C.1 *Basic Discrete-Time Fourier Series Pairs*

Time Domain	*Frequency Domain*
$x[n] = \sum_{k=0}^{N-1} X[k] e^{jkn\Omega_o}$ $Period = N$	$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jkn\Omega_o}$ $\Omega_o = \frac{2\pi}{N}$
$x[n] = \begin{cases} 1, & \lvert n \rvert \le M \\ 0, & M < \lvert n \rvert \le N/2 \end{cases}$ $x[n] = x[n+N]$	$X[k] = \frac{\sin\left(k\frac{\Omega_o}{2}(2M+1)\right)}{N\sin\left(k\frac{\Omega_o}{2}\right)}$
$x[n] = e^{jp\Omega_o n}$	$X[k] = \begin{cases} 1, & k = p, p \pm N, p \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
$x[n] = \cos(p\Omega_o n)$	$X[k] = \begin{cases} \frac{1}{2}, & k = \pm p, \pm p \pm N, \pm p \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
$x[n] = \sin(p\Omega_o n)$	$X[k] = \begin{cases} \frac{1}{2j}, & k = p, p \pm N, p \pm 2N, \dots \\ \frac{-1}{2j}, & k = -p, -p \pm N, -p \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
$x[n] = 1$	$X[k] = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
$x[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$X[k] = \frac{1}{N}$

C.2 *Basic Fourier Series Pairs*

Time Domain	*Frequency Domain*
$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_o t}$ $Period = T$	$X[k] = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_o t}\,dt$ $\omega_o = \frac{2\pi}{T}$
$x(t) = \begin{cases} 1, & \lvert t \rvert \le T_o \\ 0, & T_o < \lvert t \rvert \le T/2 \end{cases}$	$X[k] = \frac{\sin(k\omega_o T_o)}{k\pi}$
$x(t) = e^{jp\omega_o t}$	$X[k] = \delta[k - p]$
$x(t) = \cos(p\omega_o t)$	$X[k] = \frac{1}{2}\delta[k-p] + \frac{1}{2}\delta[k+p]$
$x(t) = \sin(p\omega_o t)$	$X[k] = \frac{1}{2j}\delta[k-p] - \frac{1}{2j}\delta[k+p]$
$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$X[k] = \frac{1}{T}$

C.3 *Basic Discrete-Time Fourier Transform Pairs*

Time Domain	*Frequency Domain*
$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\,d\Omega$	$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$
$x[n] = \begin{cases} 1, & \|n\| \le M \\ 0, & \text{otherwise} \end{cases}$	$X(e^{j\Omega}) = \frac{\sin\left[\Omega\left(\frac{2M+1}{2}\right)\right]}{\sin\left(\frac{\Omega}{2}\right)}$
$x[n] = \alpha^n u[n], \quad \|\alpha\| < 1$	$X(e^{j\Omega}) = \frac{1}{1-\alpha e^{-j\Omega}}$
$x[n] = \delta[n]$	$X(e^{j\Omega}) = 1$
$x[n] = u[n]$	$X(e^{j\Omega}) = \frac{1}{1-e^{-j\Omega}} + \pi\sum_{p=-\infty}^{\infty}\delta(\Omega - 2\pi p)$
$x[n] = \frac{1}{\pi n}\sin(Wn), \quad 0 < W \le \pi$	$X(e^{j\Omega}) = \begin{cases} 1, & \|\Omega\| \le W \\ 0, & W < \|\Omega\| \le \pi \end{cases}$ $X(e^{j\Omega})$ is 2π periodic
$x[n] = (n+1)\alpha^n u[n]$	$X(e^{j\Omega}) = \frac{1}{(1-\alpha e^{-j\Omega})^2}$

C.4 *Basic Fourier Transform Pairs*

Time Domain	*Frequency Domain*
$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$
$x(t) = \begin{cases} 1, & \|t\| \le T_o \\ 0, & \text{otherwise} \end{cases}$	$X(j\omega) = \frac{2\sin(\omega T_o)}{\omega}$
$x(t) = \frac{1}{\pi t}\sin(Wt)$	$X(j\omega) = \begin{cases} 1, & \|\omega\| \le W \\ 0, & \text{otherwise} \end{cases}$
$x(t) = \delta(t)$	$X(j\omega) = 1$
$x(t) = 1$	$X(j\omega) = 2\pi\delta(\omega)$
$x(t) = u(t)$	$X(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$
$x(t) = e^{-at}u(t), \quad \mathrm{Re}\{a\} > 0$	$X(j\omega) = \frac{1}{a + j\omega}$
$x(t) = te^{-at}u(t), \quad \mathrm{Re}\{a\} > 0$	$X(j\omega) = \frac{1}{(a + j\omega)^2}$
$x(t) = e^{-a\|t\|}, \quad a > 0$	$X(j\omega) = \frac{2a}{a^2 + \omega^2}$
$x(t) = \frac{1}{\sqrt{2\pi}}e^{-t^2/2}$	$X(j\omega) = e^{-\omega^2/2}$

C.7 *Properties of Fourier Representations*

Property	Fourier Transform $x(t) \xleftrightarrow{FT} X(j\omega)$ $y(t) \xleftrightarrow{FT} Y(j\omega)$	Fourier Series $x(t) \xleftrightarrow{FS;\,\omega_o} X[k]$ $y(t) \xleftrightarrow{FS;\,\omega_o} Y[k]$ Period $= T$
Linearity	$ax(t) + by(t) \xleftrightarrow{FT} aX(j\omega) + bY(j\omega)$	$ax(t) + by(t) \xleftrightarrow{FS;\,\omega_o} aX[k] + bY[k]$
Time shift	$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o}X(j\omega)$	$x(t - t_o) \xleftrightarrow{FS;\,\omega_o} e^{-jk\omega_o t_o}X[k]$
Frequency shift	$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$	$e^{jk_o\omega_o t}x(t) \xleftrightarrow{FS;\,\omega_o} X[k - k_o]$
Scaling	$x(at) \xleftrightarrow{FT} \frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$	$x(at) \xleftrightarrow{FS;\,a\omega_o} X[k]$
Differentiation in time	$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$	$\frac{d}{dt}x(t) \xleftrightarrow{FS;\,\omega_o} jk\omega_o X[k]$
Differentiation in frequency	$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(j\omega)$	—
Integration/ Summation	$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{FT} \frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$	—
Convolution	$\int_{-\infty}^{\infty} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FT} X(j\omega)Y(j\omega)$	$\int_{0}^{T} x(\tau)y(t - \tau)\,d\tau \xleftrightarrow{FS;\,\omega_o} TX[k]Y[k]$
Multiplication	$x(t)y(t) \xleftrightarrow{FT} \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\nu)Y(j(\omega - \nu))\,d\nu$	$x(t)y(t) \xleftrightarrow{FS;\,\omega_o} \sum_{l=-\infty}^{\infty} X[l]Y[k - l]$
Parseval's Theorem	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$	$\frac{1}{T}\int_{0}^{T} \lvert x(t)\rvert^2\,dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$
Duality	$X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$
Symmetry	$x(t)$ real $\xleftrightarrow{FT} X^*(j\omega) = X(-j\omega)$ $x(t)$ imaginary $\xleftrightarrow{FT} X^*(j\omega) = -X(-j\omega)$ $x(t)$ real and even $\xleftrightarrow{FT} \text{Im}\{X(j\omega)\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FT} \text{Re}\{X(j\omega)\} = 0$	$x(t)$ real $\xleftrightarrow{FS;\,\omega_o} X^*[k] = X[-k]$ $x(t)$ imaginary $\xleftrightarrow{FS;\,\omega_o} X^*[k] = -X[-k]$ $x(t)$ real and even $\xleftrightarrow{FS;\,\omega_o} \text{Im}\{X[k]\} = 0$ $x(t)$ real and odd $\xleftrightarrow{FS;\,\omega_o} \text{Re}\{X[k]\} = 0$

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C.7 *(continued)*

Property	Discrete-Time FT $x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $y[n] \xleftrightarrow{DTFT} Y(e^{j\Omega})$	Discrete-Time FS $x[n] \xleftrightarrow{DTFS;\,\Omega_o} X[k]$ $y[n] \xleftrightarrow{DTFS;\,\Omega_o} Y[k]$ Period = N
Linearity	$ax[n] + by[n] \xleftrightarrow{DTFT} aX(e^{j\Omega}) + bY(e^{j\Omega})$	$ax[n] + by[n] \xleftrightarrow{DTFS;\,\Omega_o} aX[k] + bY[k]$
Time shift	$x[n - n_o] \xleftrightarrow{DTFT} e^{-j\Omega n_o}X(e^{j\Omega})$	$x[n - n_o] \xleftrightarrow{DTFS;\,\Omega_o} e^{-jk\Omega_o n_o}X[k]$
Frequency shift	$e^{j\Gamma n}x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega-\Gamma)})$	$e^{jk_o\Omega_o n}x[n] \xleftrightarrow{DTFS;\,\Omega_o} X[k - k_o]$
Scaling	$x_z[n] = 0, \quad n \neq 0, \pm p, \pm 2p, \pm 3p, \ldots$ $x_z[pn] \xleftrightarrow{DTFT} X_z(e^{j\Omega/p})$	$x_z[n] = 0, \quad n \neq 0, \pm p, \pm 2p, \pm 3p, \ldots$ $x_z[pn] \xleftrightarrow{DTFS;\,p\Omega_o} pX_z[k]$
Differentiation in time	—	—
Differentiation in frequency	$-jnx[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega}X(e^{j\Omega})$	—
Integration/ Summation	$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{DTFT} \frac{X(e^{j\Omega})}{1 - e^{-j\Omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$	—
Convolution	$\sum_{l=-\infty}^{\infty} x[l]y[n - l] \xleftrightarrow{DTFT} X(e^{j\Omega})Y(e^{j\Omega})$	$\sum_{l=0}^{N-1} x[l]y[n - l] \xleftrightarrow{DTFS;\,\Omega_o} NX[k]Y[k]$
Multiplication	$x[n]y[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Gamma})Y(e^{j(\Omega-\Gamma)})\,d\Gamma$	$x[n]y[n] \xleftrightarrow{DTFS;\,\Omega_o} \sum_{l=0}^{N-1} X[l]Y[k - l]$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\Omega})\rvert^2\,d\Omega$	$\frac{1}{N}\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \sum_{k=0}^{N-1} \lvert X[k]\rvert^2$
Duality	$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$	$X[n] \xleftrightarrow{DTFS;\,\Omega_o} \frac{1}{N}x[-k]$
Symmetry	$x[n]$ real $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = X(e^{-j\Omega})$ $x[n]$ imaginary $\xleftrightarrow{DTFT} X^*(e^{j\Omega}) = -X(e^{-j\Omega})$ $x[n]$ real and even $\xleftrightarrow{DTFT} \mathrm{Im}\{X(e^{j\Omega})\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFT} \mathrm{Re}\{X(e^{j\Omega})\} = 0$	$x[n]$ real $\xleftrightarrow{DTFS;\,\Omega_o} X^*[k] = X[-k]$ $x[n]$ imaginary $\xleftrightarrow{DTFS;\,\Omega_o} X^*[k] = -X[-k]$ $x[n]$ real and even $\xleftrightarrow{DTFS;\,\Omega_o} \mathrm{Im}\{X[k]\} = 0$ $x[n]$ real and odd $\xleftrightarrow{DTFS;\,\Omega_o} \mathrm{Re}\{X[k]\} = 0$

D.1 *Basic Laplace Transforms*

Signal $x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$	*Transform* $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$	*ROC*
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} > 0$
$tu(t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\} > 0$
$\delta(t-\tau), \quad \tau \geq 0$	$e^{-s\tau}$	for all s
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} > -a$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} > -a$
$[\cos(\omega_1 t)]u(t)$	$\frac{s}{s^2+\omega_1^2}$	$\mathrm{Re}\{s\} > 0$
$[\sin(\omega_1 t)]u(t)$	$\frac{\omega_1}{s^2+\omega_1^2}$	$\mathrm{Re}\{s\} > 0$
$[e^{-at}\cos(\omega_1 t)]u(t)$	$\frac{s+a}{(s+a)^2+\omega_1^2}$	$\mathrm{Re}\{s\} > -a$
$[e^{-at}\sin(\omega_1 t)]u(t)$	$\frac{\omega_1}{(s+a)^2+\omega_1^2}$	$\mathrm{Re}\{s\} > -a$

■ D.1.1 Bilateral Laplace Transforms for Signals That Are Nonzero for $t < 0$

Signal	*Bilateral Transform*	*ROC*
$\delta(t-\tau), \tau < 0$	$e^{-s\tau}$	for all s
$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} < 0$
$-tu(-t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\} < 0$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} < -a$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} < -a$

D.2 *Laplace Transform Properties*

Signal	*Unilateral Transform* $x(t) \xleftrightarrow{\mathcal{L}_u} X(s)$ $y(t) \xleftrightarrow{\mathcal{L}_u} Y(s)$	*Bilateral Transform* $x(t) \xleftrightarrow{\mathcal{L}} X(s)$ $y(t) \xleftrightarrow{\mathcal{L}} Y(s)$	ROC $s \in R_x$ $s \in R_y$
$ax(t) + by(t)$	$aX(s) + bY(s)$	$aX(s) + bY(s)$	At least $R_x \cap R_y$
$x(t - \tau)$	$e^{-s\tau}X(s)$ if $x(t - \tau)u(t) = x(t - \tau)u(t - \tau)$	$e^{-s\tau}X(s)$	R_x
$e^{s_o t}x(t)$	$X(s - s_o)$	$X(s - s_o)$	$R_x + \text{Re}\{s_o\}$
$x(at)$	$\frac{1}{a}X\left(\frac{s}{a}\right), \quad a > 0$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$
$x(t) * y(t)$	$X(s)Y(s)$ if $x(t) = y(t) = 0$ for $t < 0$	$X(s)Y(s)$	At least $R_x \cap R_y$
$-tx(t)$	$\frac{d}{ds}X(s)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$	$sX(s)$	At least R_x
$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{s}\int_{-\infty}^{0^-} x(\tau)\, d\tau + \frac{X(s)}{s}$	$\frac{X(s)}{s}$	At least $R_x \cap \{\text{Re}\{s\} > 0\}$