Homework #8

Not Due

Problems 1

An orthogonal set of signals is characterized by the property that the inner product of any pair of signals in the set is zero. Fig. 1 shows a pair of signals $s_1(t)$ and $s_2(t)$ that satisfy this condition. Construct the signal constellation for $s_1(t)$ and $s_2(t)$.

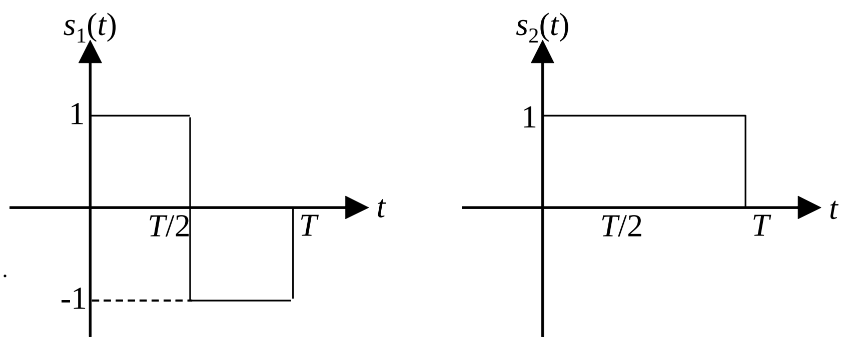

Fig. 1. Prob 1.

Problems 2

Fig. 2 shows a pair of signals $s_1(t)$ and $s_2(t)$ that are orthogonal to each other over the observation interval $0 \le t \le 3T$. The received signal is defined by

$$x(t) = s_k(t) + w(t), \quad \text{for} \begin{cases} 0 \le t \le 3T, \\ k = 1, 2, \end{cases}$$

where $w(t)$ is white Gaussian noise of zero mean and power spectral density $\frac{N_0}{2}$.

(a) Design a receiver that decides in favor of signals $s_1(t)$ or $s_2(t)$, assuming that these two signals are equiprobable.
(b) Calculate the average probability of symbol error incurred by this receiver for $E/N_0 = 4$, where E is the signal energy.

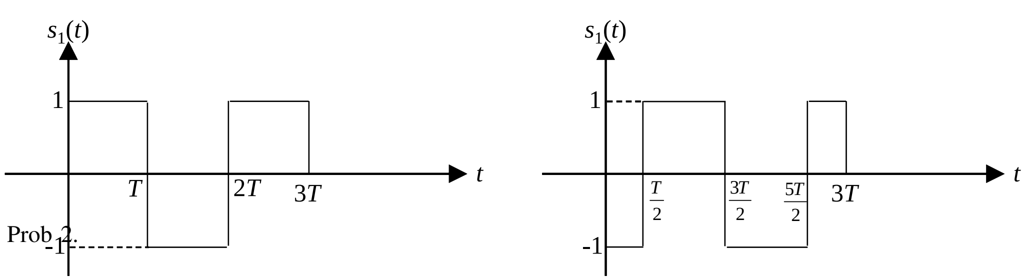

Fig. 2. Prob 2.

Problems 3

In the Bayes test, applied to a binary hypothesis testing problem where we have to choose one of two possible hypotheses H_0 and H_1, we minimize the risk R defined by

$$R = C_{00} p_0 Pr\left(\text{say } H_0 | H_0 \text{ is true}\right) + C_{10} p_0 Pr\left(\text{say } H_1 | H_0 \text{ is true}\right)$$
$$+ C_{11} p_1 Pr\left(\text{say } H_1 | H_1 \text{ is true}\right) + C_{01} p_1 Pr\left(\text{say } H_0 | H_1 \text{ is true}\right).$$

The terms C_{00}, C_{10}, C_{11}, and C_{01} denote the costs assigned to the four possible outcomes of the experiment: The first subscript indicates the hypothesis chosen, and the second the hypothesis that is true. Assume that $C_{10} > C_{00}$ and $C_{01} > C_{11}$. The p_0 and p_1 denote the a priori probabilities of hypotheses H_0 and H_1, respectively.

(a) Given the observation vector $\mathbf{x}$, show that the partitioning of the observation space so as to minimize the risk R leads to the likelihood ratio test:

$$\text{say } H_0 \text{ if } \Lambda(\mathbf{x}) < \lambda$$
$$\text{say } H_1 \text{ if } \Lambda(\mathbf{x}) > \lambda,$$

where $\Lambda(\mathbf{x})$ is the likelihood ratio

$$\Lambda(\mathbf{x}) = \frac{f_X(\mathbf{x}|H_1)}{f_X(\mathbf{x}|H_0)}$$

and λ is the threshold of the test defined by

$$\lambda = \frac{p_0(C_{10} - C_{00})}{p_1(C_{01} - C_{11})}.$$

(b) What are the cost values for which the Bayes' criterion reduces to the minimum probability of error criterion?