M.-H. Hsieh and C.-H. Wei," Channel Estimation for OFDM Systems based on Comb-Type Pilot Arrangement in Frequency-Selective Fading Channels", IEEE Trans. Consumer Electronics, Feb.1998, pp. 217-225.
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11.1 Multicarrier Modulation and OFDM

- **The basic idea of multicarrier modulation follows naturally from the competing desires for high data rate and ISI- free channels.**
- **To avoid ISI , the symbol time T_s has to be significantly larger than the channel's delay spread τ. For wideband channels that provide the high data rates needed by today's applications , the desired symbol time is usually much smaller than the delay spread, so the ISI is severe.**
- **To overcome this problem, multicarrier modulation divides the high-rate transmit stream into L low-rate substreams , each of which has $T_s / L >> \tau$ and is effectively ISI-free .**

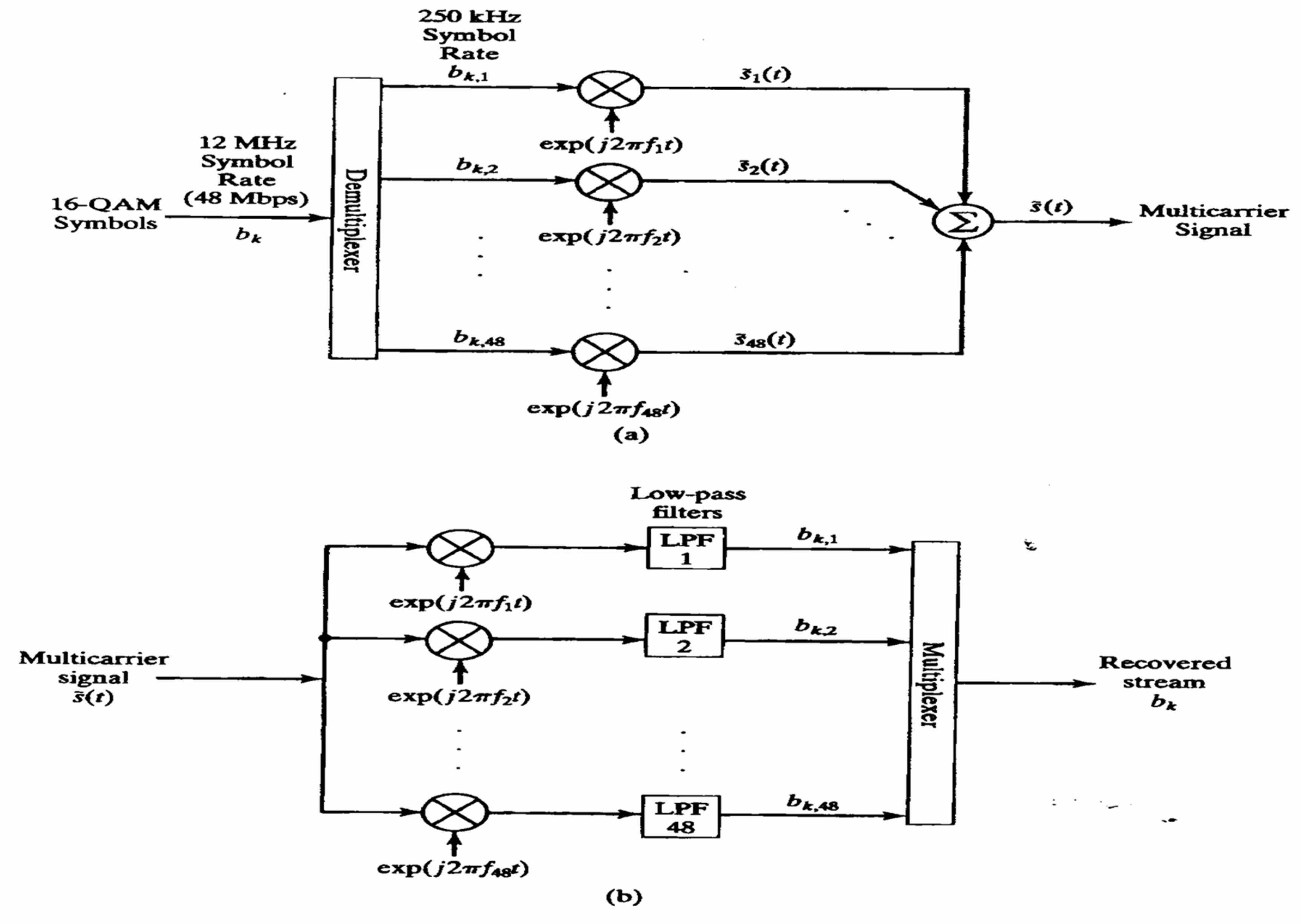
- **These individual substreams are then sent over *L* parallel subchannels , maintaining the total desired data rate.**
- **Typically , the subchannels are orthogonal under ideal propagation conditions , in which case multicarrier modulation is often referred to as orthogonal frequency division multiplexing (OFDM).**
- **The number of substreams is chosen to ensure that each subchannel has a bandwidth less than the coherence bandwidth of the channel ,so the subchannels experience relatively flat fading . Thus the ISI on each subchannel is small.**

  Moreover , in the digital implementation of OFDM , the ISI can be completely eliminated through the use of a cyclic prefix.
- **A simple illustration of a multicarrier transmitter and receiver is given in Fig.11.1 .**

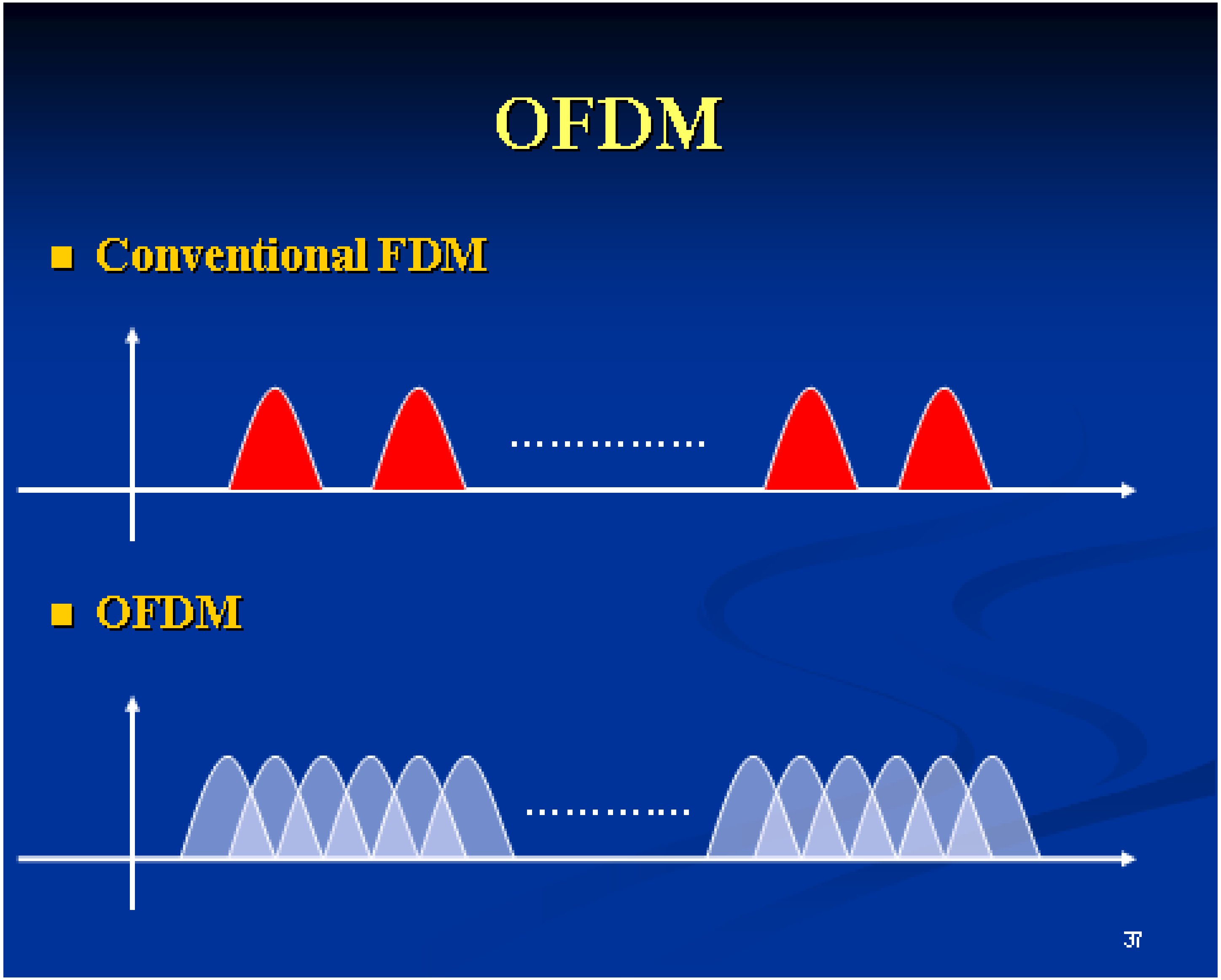
- Orthogonal frequency division multiplexing(OFDM) is a multicarrier modulation technique that has recently found widespread applications in high-data-rate communication systems ,such as digital subscriber lines (DSL) , wireless LAN (IEEE 802.11a/g/n), digital video broadcasting, and now WiMAX and other emerging wireless broadband systems (such as 3G LTE, 4G cellular systems)
- OFDM’s popularity for high data rate applications

  stems from its efficient and flexible management of ISI in highly dispersive channels.

Fig.11.1 Multicarrier system (a) transmitter (b) receiver



Conventional FDM signal vs. OFDM signal in frequency domain

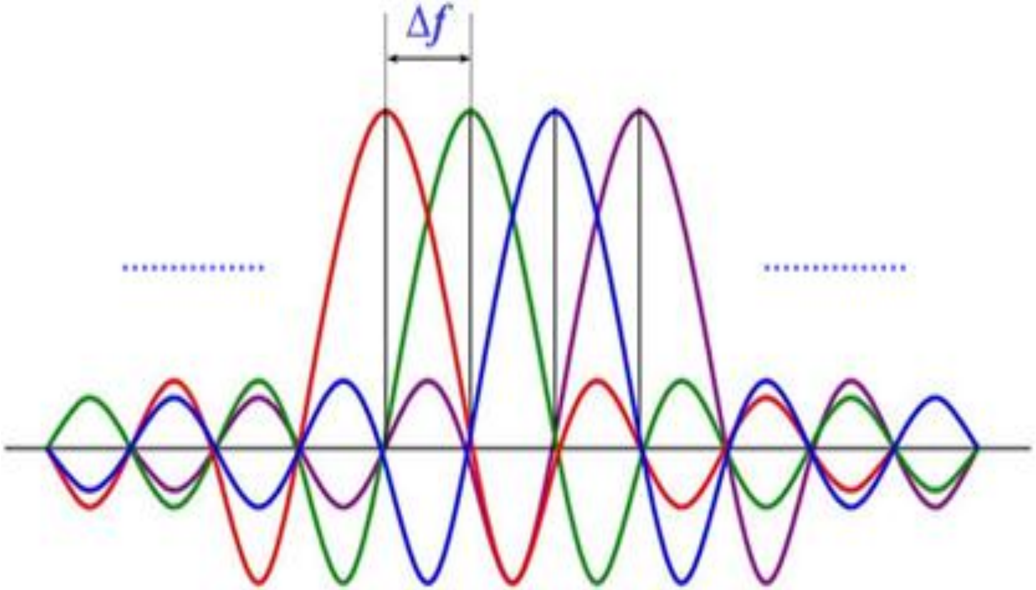


OFDM

- **Conventional FDM**

- **OFDM**

OFDM



11.2 OFDM and DFT

- **The use of discrete Fourier transform (DFT) to implement an OFDM system was proposed by Weistein and Ebert in 1971. The fast Fourier transform (FFT) algorithm can be employed to efficiently to compute DFT. The FFT and its inverse, the IFFT, can create a multitude of orthogonal subcarriers using a single carrier.**
- **DFT and IDFT**

The DFT transform a set of samples in the time domain into an equivalent set of samples in the frequency domain. The inverse discrete Fourier transform (IDFT) performs the reverse operation.

- **The transform pair is described as**

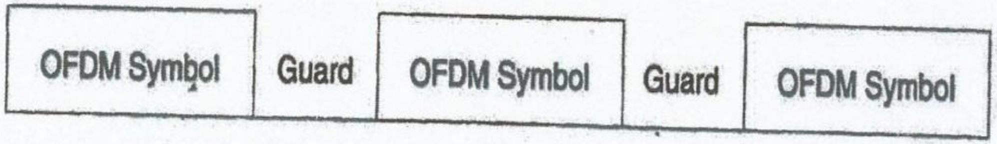
DFT : $$B_m = (1/\sqrt{N})\sum_{n=0}^{N-1} b_n \exp(-j2\pi mn/N),$$
$$l = 0,1,2,..,N-1$$

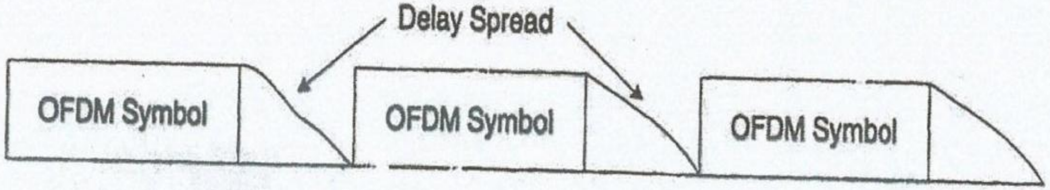
IDFT : $$b_n = (1/\sqrt{N})\sum_{m=0}^{N-1} B_m \exp(j2\pi ln/N),$$
$$n = 0,1,2,..,N-1 \qquad (11.1)$$

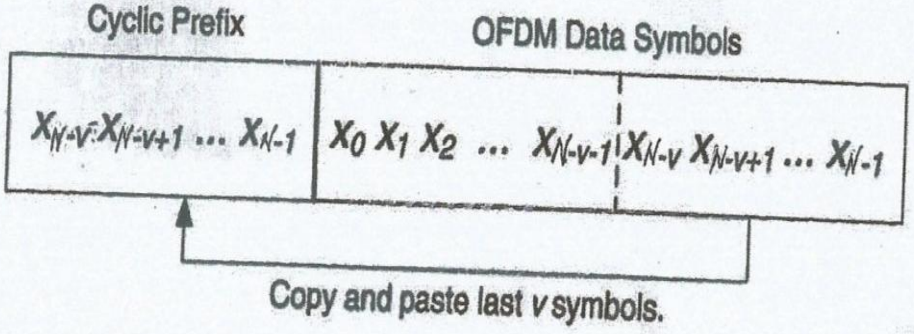
- **Fig.11.xx shows a block diagram of OFDM transmitter and receiver.**

11.3 Block Transmission with Guard Intervals

- **An OFDM symbol is obtained by grouping N data symbols. Each OFDM symbol lasts for a duration of T seconds , where $T = N\ T_s$.**
- **To keep each OFDM symbol independent of other after passing through a wireless channel , a guard interval between OFDM symbols is introduced.**
- **After receiving a series of OFDM symbols, as long as T_g is larger than τ , each OFDM symbol will interfere only with itself.**


OFDM Symbol
Guard
OFDM Symbol
Guard
OFDM Symbol


Delay Spread
OFDM Symbol
OFDM Symbol
OFDM Symbol


Cyclic Prefix
OFDM Data Symbols
$X_{N-v}\ X_{N-v+1} \dots X_{N-1}$
$X_0\ X_1\ X_2 \dots X_{N-v-1}$
$X_{N-v}\ X_{N-v+1} \dots X_{N-1}$
Copy and paste last v symbols.

11.4 Circular Convolution and DFT

- **The n-point circular convolution of $x[n]$ and $h[n]$ is defined as**

  $$y[n] = x[n] \star h[n] = h[n] \star x[n] = \Sigma_k h[k]\, x[n-k]_N \qquad (11.2)$$

  where $x[n-k]_N$ denotes $[n-k]$ modulo N. In other words, $x[n-k]_N$ is a periodic version of $x[n-k]$.

  It can be verified that if

  $$\text{DFT}\{y[n]\} = \text{DFT}\{h[n] \star x[n]\} \qquad (11.3)$$

  then $Y_m = H_m\, X_{\mathrm{m}}$, $\quad 0 \leqq m \leqq N-1 \qquad (11.4)$

  and $X_{\mathrm{m}} = Y_m / H_m \qquad (11.5)$

- **When an input data stream $x[n]$ is sent through a linear time-invariant Finite Impulse Response (FIR) channel $h[n]$, the output is the linear convolution of the input and the channel :**

$$y[n] = x[n] * h[n] = h[n] * x[n] = \Sigma_k h[k]\, x[n-k] \tag{11.6}$$

- **Thus, the channel output is not a circular convolution . However, the linear convolution can be turned into a circular convolution by adding a special prefix to the input called a cyclic prefix.**

11.5 Cyclic Prefix

- **Consider and a discrete –time channel with finite impulse response $h[n] = h_0, h_1, h_2, \ldots, h_v$, of length $v+1$**
- **The input sequence of length N is**

  $$x[n] = x_0, x_1, \ldots, x_{N-1}$$

- **The cyclic prefix of $x[n]$ is defined as $x_{N-v}, \ldots, x_{N-1}$.**

  It consists of the last v values of the $x[n]$ sequence .

  For each input sequence of length N, these last µsamples are appended to the beginning of the sequence.

  This yields a new sequence, $x_{cp}[n]$, of length $N+v$,

  $$0 \leqq v \leqq N-1,$$

  where $x_{cp}[n] = x_{-v}, \ldots, x_{N-1} = x_{N-v}, \ldots, x_{N-1}, x_0, \ldots, x_{N-1}$

- **It is noted that $v + 1 = \tau_d / T_s$, where τ_d is the channel delay spread and T_s is the sampling interval associate with the sequence.**

- **Suppose $x[n]$ is input to a discrete-time channel with impulse response $h[n]$. The channel output $y[n]$, $0 \leqq n \leqq N-1$, is then**

  $$y[n] = x_{cp}[n] \ast h[n] = \Sigma_{k=0}^{v} h[k]\, x_{cp}[n-k]$$

  $$= \Sigma_{k=0}^{v} h[k]\, x[n-k]_N$$

  $$= x[n] \star h[n] \qquad (11.7)$$

  Thus , by appending a cyclic prefix to the channel input , the linear convolution associated with the channel impulse response becomes a circular convolution.

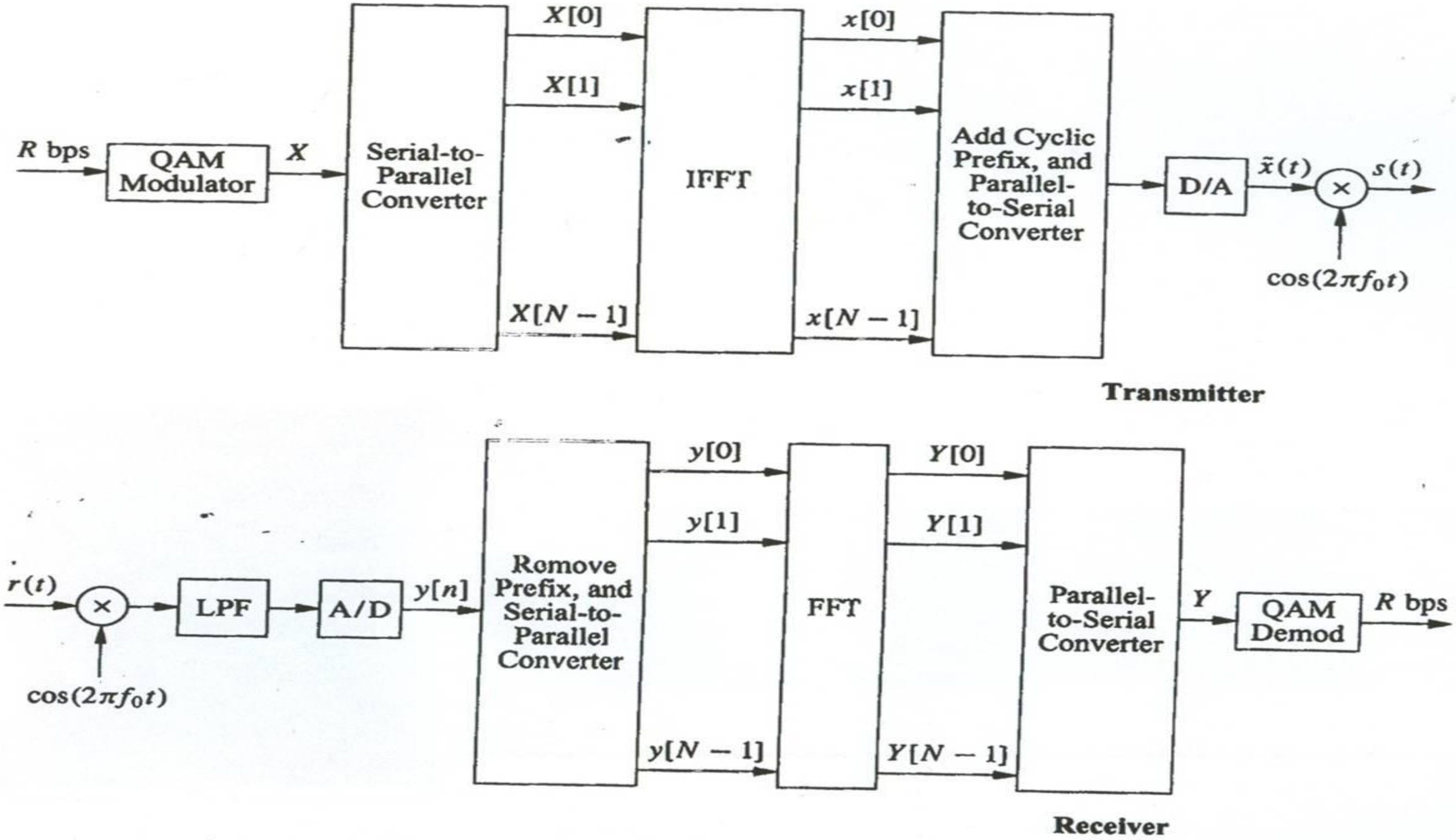
- **Taking the DFT of the channel output in the absence of noise then yields**

  $$Y_m = X_m H_m \qquad 0 \leqq i \leqq N-1$$

- **The input sequence $x[n]$ can be recovered from the channel output $y[n]$, for the known $h[n]$, by**

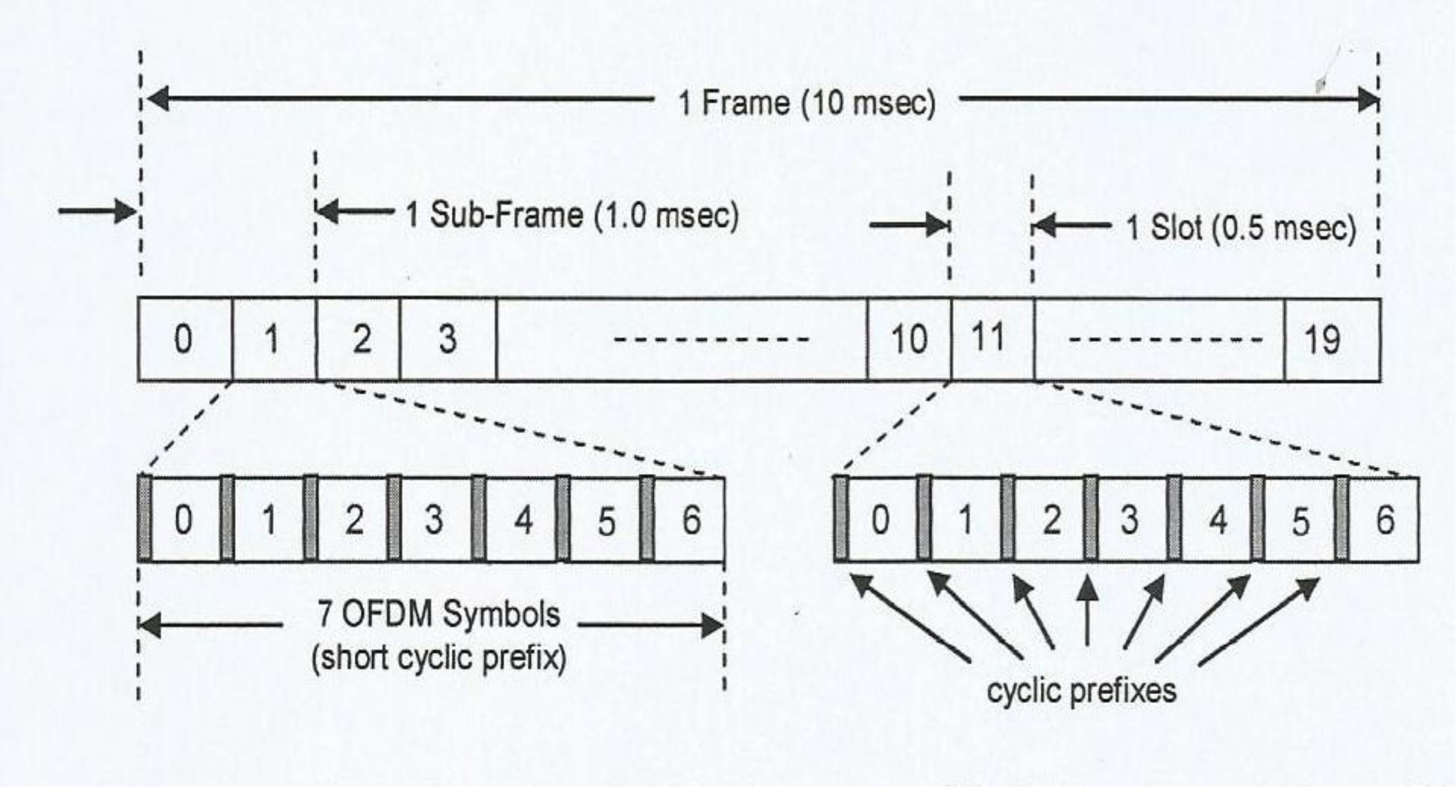
$$x[n] = \text{IDFT}\{Y_m / H_m\} \qquad (11.8)$$

11.6 OFDM Implementation

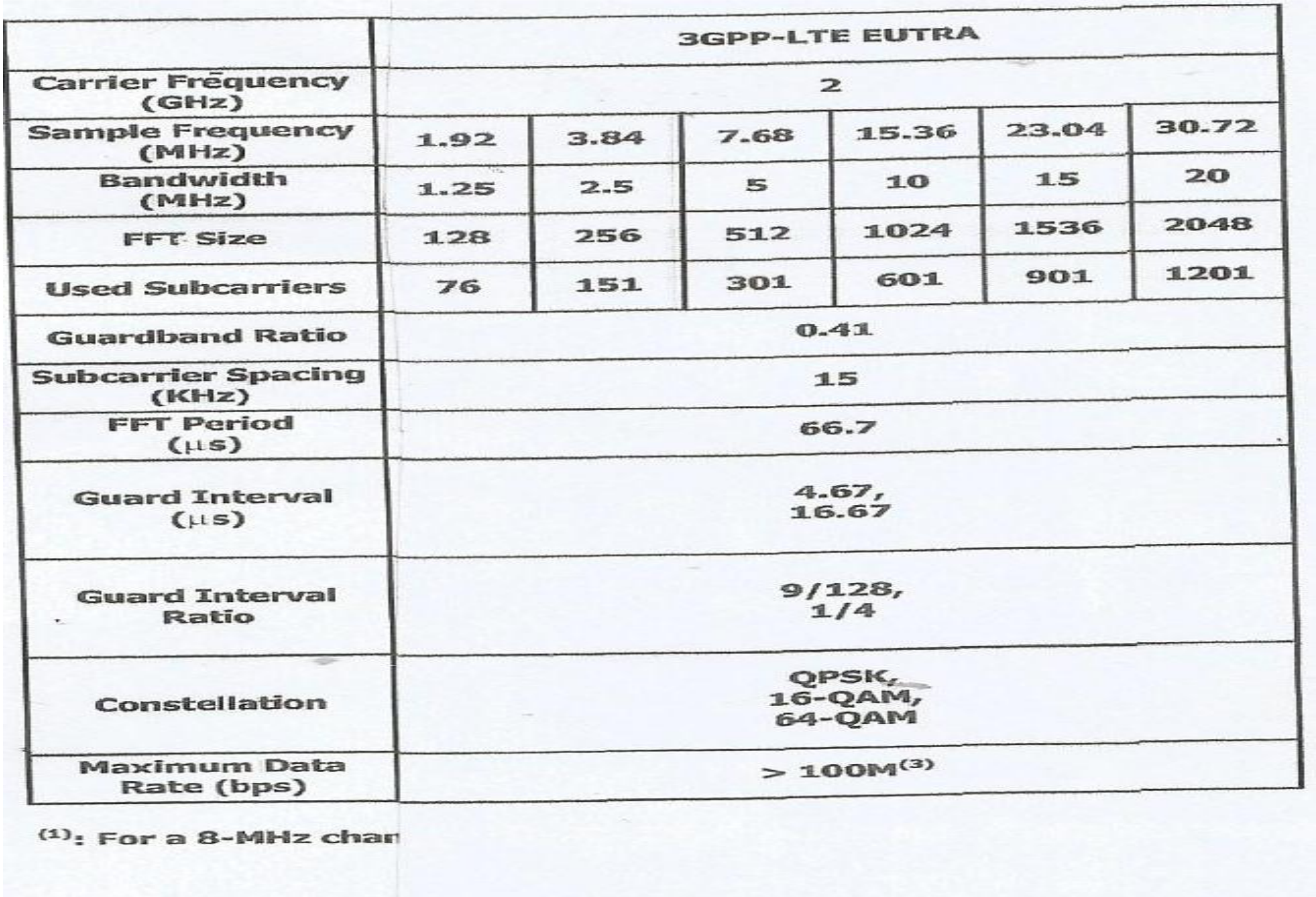


OFDM with IFFT/FFT implementation.

- **LTE genetic frame structure**



Example: OFDM frame in LTE



	3GPP-LTE EUTRA					
Carrier Frequency (GHz)	2					
Sample Frequency (MHz)	1.92	3.84	7.68	15.36	23.04	30.72
Bandwidth (MHz)	1.25	2.5	5	10	15	20
FFT Size	128	256	512	1024	1536	2048
Used Subcarriers	76	151	301	601	901	1201
Guardband Ratio	0.41					
Subcarrier Spacing (KHz)	15					
FFT Period (μs)	66.7					
Guard Interval (μs)	4.67, 16.67					
Guard Interval Ratio	9/128, 1/4					
Constellation	QPSK, 16-QAM, 64-QAM					
Maximum Data Rate (bps)	> 100M(3)					

(1): For a 8-MHz char

Example : OFDM frame for IEEE 802.11a Wireless LAN

- In 802.11a , $N = 64$ subcarriers :
  48 are actually used for data transmission , the outer 12 are zeroed in order to reduce adjacent channel interference , and 4 are used as pilot symbols for channel estimation.
- The cyclic prefix consists of $v = 16$ samples , so the total number of subcarriers associated with each OFDM symbol is 80.
- Coded bits are packetized , and the transmitter gets periodic feedback from the receiver about the packet error rate , which it uses to pick an appropriate error correction code and modulation technique. The same code and modulation must be used for all the subcarriers at any given time.
- The error correction code is a convolutional code with one of three possible code rates : $r = ½$, 2/3, or 3/4 .
- The modulation types that can be used on the subchannels are BPSK, QPSK, 16-QAM , or 64-QAM .

- **The total system bandwidth of 300 MHz is divided into 20-MHz channels that can be assigned to different users.**

  Each channel bandwidth B (and sample rate $1 / T_s$) is 20 MHz and since there are 64 subcarriers evenly spaced over that bandwidth , the subcarrier bandwidth is :

$$B_{sb} = 20 \text{ MHz} \ / \ 64 = 312.5 \ k\text{Hz}$$

Since $v = 16$ and $T_s = 0.05$ μsec ,

the maximum delay spread for which ISI is removed is

$$\tau_d < v \, T_s = 16 / (20 \text{ MHz}) = 0.8 \ \mu\text{sec}$$

which corresponds to delay spread in an indoor environment.

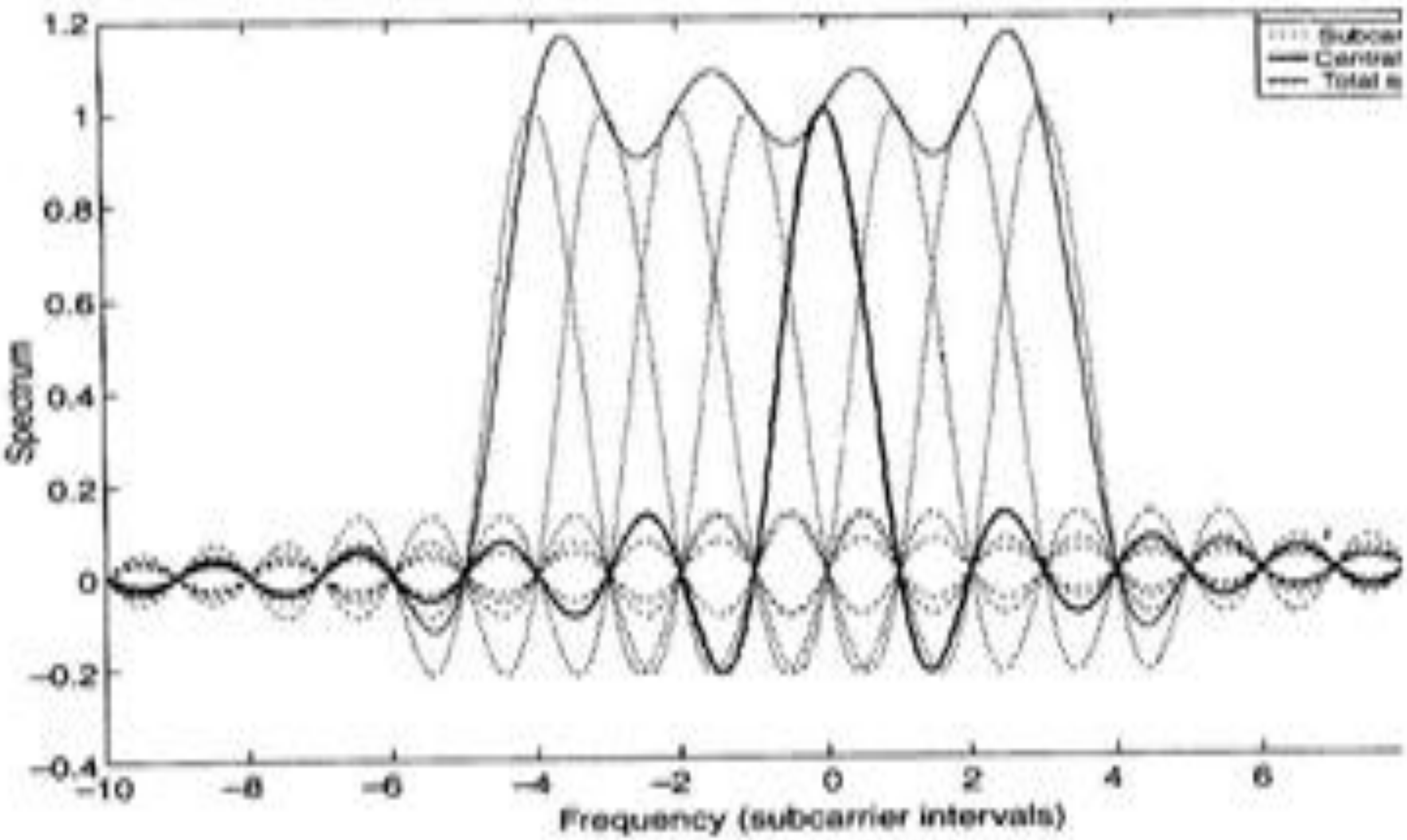
The symbol time per subchannel is equal to the OFDM symbol time , that is

$$T = 80 \, T_s = 80 / (20 \times 10^6) = 4 \ \mu\text{sec}$$

The data rate per channel is $(\log_2 M) / T_s$ for MQAM modulation

11.7 Peak-to-Average Power Ratio (PAPR)

- **A major problem with the multicarrier modulation in general and OFDM systems in particular is the high peak-to-average power ratio (PAPR) that is inherent in the transmitted signal. Large signal peaks occur in the transmitted signal when the signals in the *K* subchannels add constructively in phase .**
- **Such large signal peaks may saturate the power amplifier at the transmitter, thus, causing intermodulation distortion in the transmitted signal.**
- **Let us denote the collection of all data symbols X_n , $n = 0,1,\ldots,$ N-1 , as a vector $X = [\ X_0\ ,\ X_1\ ,\ \ldots,\ X_{N\text{-}1}\ ]^T$ that will be termed as data block.**



OFDM spectrum

The complex baseband representation of an OFDM signal of N subcarriers is given by

$$x(t) = (1/\sqrt{N}) \sum_{n=0}^{N-1} X_n \exp(j2\pi n \Delta f\, t) \quad ,0 \leqq t \leqq T \tag{11.11}$$

where Δf is the subcarrier spacing , and NT denotes the useful data block period . $\Delta f = 1/NT$.

The PAPR of the transmit signal is defined as

$$\text{PAPR} = \max_{0 \leqq t \leqq T} |x(t)|^2 \Big/ (1/NT) \int_0^{NT} |x(t)|^2\, dt$$

In discrete time signal processing, we usually oversample $x(t)$ by a factor of L ($L > 1$, say $L = 4$)

The " *J*-times oversampled" time-domain signal samples are represented as a vector

$$x' = [\, x'_0 , x'_1 , \ldots , x'_{NL-1} \,]$$

and obtained as

$$x'_k = x(kT/L)$$
$$= (1/\sqrt{N}) \Sigma_{n=0}^{N-1} X_n \exp(\, j2\pi kn \Delta f \; T/L \,),$$
$$k = 0,1,2,\ldots, NL-1 \qquad (11.13)$$

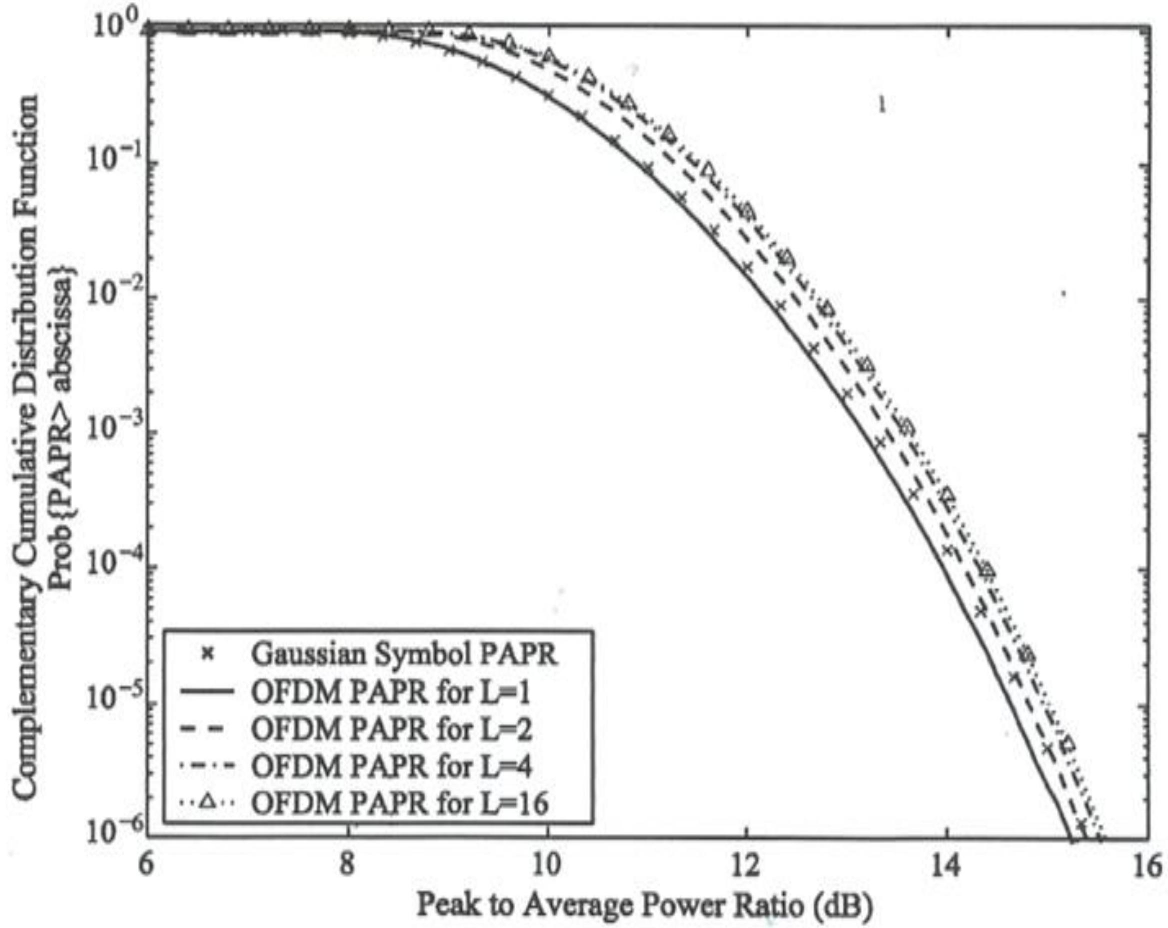
The PAPR computed from the *J*-times oversampled time-domain signal samples is then given by

$$\text{PAPR} = \max_{0 \leqq k \leqq NL-1} | x'_k |^2 \,/\, E | x'_k |^2$$

where E$|x'k|^2$ denotes the ensemble average of the of the samples.

The cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. In the literature , the complementary CDF (CCDF) is commonly used instead of CDF itself. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold , as illustrated in Fig.11.xx

There are several methods proposed to mitigating the PAPR problem. Some of the promising approaches are : clipping method, coding scheme , selective mapping, and partial transmitted sequence [references].


Complementary Cumulative Distribution Function
Prob{PAPR> abscissa}
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
6
8
10
12
14
16
Peak to Average Power Ratio (dB)
Gaussian Symbol PAPR
OFDM PAPR for L=1
OFDM PAPR for L=2
OFDM PAPR for L=4
OFDM PAPR for L=16

Amplitude Clipping and Filtering

The simplest technique for PARR reduction might be amplitude clipping. Amplitude clipping limits the peak envelope of the input signal to a predetermined value or otherwise passes the input signal through unperturbed , that is ,

$$B(x) = \begin{cases} x , & |x| \leqq A \\ A \exp[\, j\varphi(x)] , & |x| > A \end{cases} \qquad (11.15)$$

where A is preset clipping level.

$\varphi(x) = x / |x|$ **is the phase of** x **.**

Clipping method introduces both in-band distortion and out-of-band radiation into OFDM signal, which degrades system performance including BER and spectral efficiency.

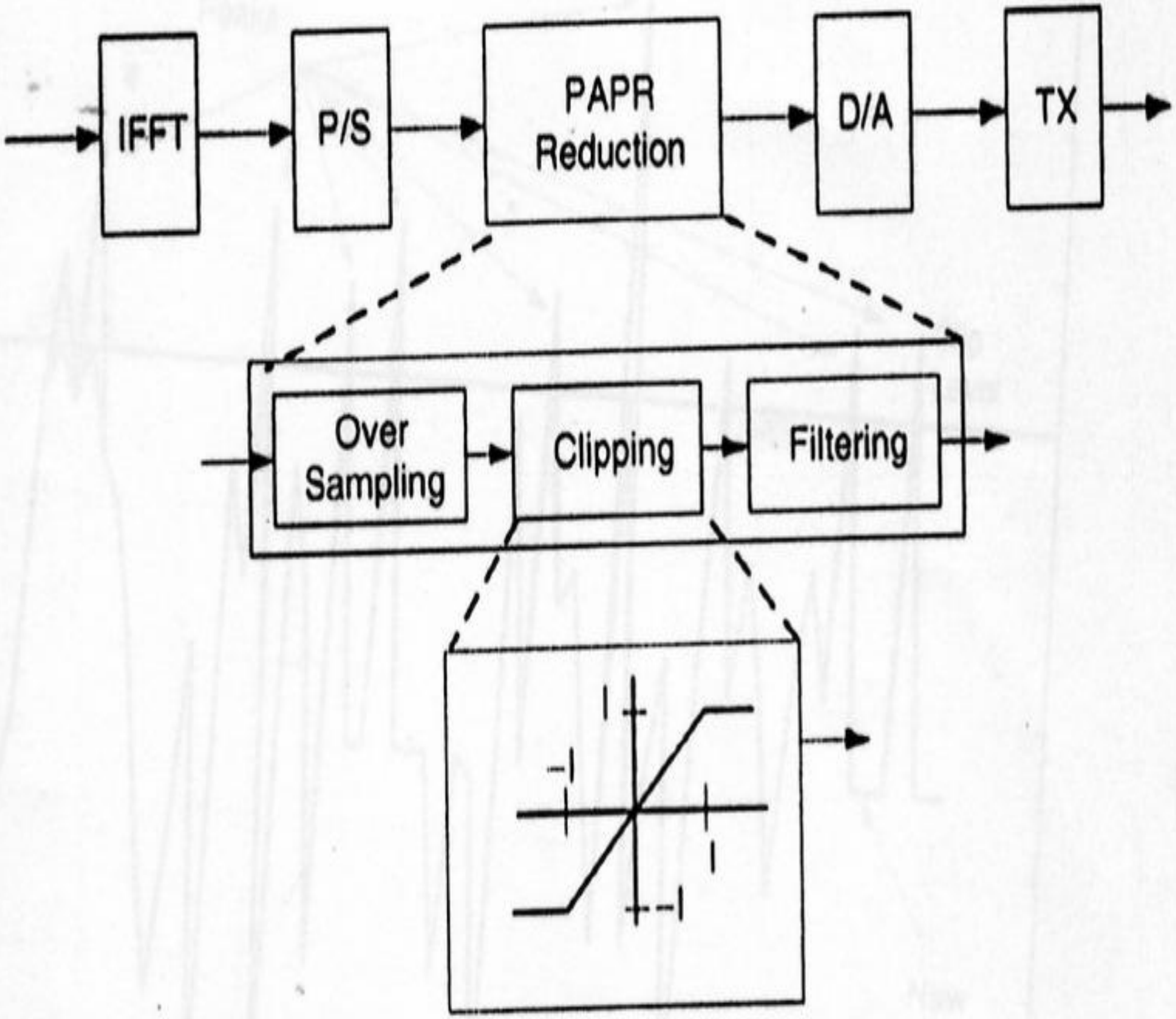
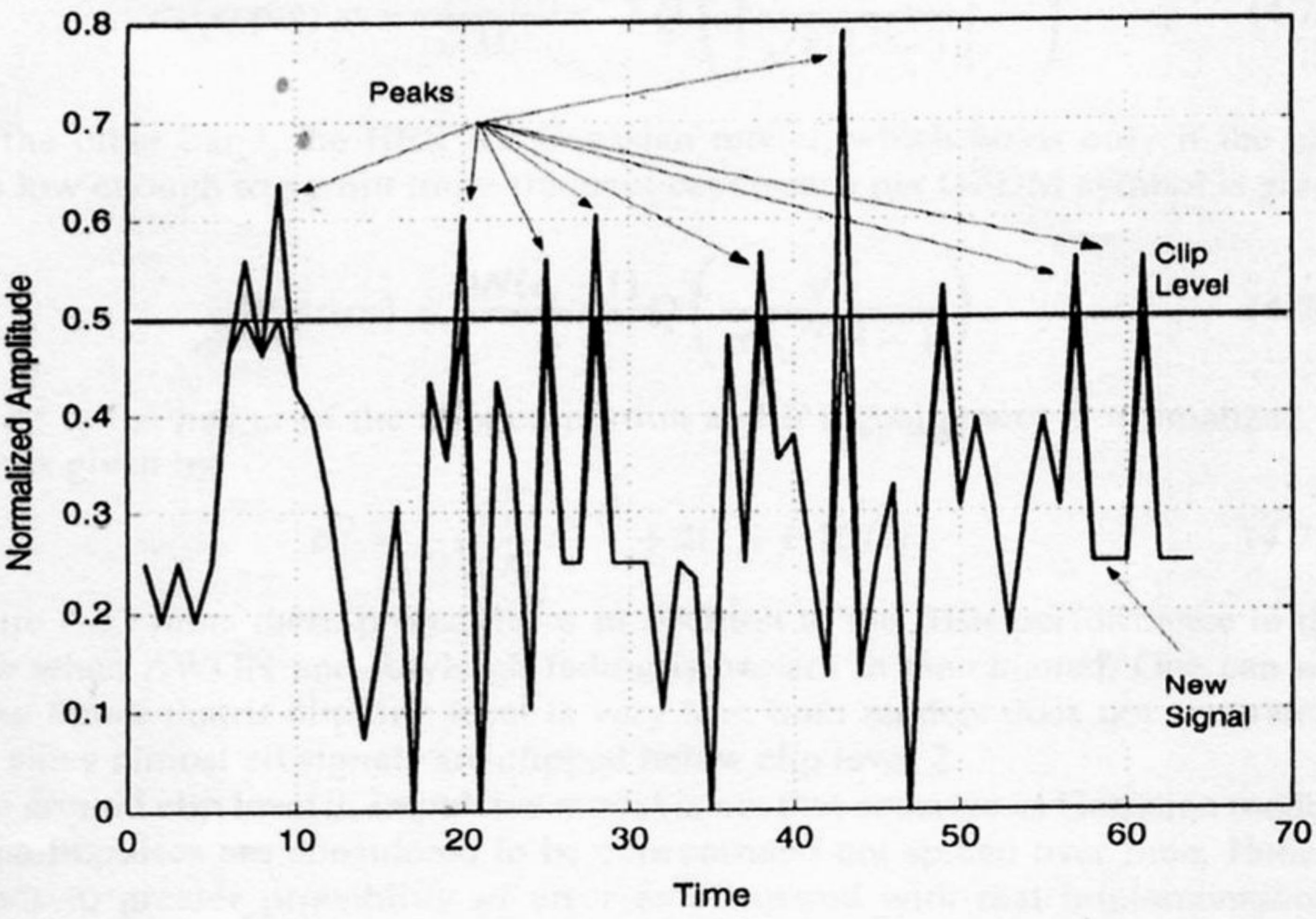
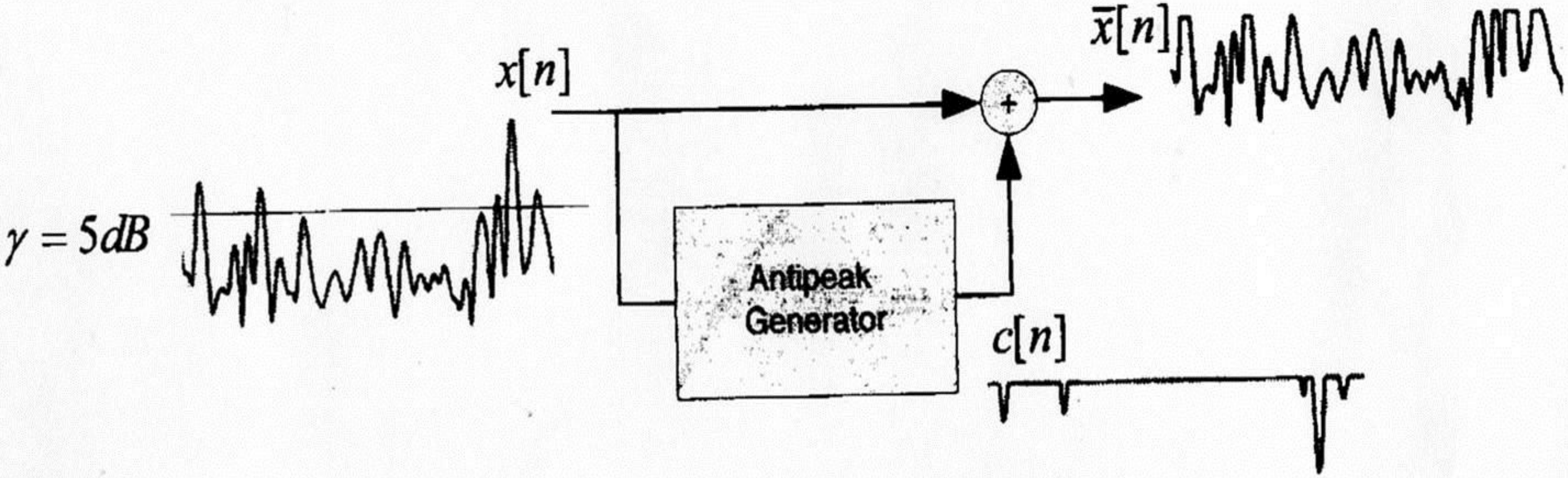


Fig. 4.51 Clipping and Filtering



ig. 4.52 Clipping



A peak cancellation as a model of soft limiter when $\gamma = 5dB$

Filtering can reduce out-of-band radiation after clipping although it can not reduce in-band distortion.

An alternate approach to minimize the out-of-band radiation is using peak windowing schemes such as Gaussian window or Kaiser window, etc. .

However, clipping may cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points (after digital analog conversion).

To reduce peak regrowth, a repeated clipping-and-filtering operation can be used to obtain a desirable PAPR at a cost of computational complexity increase.

Each iteration requires two FFT/IFFT operations ,and after the last iteration , one extra IFFT is required to convert the clipped OFDM symbol to time domain.

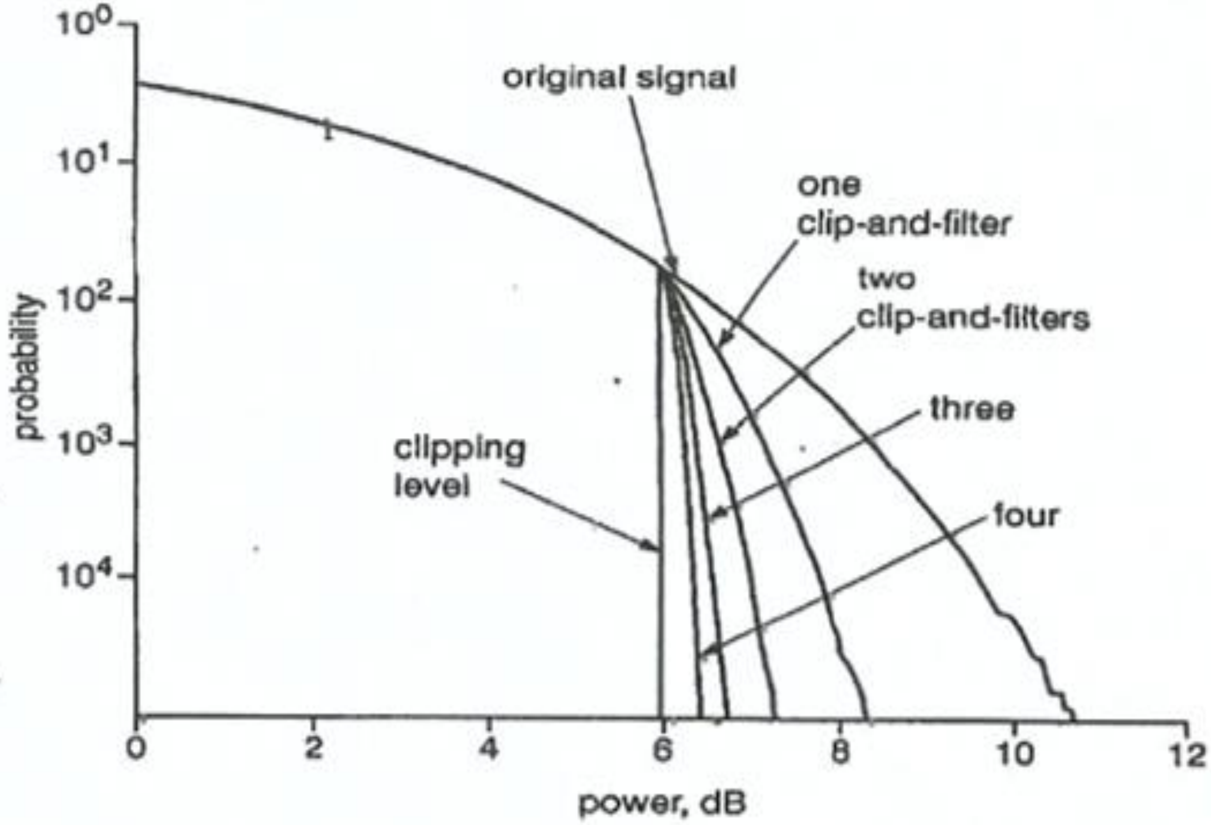


Fig. 2 *Cumulative distribution for signal after PAPR reduction*

11.8 Frequency Offset and Intercarrier Interference

In OFDM system, the frequency-selective channel is divided into many frequency-flat subchannels. The cyclic prefix is a kind of guard interval that effectively mitigates the effects of delay dispersion due to multipath fading. The cyclic prefix also provides an excellent way of ensuring orthogonality of the subcarriers in a frequency-selective slow fading environment. Thus, no intercarrier interference (ICI) due to frequency selectivity of the channel will incur.

However, in time-selective environment (e.g. wireless mobile channel), a frequency offset in one subcarrier may cause intercarrier interference (ICI) in many adjacent subcarriers (see Fig.10.xx). The frequency offset can be due to the Doppler shift , sampling clock offset, and/or the mismatch between the oscillator frequencies at transmitter and receiver.

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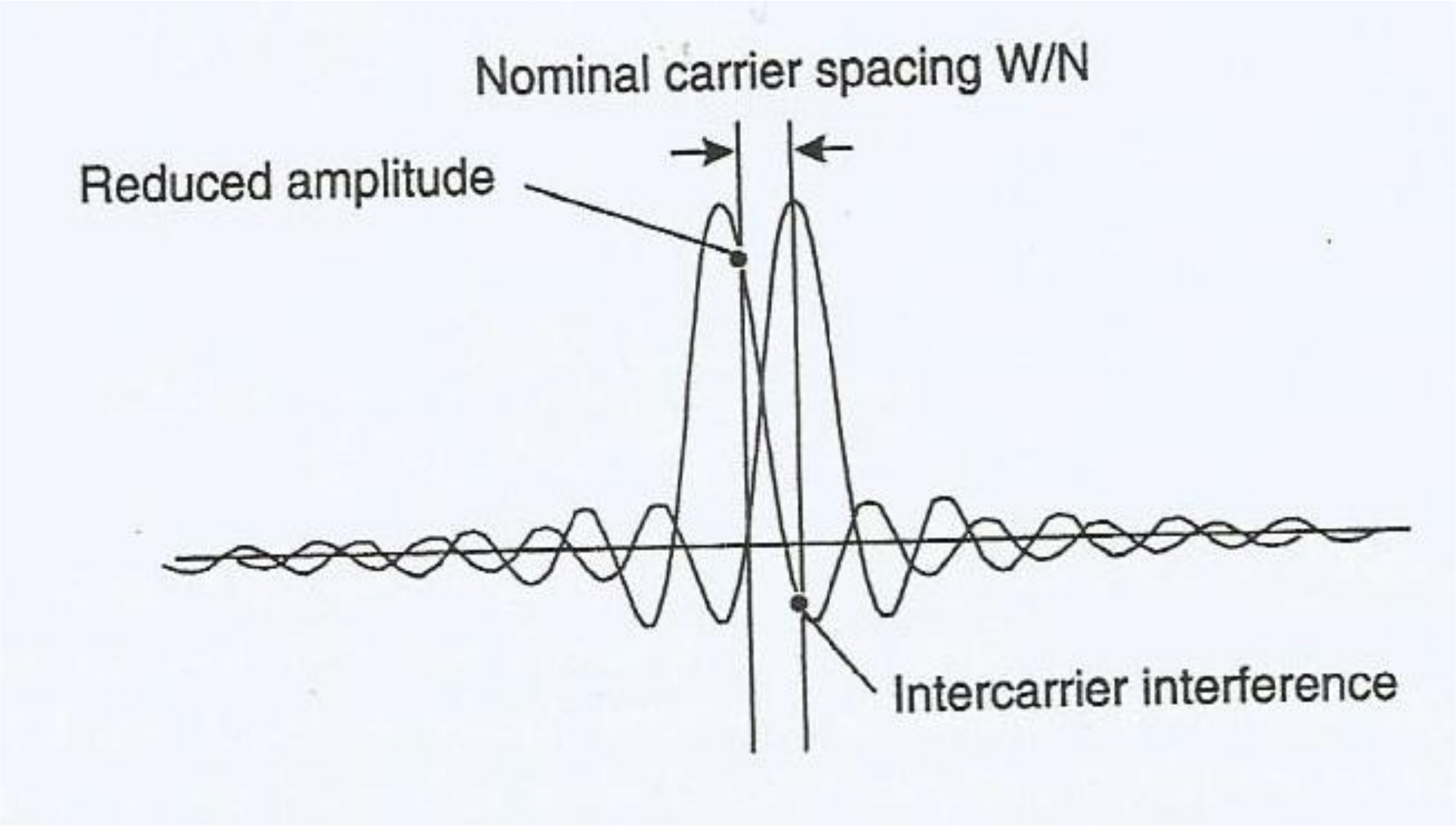

Nominal carrier spacing W/N
Reduced amplitude
Intercarrier interference

We are to consider an OFDM symbol consisting of $N+\nu$ samples. The *n-th* time-domain sample of the transmitted symbol can be expressed as

$$x_n = x^{\sim}(t)\ \Big|_{t = \nu Ts + n Ts} = x^{\sim}(\nu T_s + n T_s)$$

$$= (1/N)\ \Sigma_{k=N/2+1}^{N/2} X_k \exp(j2\pi nk/N)\ ,$$

$$n = -\nu,\ldots,0,1,\ldots,N-1$$

where $x^{\sim}(t)$ is the continuous-time signal $x^{\sim}(t)$ at the output of the D/A converter

The received signal, with a carrier frequency offset Δf, is in the form of

$$z_n = z(t) \exp(j2\pi\, \Delta f\ t)\ \Big|_{t = \nu Ts + n Ts} \qquad (11.37)$$

where

$$z(t) = h(t) * x(t) + w(t) \qquad (11.35)$$

and $h(t)$ is the impulse response of the channel.

- **The received frequency-domain signal then becomes**

$$Z_k = (X_k H_k) \; [\sin(\pi \varepsilon) \,/\, N \sin(\pi\varepsilon/N)] \; \exp\{ j2\pi\nu \varepsilon / N \} \; \exp\{ j \pi\varepsilon (N-1) /N \} + \text{ICI} + V_k \qquad (10.38)$$

where V_k is the channel noise component in the *k-th* subcarrier , and ε is the relative frequency offset (the ratio of the actual frequency to intercarrier spacing), i.e. $\varepsilon = \Delta f \,/\, (1/NT_s)$

The intercarrier Interference (ICI) is given by

$$\text{ICI} = \sum_{\substack{-N/2+1 \\ l \neq k}}^{N/2} X_l H_l \; \{ \sin[\pi(\varepsilon + l - k)] / N \sin[\pi(\varepsilon + l - k)/N] \} \; \exp\{ j (2\pi \nu \varepsilon /N) \} \; \exp\{ j [\pi (\varepsilon + l - k) (N - 1) / N] \} \qquad (10.39)$$

- **The carrier frequency offset ,ε , results in attenuation in magnitude , phase shift and ICI in the received frequency domain signals.**

  Note that the phase shift is identical in every subcarrier and is also proportional to the symbol index *i* .

11.9 OFDMA

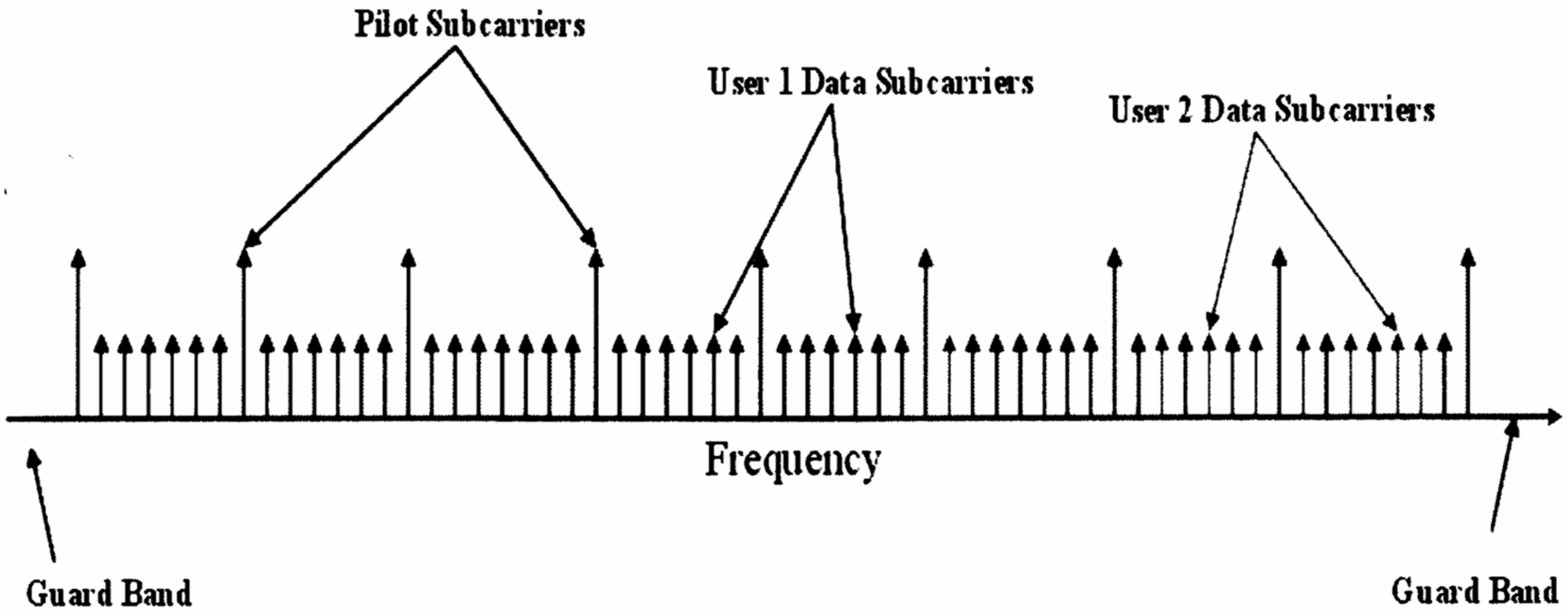
- **OFDMA is a multi-user version of the OFDM digital modulation scheme. Multiple access is achieved in OFDMA by assigning subsets of subcarriers to individual users as shown above.**

  This allows simultaneous low data rate transmission from several users.

- **Based on feedback information about the channel conditions, adaptive user-to-subcarrier assignment can be achieved.**

  If the assignment is done sufficiently fast , this further improves the OFDM robustness to fast fading and narrow-band cochannel interference.

- **OFDMA can also be seen as an alternative to combining OFDM with time-division multiple access .**


Pilot Subcarriers
User 1 Data Subcarriers
User 2 Data Subcarriers
Frequency
Guard Band
Guard Band

11.10 SC-FDMA

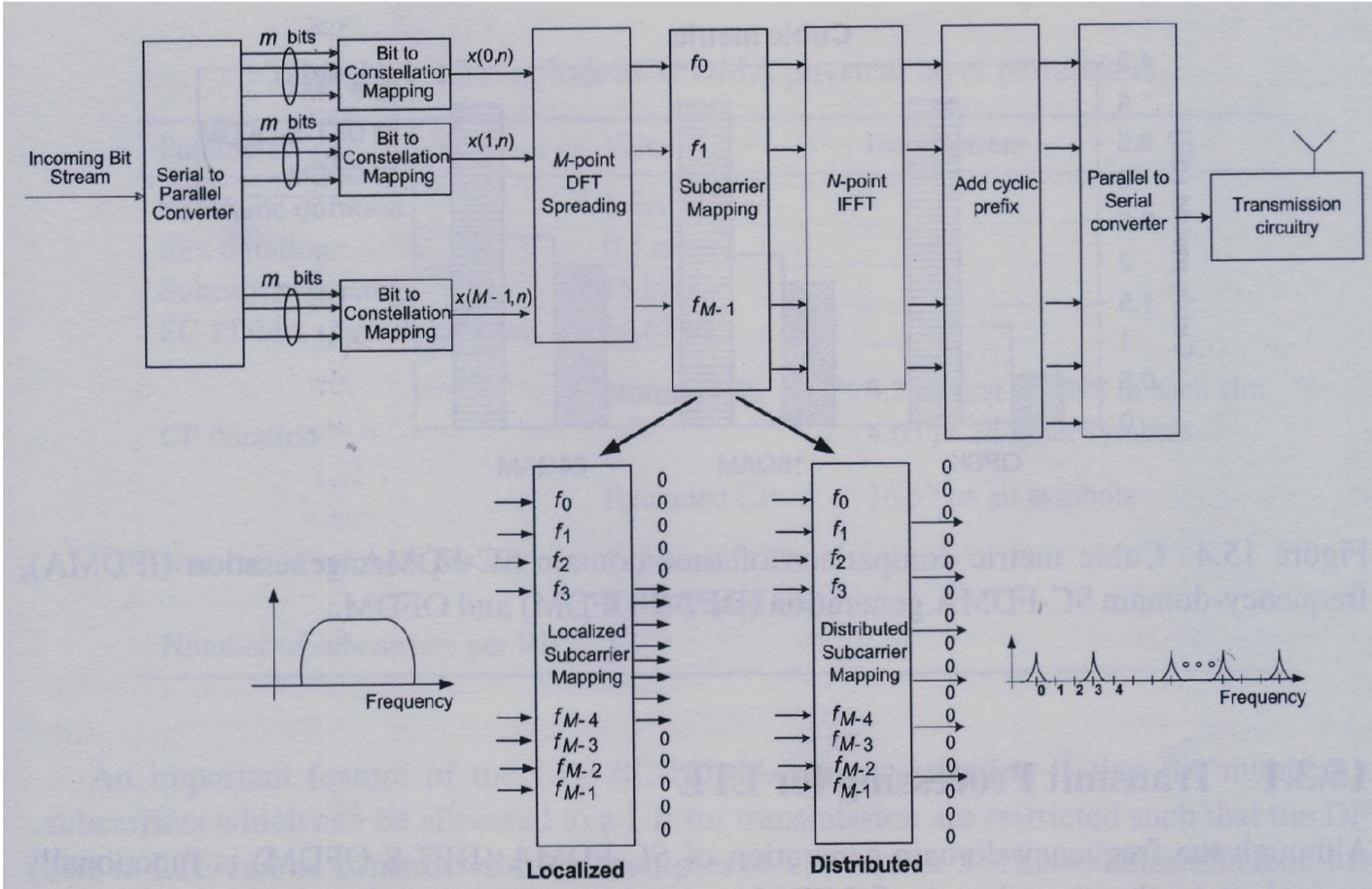
- Single-carrier FDMA (SC-FDMA) combines the desirable characteristics of OFDM with the low PAPR of single-carrier transmission schemes. Just like in OFDM , SC-FDMA divides the transmission bandwidth into multiple parallel subcarriers, with the orthogonality between the subcarriers being maintained in frequency-selective fading channel by the use of cyclic prefix or guard period .
- The main objective of SC-FDMA is to introduce transmission with lower PAPR than OFDMA. Since OFDMA shows envelope fluctuations, and signal with high PAPR requires highly linear power amplifiers to reduce the distortion. The design of mobile terminals are complex and they become power hungry.
-

- **In OFDM, FFT is applied on the receiver side on each block of symbols, and IFFT on the transmitter side.**

  In SC-FDMA, both FFT and IFFT are applied on the transmitter side, and also on the receiver side.
- **In OFDM , equalization is achieved on the receiver side after the FFT calculation, by multiplying each Fourier coefficient by a complex number. Thus, frequency-selective fading and phase distortion can be combated by utilizing a simple equalizer per subcarrier after FFT. But ,SC-FDMA utilizes complex a complex equalizer before sending the resultant to IFFT. IFFT remove the effect of the FFT at the transmitter.**
- **In SC-FDMA, multiple access is made possible by inserting silent Fourier-coefficients on the transmitter side before the IFFT, and removing them on the receiver side before the IFFT. Different users are assigned to different Fourier-coefficients (sub-carriers).**

- **SC-FDMA first runs an FFT over the groups of input bits to spread them over all sub-carriers and then uses the result for the IDFT which creates the time signal. This is why SC-FDMA is sometimes also referred to as FFT spread OFDM.**
- **While SC-FDMA adds additional complexity at both the transmitter and receiver side, the 3GPP standardization body has nevertheless decided for it because of its useful properties : low PAPR and low sensitivity to carrier frequency offset .**

SC-FDMA frequency domain transmit processing



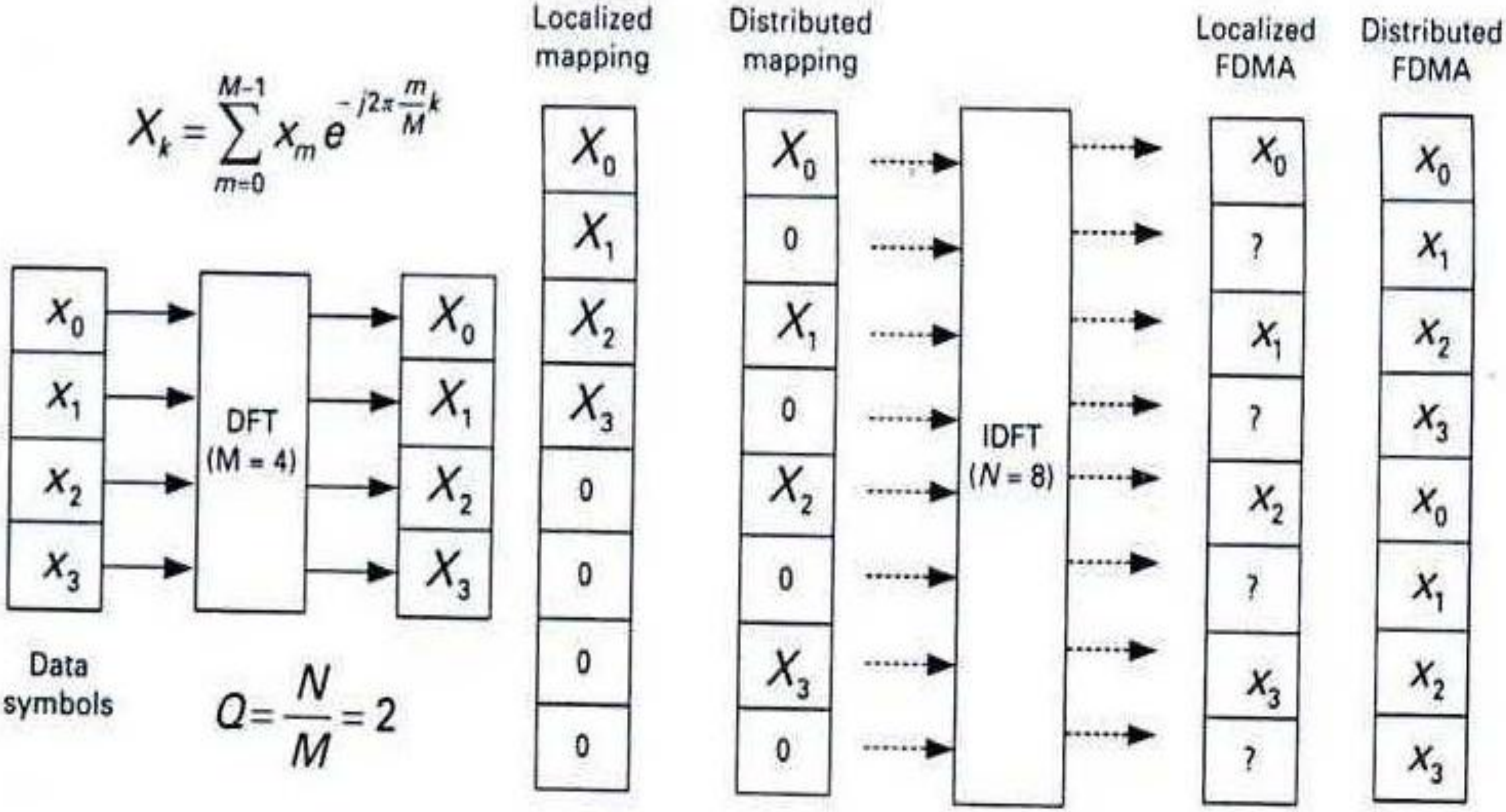


Figure 4.1. Subcarrier mapping for Localized and Distributed FDMA.

- The N subcarriers of the user into M subcarrier is mapped in either distributed or localized manner , as illustrared in Fig. xx.

  Distributed mapping (. also denoted as interleaved FDMA or IFDMA)

  Localized mapping maps subcarriers allocated to user adjacent to each other.

Historical background

1966 R.W Chang shows that multicarrier modulation can solve the multipath problem without reducing data rate. This is generally considered the first official publication on multicarrier modulation.

R.W.Chang"Synthesis *of bandlimited orthogonal signals for multichannel data transmission" Bell Systems Technical Journal, 45:1775-1796, December 1966.*

1971 Weinstein and Ebert show that multicarrier modulation can be accomplished using DFT.

S.Weinstein and P. Ebert," Data transmission by frequency-division multiplexing using the discrete Fourier transform", IEEE. Trans. Communications,19(5): 628- 634, October 1971.

1985 Cimini at Bell Labs identifies many of the key issues in OFDM transmission and does a proof- of-concept design.

L.J.Cimini,"Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing", IEEE Trans. Communications,33(7):665-675, July, 1985

1993 DSL adopts OFDM, also called discrete multitone,following successful field trials/competitions at Bellcore versus equalizer-based systems.

1999 The IEEE 802.11 committee on wireless LANs releases the 802.11a standard for OFDM operation in 5GHz UNI band.