Probability and Statistics HW1

Issue day at 3/7
Due day at 3/14

1. Fig. 1 illustrates a switch system which contains 5 switches. And each switch is turned on(connect) or off(disconnect) randomly and independently. Supposed that switch(A), switch(B) , switch(C) , switch(D) , switch(E) is turned on with the probability 0.8, 0.9, 0.9, 0.7, and 0.8, respectively.

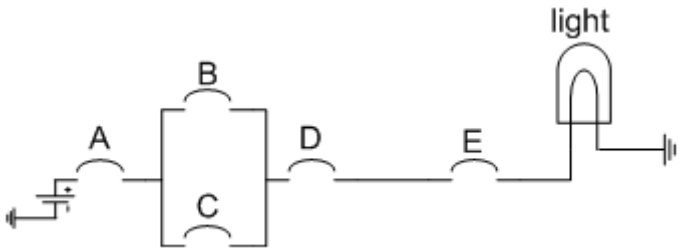

Fig. 1

(a). Find the probability that the light will be turned on

(b). If the light is turned off, which switch is most likely to be turned off?

Suppose we know that there is only one switch which is turned off.

2. Let A and B be two sets.

(a). Show that

$$A^C = \left(A^C \cap B\right) \cup \left(A^C \cap B^C\right), \quad B^C = \left(A \cap B^C\right) \cup \left(A^C \cap B^C\right)$$

(b). Show that

$$\left(A \cap B\right)^C = \left(A^C \cap B\right) \cup \left(A^C \cap B^C\right) \cup \left(A \cap B^C\right)$$

(c). Consider rolling a fair six-sided die. Let ***A*** be the set of outcomes where the roll is an odd number. Let ***B*** be the set of outcomes where the roll is less than 4. Calculate the sets on both sides of the equality in part (b), and verify that the equality holds.

(From textbook chap1. Problem 2)

3. A partition of the sample space Ω is a collection of disjoint events $S_1,\ldots,S_n$ such that $\Omega=\bigcup_{i=1}^{n} S_i$.

   (a). Show that for any event ***A***, we have

$$P(A)=\sum_{i=1}^{n} P(A\cap S_i)$$

   (b).Use part (a) to show that for any events ***A***, ***B***, and ***C***, we have

$$P(A)=P(A\cap B)+P(A\cap C)+P(A\cap B^C\cap C^C)-P(A\cap B\cap C)$$

   (From textbook chap1. Problem 9)

4. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

   (a). Find the probability that doubles are rolled (rolls the same outcome)

   (b).Given that the roll results in a sum of 4 or less, find the conditional Probability that doubles are rolled.

   (c). Find the probability that at least one die roll is a 6.

   (d). Given that two dice land on different numbers, find the conditional probability that at least one die roll is a 6.

   (From textbook chap1. Problem 14)