Modulation II

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Interference for Linear Modulation

Like to study the behavior of AM and FM systems from single-tone interference

Assume a single tone interference: $A_i \cos\left[2\pi\left(f_c + f_i\right)t\right]$

Interference in linear modulation

$$\begin{cases} \text{Message: } \dfrac{A_m}{A_c}\cos\left(2\pi f_m t\right) \\ \text{Interference: } A_i \cos\left[2\pi\left(f_c + f_i\right)t\right] \end{cases}$$

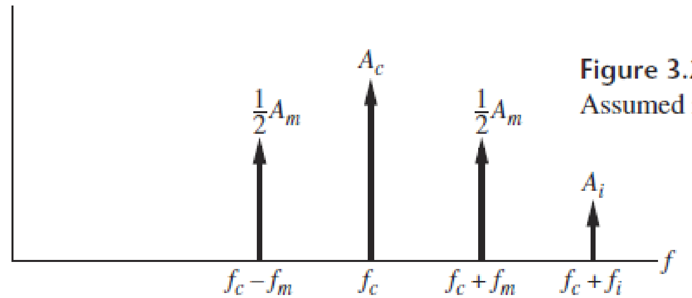


Figure 3.22
Assumed received-signal spectrum.

$$\Rightarrow x_c\left(t\right) = A_c \cos\left(2\pi f_c t\right) + A_i \cos\left[2\pi\left(f_c + f_i\right)t\right] + A_m \cos\left(2\pi f_m t\right)\cos\left(2\pi f_c t\right)$$

$$x_c\left(t\right) = A_c\left[1 + a m_n\left(t\right)\right]\cos\left(2\pi f_c t\right)$$

1. Coherent detection: linear detector

$$y_D\left(t\right) = A_m \cos\left(2\pi f_m t\right) + A_i \cos\left(2\pi f_i t\right)$$

(interference is additive, not a big problem)

2. Envelope detection: nonlinear detector

$$x_r\left(t\right) = \text{Re}\left\{\left(A_c + A_i e^{j2\pi f_i t} + \frac{1}{2}A_m e^{j2\pi f_m t} + \frac{1}{2}A_m e^{-j2\pi f_m t}\right)e^{j2\pi f_c t}\right\}$$

Interference for Linear Modulation

$$x_r(t) = \text{Re}\left\{\left(A_c + A_i e^{j2\pi f_i t} + \frac{1}{2}A_m e^{j2\pi f_m t} + \frac{1}{2}A_m e^{-j2\pi f_m t}\right)e^{j2\pi f_c t}\right\}$$

Hard to analyze output of nonlinear detector $\Rightarrow$ use phasor

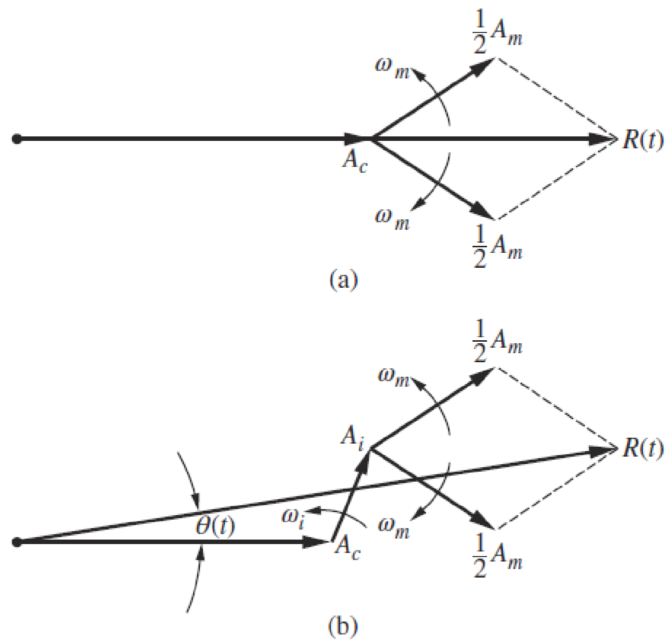


Figure 3.23
Phasor diagrams illustrating interference.
(a) Phasor diagram without interference.
(b) Phasor diagram with interference.

Interference for Linear Modulation

Analysis can be simplified by looking at different scenarios. First rewrite $x_r(t)$ as

$$x_r(t) = A_c \cos(2\pi f_c t) + A_m \cos(2\pi f_m t)\cos(2\pi f_c t) + A_i \cos\left[2\pi(f_c + f_i)t\right]$$

$$= A_c \cos(2\pi f_c t) + A_m \cos(2\pi f_m t)\cos(2\pi f_c t)$$

$$+ A_i\left[\cos(2\pi f_c t)\cos(2\pi f_i t) - \sin(2\pi f_c t)\sin(2\pi f_i t)\right]$$

$$= \left[A_c + A_m \cos(2\pi f_m t) + A_i \cos(2\pi f_i t)\right]\cos(2\pi f_c t)$$

$$- A_i \sin(2\pi f_c t)\sin(2\pi f_i t)$$

Interference for Linear Modulation

Case (i): If $A_c >> A_i$ (typical case)

$$x_r(t) = \left[A_c + A_m\cos(2\pi f_m t) + A_i\cos(2\pi f_i t)\right]\cos(2\pi f_c t)$$

Envelope of $x_r(t) = A_c + A_m\cos(2\pi f_m t) + A_i\cos(2\pi f_i t)$

$y_D(t) = A_m\cos(2\pi f_m t) + A_i\cos(2\pi f_i t)$ (after DC term is blocked)

Effective carrier frequency is f_c

Same as coherent detector

Case (ii): If $A_c << A_i$

$$\begin{aligned}
x_r(t) &= A_c\cos(2\pi f_c t) + A_m\cos(2\pi f_m t)\cos(2\pi f_c t) + A_i\cos\left[2\pi(f_c + f_i)t\right] \\
&= A_c\cos\left[2\pi(f_c + f_i - f_i)t\right] + A_i\cos\left[2\pi(f_c + f_i)t\right] + A_m\cos(2\pi f_m t)\cos\left[2\pi(f_c + f_i - f_i)t\right] \\
&= A_c\left\{\cos\left[2\pi(f_c + f_i)t\right]\cos(2\pi f_i t) + \sin\left[2\pi(f_c + f_i)t\right]\sin(2\pi f_i t)\right\} \\
&\quad + A_i\cos\left[2\pi(f_c + f_i)t\right] + A_m\cos(2\pi f_m t)\left\{\cos\left[2\pi(f_c + f_i)t\right]\cos(2\pi f_i t)\right. \\
&\quad \left. + \sin\left[2\pi(f_c + f_i)t\right]\sin(2\pi f_i t)\right\}
\end{aligned}$$

Interference for Linear Modulation

Case (ii): If $A_c << A_i$ (cont)

$$x_r(t) = \left[A_i + A_c\cos(2\pi f_i t) + A_m\cos(2\pi f_m t)\cos(2\pi f_i t)\right]\cos\left[2\pi(f_c + f_i)t\right]$$
$$+\left[A_c\sin(2\pi f_i t) + A_m\cos(2\pi f_m t)\sin(2\pi f_i t)\right]\sin\left[2\pi(f_c + f_i)t\right]$$

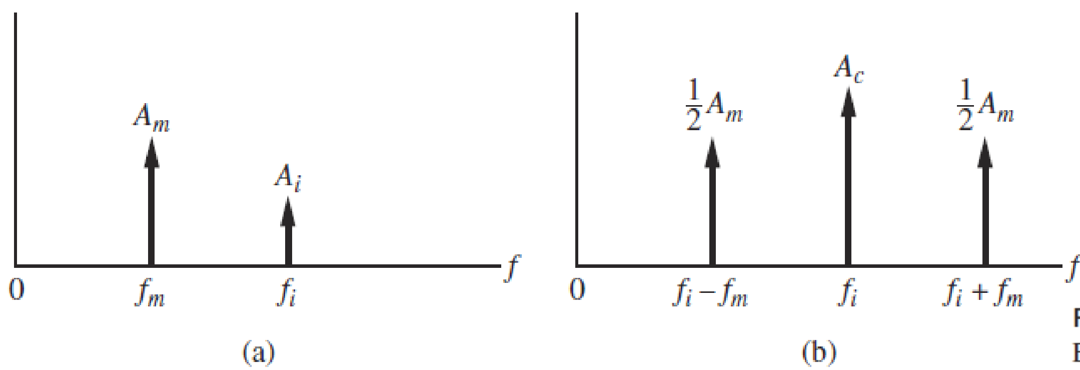
Since $A_c << A_i$, last term is negligible, then

$$\approx\left[A_i + A_c\cos(2\pi f_i t) + A_m\cos(2\pi f_m t)\cos(2\pi f_i t)\right]\cos\left[2\pi(f_c + f_i)t\right]$$

Envelope of $x_r(t) \approx A_i + A_c\cos(2\pi f_i t) + A_m\cos(2\pi f_m t)\cos(2\pi f_i t)$

$$\Rightarrow\ y_D(t) \approx A_c\cos(2\pi f_i t) + A_m\cos(2\pi f_m t)\cos(2\pi f_i t) \quad \text{(assming DC term blocked)}$$

Message is lost! Effective carrier frequency becomes $f_c + f_i$



This is called threshold effect; when the interference > a threshold, message is lost

Figure 3.24
Envelope detector output spectra. (a) $A_c \gg A_i$. (b) $A_c \ll A_i$.

Interference for Angle Modulation

Assume interference tone at $f_c + f_i$

Assume an unmodulated carrier + interference, input to discriminator:

$$x_t(t) = A_c \cos\left(2\pi f_c t + \phi(t)\right) + A_i \cos\left[2\pi\left(f_c + f_i\right)t\right]$$

assuming $\phi(t) = 0$

$$= A_c \cos\left(2\pi f_c t\right) + A_i \cos\left(2\pi f_i t\right)\cos\left(2\pi f_c t\right) - A_i \sin\left(2\pi f_i t\right)\sin\left(2\pi f_c t\right)$$

$$= \left[A_c + A_i \cos\left(2\pi f_i t\right)\right]\cos\left(2\pi f_c t\right) - \left[A_i \sin\left(2\pi f_i t\right)\right]\sin\left(2\pi f_c t\right)$$

$$= R(t)\cos\left(2\pi f_c t + \psi(t)\right),$$

where

$$R(t) = \sqrt{\left[A_c + A_i \cos\left(2\pi f_i t\right)\right]^2 + \left[A_i \sin\left(2\pi f_i t\right)\right]^2}$$

$$\psi(t) = \tan^{-1}\left(\frac{A_i \sin\left(2\pi f_i t\right)}{A_c + A_i \cos\left(2\pi f_i t\right)}\right)$$

Interference for Angle Modulation

How does the interference behave after demodulation?

Case (i): If $A_c >> A_i$

$$\begin{cases} R(t) \approx A_c + A_i \cos(2\pi f_i t) \\ \psi(t) \approx \tan^{-1}\left(\dfrac{A_i \sin(2\pi f_i t)}{A_c}\right) \approx \dfrac{A_i \sin(2\pi f_i t)}{A_c} \end{cases}$$

Assume ideal discriminator that extracts $\psi(t)$, $\dfrac{d\psi(t)}{dt}$

$$\begin{cases} \text{For PM: } y_D(t) = K_D \psi(t) = K_D \dfrac{A_i}{A_c} \sin(2\pi f_i t) \\ \text{For FM: } y_D(t) = \dfrac{1}{2\pi} K_D \dfrac{d\psi(t)}{dt} = \dfrac{1}{2\pi} K_D \dfrac{d}{dt}\left(\dfrac{A_i \sin(2\pi f_i t)}{A_c}\right) \\ \qquad = K_D \dfrac{A_i}{A_c} f_i \cos(2\pi f_i t) \end{cases}$$

When f_i is small, interference in FM is smaller than that in PM.

FM: Values of $f_i > W$ is removed by LPF

Interference for Angle Modulation

In summary, for $A_c >> A_i$, the single tone $(f_c + f_i)$ interference at the demodulator output:

$$\begin{cases} \text{PM: } K_D \dfrac{A_i}{A_c} \sin(2\pi f_i t) \\ \text{FM: } K_D \dfrac{A_i}{A_c} f_i \cos(2\pi f_i t) \end{cases}$$

If they all have the same constants, then $K_D \dfrac{A_i}{A_c}$ (PM) $= K_D \dfrac{A_i}{A_c}$ (FM)

No dependency on f_i for PM

Amplitude of interference increases linearly with f_i for FM

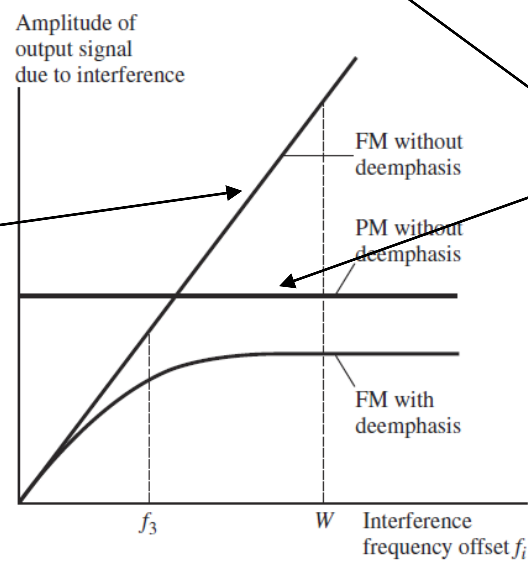


Figure 4.31
Amplitude of discriminator output due to interference.

Interference for Angle Modulation

Case (ii): If $A_c << A_i$, it is difficult to analyze. Can gain insight by looking at phasor

Again, an unmodulated carrier + interference, input to discriminator:

$$x_t(t) = A_c\cos\left(2\pi f_c t + \phi(t)\right) + A_i\cos\left[2\pi\left(f_c + f_i\right)t\right]$$

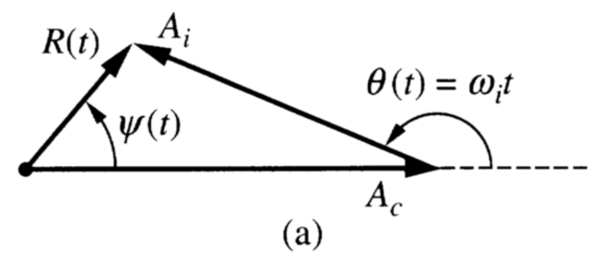
$$\Rightarrow x_r(t) = \mathrm{Re}\left\{\left(A_c + A_i e^{j2\pi f_i t}\right)e^{j2\pi f_c t}\right\}$$ (assume $\phi(t) = 0$ for unmodulated carrier)

(term inside parenthesis is phasor)

Carrier phase: reference $= 0$

Interference phase: $\theta(t) = 2\pi f_i t$

Resultant phase: $\psi(t)$



Phasor diagram for carrier plus single-tone interference. (a) Phasor diagram for general $\theta(t)$. (b) Phasor diagram for $\theta(t) \approx 0$.

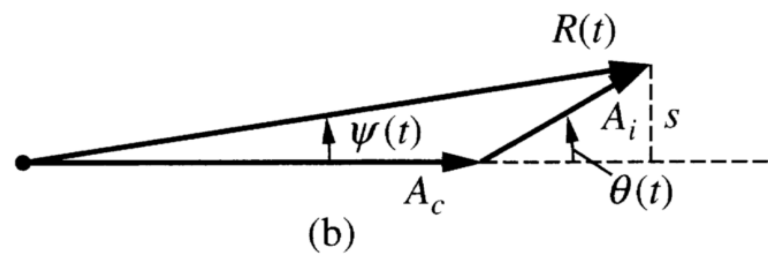
(1) $\theta(t) \approx 0$

$$s \approx A_i\theta(t) \quad \left(\text{since } A_i\sin\theta(t) \text{ and } \sin\theta(t) \approx \theta(t) \text{ for small } \theta(t)\right)$$

$$\approx \left(A_c + A_i\right)\psi(t)$$

$$\Rightarrow \psi(t) \approx \frac{A_i}{A_c + A_i}\theta(t) = \frac{A_i}{A_c + A_i}2\pi f_i t$$

Since $y_D(t) = \dfrac{K_D}{2\pi}\dfrac{d\psi(t)}{dt}$, $y_D(t) = K_D\dfrac{A_i}{A_c + A_i}f_i$



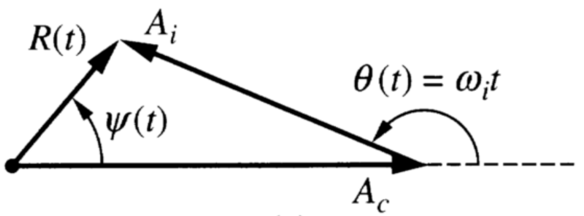
Interference for Angle Modulation

(2) $\theta(t) \approx \pi,\ A_i \underset{\sim}{<} A_c$ ($\underset{\sim}{<}$ defined as slightly less)

$$s = A_i \sin(\pi - \theta(t))$$
$$\approx A_i(\pi - \theta(t))$$
$$\approx (A_c - A_i)\psi(t) \qquad \left(\psi(t) = \tan\left(\frac{s}{A_c - A_i}\right) \approx \frac{s}{A_c - A_i} \text{ for small } \psi(t)\right)$$

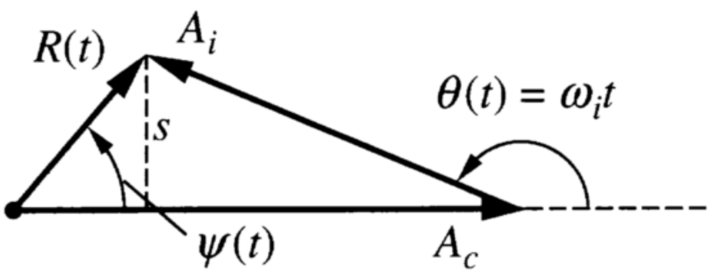
$$\Rightarrow \psi(t) \approx \frac{s}{A_c - A_i} \approx \frac{A_i(\pi - \theta(t))}{A_c - A_i} = \frac{A_i(\pi - 2\pi f_i t)}{A_c - A_i}$$

Since $y_D(t) = \frac{K_D}{2\pi}\frac{d\psi(t)}{dt}$, $y_D(t) = -K_D \frac{A_i}{A_c - A_i} f_i$



(a)

Phasor diagram for carrier plus single-tone interference. (a) Phasor diagram for general $\theta(t)$. (c) Phasor diagram for $\theta(t) \approx \pi$ and $A_i \underset{\sim}{<} A_c$.



(c)

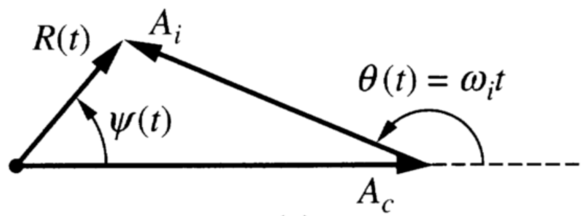
Interference for Angle Modulation

(3) $\theta(t) \approx \pi, \ A_i \underset{\sim}{>} A_c$ ($\underset{\sim}{<}$ defined as slightly less)

$$s = A_i \sin\left(\pi - \theta(t)\right)$$
$$\approx A_i\left(\pi - \theta(t)\right)$$
$$\approx \left(A_i - A_c\right)\left(\pi - \psi(t)\right)$$

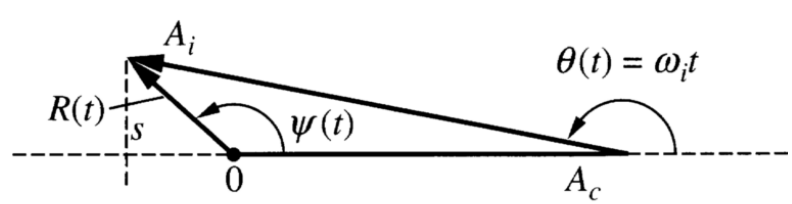
$$\Rightarrow \psi(t) \approx \pi - \frac{s}{A_i - A_c} \approx \pi - \frac{A_i\left(\pi - \theta(t)\right)}{A_i - A_c}$$
$$= \pi - \frac{A_i\left(\pi - 2\pi f_i t\right)}{A_i - A_c}$$

Since $y_D(t) = \frac{K_D}{2\pi}\frac{d\psi(t)}{dt}, \ y_D(t) = K_D \frac{A_i}{A_i - A_c} f_i$



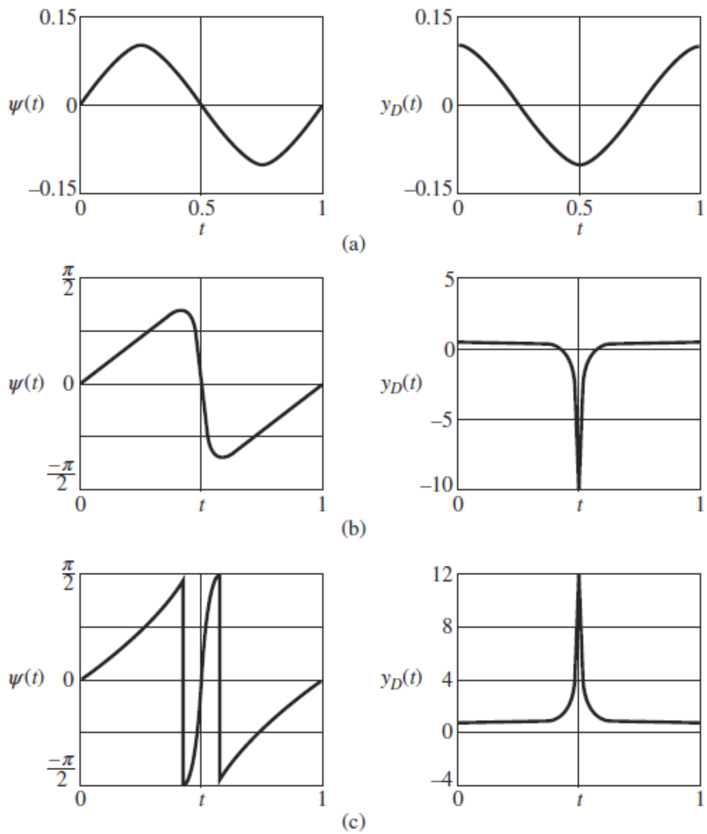
(a)

Phasor diagram for carrier plus single-tone interference. (a) Phasor diagram for general $\theta(t)$. (d) Phasor diagram for $\theta(t) \approx \pi$ and $A_i \underset{\sim}{>} A_c$.



(d)

Demodulated Waveform for Angle Modulation with Interference



$A_C >> A_i$

Both are sinsusoidal

$A_i \lesssim A_c$

$$y_D(t) = -K_D \frac{A_i}{A_c - A_i} f_i$$

Negative spike in $y_D(t)$ (negative when $f_i > 0$)

$A_i \gtrsim A_c$

$$y_D(t) = K_D \frac{A_i}{A_i - A_c} f_i$$

Positive spike in $y_D(t)$ (postive when $f_i > 0$)

Figure 4.30
Phase deviation and discriminator output due to interference. (a) Phase deviation and discriminator output for $A_i = 0.1A_c$. (b) Phase deviation and discriminator output for $A_i = 0.9A_c$. (c) Phase deviation and discriminator output for $A_i = 1.1A_c$.

Interference for Angle Modulation

- Recall for FM, output of demodulator from a single-tone interferer at $f_c + f_i$ is

  $y_D(t) = K_D \frac{A_i}{A_c} f_i \cos\left(2\pi f_i t\right),$ hence amplitude of the interference grows linearly with f_i

- When $A_i \ll A_c$, interference on FM for large f_i can be reduced by using a LPF deemphasis filter at output of FM discriminator
  - 3dB frequency of filter usually less than message signal bandwidth W
  - reduces interference at large f_i so amplitude of output of modulator is constant at large f_i
- Problem: at low freq, de-emphasis filter will distort the message (i.e. $f_3 < W$)
  - can be avoided by using a highpass preemphasis filter
  - transfer function equals reciprocal of the deemphasis filter

Interference for Angle Modulation

Amplitude of interference increases linearly with f_i

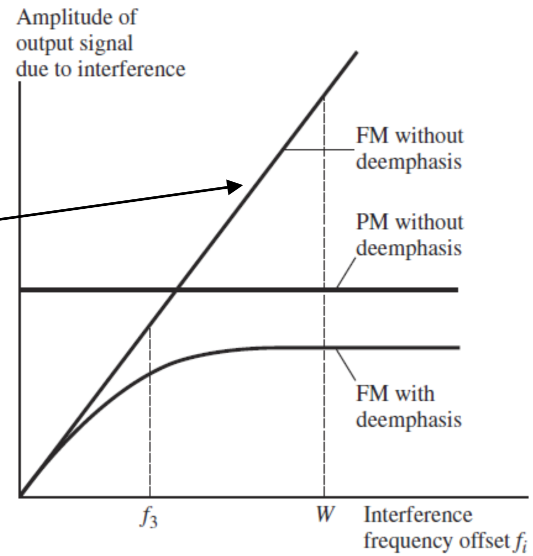


Figure 4.31
Amplitude of discriminator output due to interference.

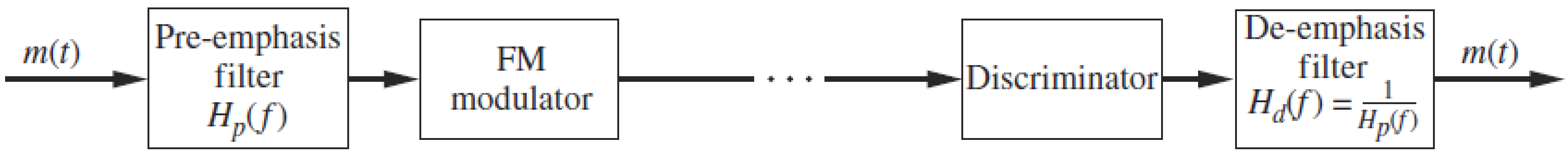


Figure 4.32
Frequency modulation system with pre-emphasis and de-emphasis.

Phase-Locked Loop (PLL) and Feedback Demodulators

- Tracks the instantaneous angle (phase and frequency) of the input signal. Includes
  - Phase detector (comparator)
  - Loop filter
  - Loop amplifier
  - VCO (voltage-controlled oscillator)
- Basic operation
  - Adjust the phase of the local VCO output ($e_o(t)$) to match the input ($x_r(t)$) signal phase

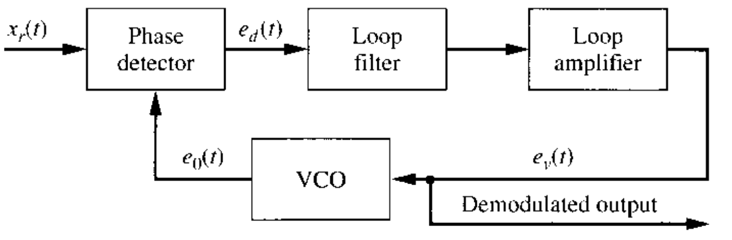


Figure 3.45
Phase-locked loop.

Phase Detector

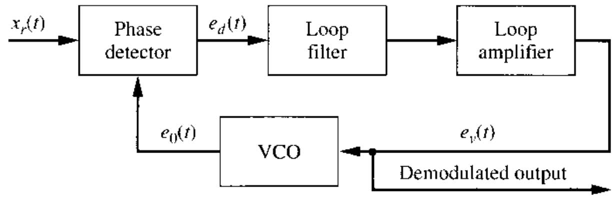


Figure 3.45
Phase-locked loop.

Input: $x_r(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$

VCO output: $e_0(t) = A_v \sin\left[2\pi f_c t + \theta(t)\right]$

Goal: $\theta(t) \overset{\text{matches}}{\rightarrow} \phi(t)$

$\Rightarrow$ Developing a relationship between $\theta(t)$ (output) and $\phi(t)$ (input) is equivalent to knowing the relationship between $e_0(t)$ and $x_r(t)$

Phase detector output: $e_d(t) = g\left(\phi(t) - \theta(t)\right)$,

$g(\bullet)$ is a characteristic function of the phase detector

(1) ideal - saw-tooth: $K_d\left(\phi(t) - \theta(t)\right)$

(2) $\sin(\bullet)$: $e_d(t) = \frac{A_c A_v K_d}{2} \sin\left[\phi(t) - \theta(t)\right]$

VCO

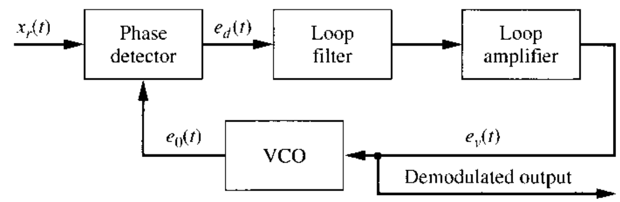


Figure 3.45
Phase-locked loop.

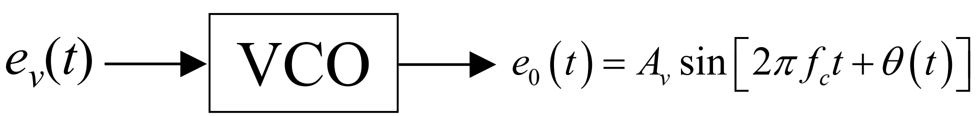
Output of phase detector is filtered, amplified, and applied to VCO.

VCO: a frequency modulator - the frequency deviation of its output,

$\frac{d\theta(t)}{dt}$, is proportional to the VCO input, i.e.

$$\frac{d\theta(t)}{dt} = K_v e_v(t) \text{ rad/s}, \; K_v : \text{ VCO constant}$$

$$\Rightarrow \theta(t) = K_v \int^t e_v(\alpha)\, d\alpha$$



PLL

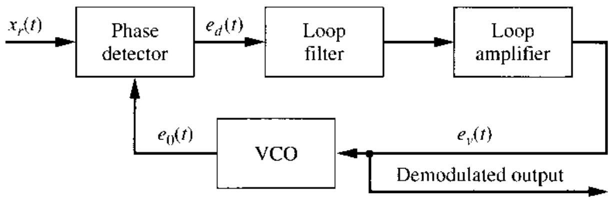


Figure 3.45 Phase-locked loop.

One of the I/O relationships inside PLL: $E_v(s) = F(s)E_d(s)$,

$F(s)$: transfer function of loop filter

$$\Leftrightarrow e_v(\alpha) = \int^{\alpha} e_d(\lambda) f(\alpha - \lambda) d\lambda$$

$$\theta(t) = K_v \int^{t} e_v(\alpha) d\alpha$$

Assuming $e_d(t) = \frac{A_c A_v K_d}{2} \sin[\phi(t) - \theta(t)] = \frac{A_c A_v K_d}{2} \sin[\psi(t)]$

$$\Rightarrow \theta(t) = K_v \int^{t} e_v(\alpha) d\alpha = K_v \int^{t} \int^{\alpha} e_d(\lambda) f(\alpha - \lambda) d\lambda d\alpha$$

$$= \frac{A_c A_v K_d K_v}{2} \int^{t} \int^{\alpha} \sin[\psi(\lambda)] f(\alpha - \lambda) d\lambda d\alpha$$

$$= K_t \int^{t} \int^{\alpha} \sin[\psi(\lambda)] f(\alpha - \lambda) d\lambda d\alpha$$

PLL

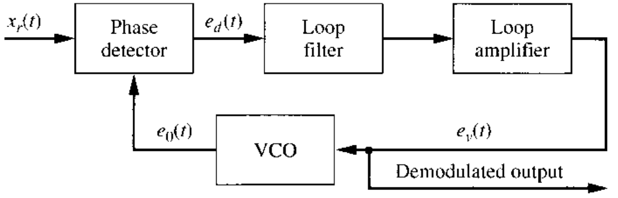


Figure 3.45 Phase-locked loop.

$$\theta(t) = K_t \int^t \int^\alpha \sin\left[\psi(\lambda)\right] f(\alpha - \lambda)\, d\lambda\, d\alpha$$

So, if phase error $\psi(t) \triangleq \phi(t) - \theta(t)$ is small, then

$$\theta(t) = K_t \int^t \int^\alpha \psi(\lambda) f(\alpha - \lambda)\, d\lambda\, d\alpha$$

i.e. PLL becomes a linear feedback system.

If $\theta(t) \approx \phi(t) \Rightarrow \frac{d\theta(t)}{dt} \approx \frac{d\phi(t)}{dt}$

i.e. VCO freq deviation is a good estimate of the input freq deviation.

Recall FM: $x_c(t) = A_c \cos\left[2\pi f_c t + \underbrace{2\pi \int^t m(\alpha)\, d\alpha}_{\phi(t)}\right]$

$$\therefore \frac{d\theta(t)}{dt} \propto m(t)$$

Summary of PLL

Nonlinear PLL model

$$\theta(t)=K_t\int^t\int^\alpha \sin\left[\psi(\lambda)\right]f(\alpha-\lambda)\,d\lambda\,d\alpha$$

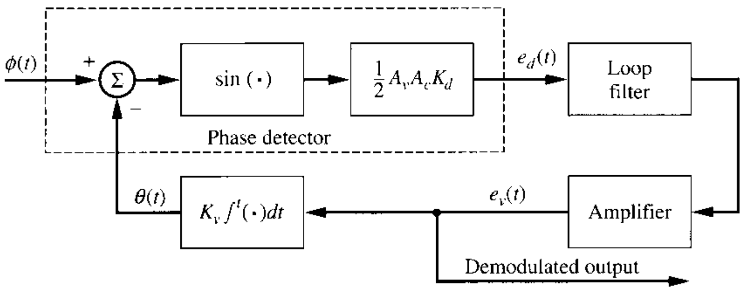


Figure 3.46
Nonlinear PLL model.

Linear PLL model

$$\theta(t)=K_t\int^t\int^\alpha \sin\left[\psi(\lambda)\right]f(\alpha-\lambda)\,d\lambda\,d\alpha$$

$$\text{If } \theta(t)\approx\phi(t)\Rightarrow \sin\left[\psi(t)\right]\approx\psi(t)$$

$$\theta(t)=K_t\int^t\int^\alpha \psi(\lambda)f(\alpha-\lambda)\,d\lambda\,d\alpha$$

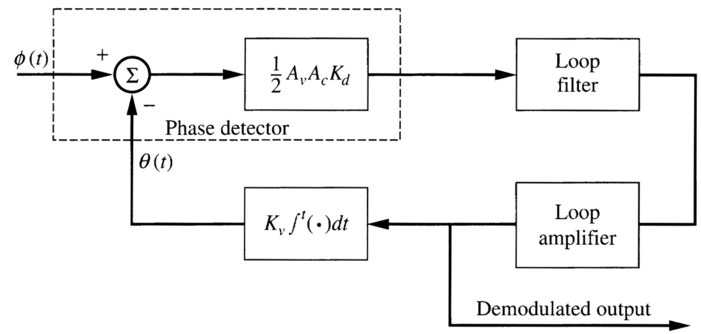


Figure 3.47
Linear PLL model.

Divide analysis into 2 parts:

- Tracking Mode using linear model (steady-state response)
- Acquisition Mode using nonlinear model (transient response)

PLL Tracking Mode: Linear Model

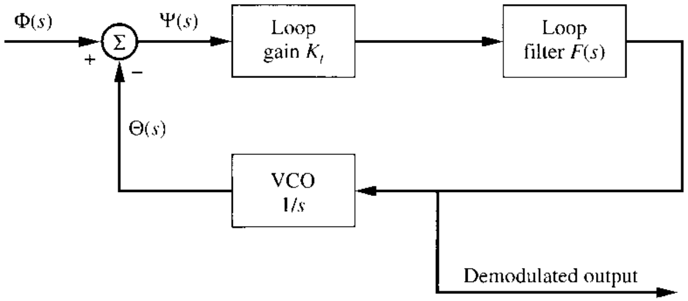


Figure 3.48
Linear PLL model in the frequency domain.

Linear PLL model

$$\theta(t) = K_t \int^t \int^\alpha \left[\phi(\lambda) - \theta(\lambda)\right] f(\alpha - \lambda)\, d\lambda\, d\alpha$$

$$= K_t \int^t \left[\phi(\alpha) - \theta(\alpha)\right] * f(\alpha)\, d\alpha$$

$\Leftrightarrow \Theta(s) = K_t \left[\Phi(s) - \Theta(s)\right] \dfrac{F(s)}{s}$ (assuming zero initial cond.)

$\Rightarrow$ transfer function for PLL: $H(s) \triangleq \dfrac{\Theta(s)}{\Phi(s)} = \dfrac{K_t F(s)}{s + K_t F(s)}$

transfer function relating phase error to input phase:

$$G(s) = \frac{\Psi(s)}{\Phi(s)} = \frac{\Phi(s) - \Theta(s)}{\Phi(s)} = 1 - H(s) = \frac{s}{s + K_t F(s)}$$

PLL Tracking Mode: Linear Model

$$H(s) \triangleq \frac{\Theta(s)}{\Phi(s)} = \frac{K_t F(s)}{s + K_t F(s)}$$

$$G(s) = \frac{\Psi(s)}{\Phi(s)} = \frac{\Phi(s) - \Theta(s)}{\Phi(s)} = 1 - H(s) = \frac{s}{s + K_t F(s)}$$

$$\Rightarrow \Psi(s) = \Phi(s) G(s)$$

Recall $\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t) \Leftrightarrow \frac{1}{(s+\alpha)^n}$.

To study the steady-state error (response), assume input $\phi(t)$ has the general form

$$\phi(t) = \pi R t^2 + 2\pi f_\Delta t + \theta_0,\ t > 0$$

$$\Leftrightarrow \Phi(s) = \frac{2\pi R}{s^3} + \frac{2\pi f_\Delta}{s^2} + \frac{\theta_0}{s},$$

R : frequency ramp (Hz/s), f_Δ : frequency step.

Also, the freq. deviation: $\frac{1}{2\pi} \frac{d\phi(t)}{dt} = Rt + f_\Delta, \quad t > 0$

PLL Tracking Mode: Linear Model

Steady-state error (response) can be obtained using final value theorm

$$\left(\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)\right).$$

$$\Rightarrow \psi_{ss} \triangleq \lim_{t\to\infty} \psi(t) = \lim_{s\to 0} s\Psi(s) = \lim_{s\to 0} s\left[\Phi(s)G(s)\right]$$

$$= \lim_{s\to 0} s\left[\frac{2\pi R}{s^3} + \frac{2\pi f_\Delta}{s^2} + \frac{\theta_0}{s}\right]G(s)$$

$$= \lim_{s\to 0} s\left[\frac{2\pi R}{s^3} + \frac{2\pi f_\Delta}{s^2} + \frac{\theta_0}{s}\right]\left[\frac{s}{s + K_t F(s)}\right]$$

Consider different loop filter transfer function $F(s)$. Generates

- 1^{st} order PLL
- 2^{nd} order PLL
- 3^{rd} order PLL

Table 3.4 Loop Filter Transfer Functions

PLL order	Loop filter transfer function, $F(s)$
1	1
2	$1 + a/s = (s+a)/s$
3	$1 + a/s + b/s^2 = (s^2 + as + b)/s^2$

PLL Tracking Mode: Linear Model

Note 3rd order $F(s)=\frac{1}{s^2}\left(s^2+as+b\right)$ is the most general filter

If $a=b=0 \Rightarrow \quad F(s)=1$ (1st order $F(s)$)

If $a\neq 0, b=0 \Rightarrow F(s)=\frac{1}{s^2}\left(s^2+as\right)=\frac{1}{s}(s+a)$ (2nd order $F(s)$)

Since 3rd order $F(s)$: $G(s)=\frac{s}{s+K_tF(s)}=\frac{s^3}{s^3+K_ts^2+K_tas+K_tb}$

$$\psi_{ss}=\lim_{s\to 0} s\Psi(s)=\lim_{s\to 0} s\left[\Phi(s)G(s)\right]$$

$$=\lim_{s\to 0} s\left[\frac{2\pi R}{s^3}+\frac{2\pi f_\Delta}{s^2}+\frac{\theta_0}{s}\right]G(s)$$

$$=\lim_{s\to 0} s\left[\frac{2\pi R}{s^3}+\frac{2\pi f_\Delta}{s^2}+\frac{\theta_0}{s}\right]\left[\frac{s^3}{s^3+K_ts^2+K_tas+K_tb}\right]$$

$$=\lim_{s\to 0}\frac{s\left(\theta_0 s^2+2\pi f_\Delta s+2\pi R\right)}{s^3+K_ts^2+K_tas+K_tb}$$

1^{st} order PLL Tracking Mode: Linear Model ($F(s) = 1$)

Case 1: 1^{st} order PLL $\quad (F(s)=1)$

$$H(s) \triangleq \frac{\Theta(s)}{\Phi(s)} = \frac{K_t F(s)}{s + K_t F(s)}$$

$$= \frac{K_t}{s + K_t}$$

$$\Leftrightarrow h(t) = K_t e^{-K_t t} u(t)$$

$$\lim_{K_t \to \infty} K_t e^{-K_t t} u(t) = \delta(t)$$

$$\Rightarrow \text{For large gain, } \theta(t) \approx \phi(t).$$

Note:

PLL serves as a demodulator for angle-modulated signal

FM: VCO input is proportional to $\dfrac{d\theta(t)}{dt}$, i.e. the freq deviation of the PLL input signal

PM: Integrate VCO input to obtain demodulated output

1st order PLL Tracking Mode: Linear Model ($F(s) = 1$)

General form of input to PLL

$\phi(t) = \pi R t^2 + 2\pi f_\Delta t + \theta_0,\ t > 0$

$\Leftrightarrow \Phi(s) = \frac{2\pi R}{s^3} + \frac{2\pi f_\Delta}{s^2} + \frac{\theta_0}{s},$

R : frequency ramp (Hz/s), f_Δ : frequency step.

Steady-state phase error:

Recall $\Theta(s) = K_t \Psi(s) \frac{F(s)}{s}$

$$\Rightarrow \Theta(s) = K_t \Psi(s) \frac{1}{s}$$

$$\Rightarrow \Theta(s) = \frac{K_t}{s + K_t} \Phi(s) = K_t \Psi(s) \frac{1}{s}$$

$$\Rightarrow \Psi(s) = \frac{s}{s + K_t} \Phi(s)$$

If $\Phi(s) = \frac{\theta_0}{s} \Rightarrow \Psi(s) = \frac{\theta_0}{s + K_t}$

$$\psi_{ss} = \lim_{s \to 0} s\Psi(s)$$

$$= \lim_{s \to 0} \frac{s\theta_0}{s + K_t} = 0$$

Summary of Steady-State Errors

Table 3.5 Steady-State Errors

PLL order	$\theta_0 \neq 0$ $f_\Delta = 0$ $R = 0$	$\theta_0 \neq 0$ $f_\Delta \neq 0$ $R = 0$	$\theta_0 \neq 0$ $f_\Delta \neq 0$ $R \neq 0$
1 $(a = 0,\ b = 0)$	0	$2\pi f_\Delta / K_t$	∞
2 $(a \neq 0,\ b = 0)$	0	0	$2\pi R / K_t$
3$(a \neq 0,\ b \neq 0)$	0	0	0

1st order PLL Tracking Mode: Linear Model Example

Let $m(t) = Au(t)$, so that $x_c(t) = A_c \cos\left[2\pi f_c t + k_f A\int^t u(\alpha)\,d\alpha\right]$.

What is the demodulatd output using a 1st order PLL?

Recall that $H(s) = \dfrac{\Theta(s)}{\Phi(s)} = \dfrac{K_t}{s + K_t}$.

Let the input to PLL be $u(t)$ (recall $L\{u(t)\} = 1/s$):

$$\phi(t) = Ak_f\int^t u(\alpha)\,d\alpha \Leftrightarrow \Phi(s) = \frac{Ak_f}{s^2}$$

General form of input to PLL

$\phi(t) = \pi R t^2 + 2\pi f_\Delta t + \theta_0,\ t > 0$

$\Leftrightarrow \Phi(s) = \dfrac{2\pi R}{s^3} + \dfrac{2\pi f_\Delta}{s^2} + \dfrac{\theta_0}{s}$,

R : frequency ramp (Hz/s),

f_Δ : frequency step.

$$\Rightarrow \Theta(s) = \frac{Ak_f}{s^2}\frac{K_t}{s + K_t}$$

Recall $\theta(t) = K_v\int^t e_v(\alpha)\,d\alpha \Rightarrow \dfrac{d\theta(t)}{dt} = K_v e_v(t)$

1st order PLL Tracking Mode: Linear Model Example

$$\Leftrightarrow E_v(s) = \frac{s}{K_v}\Theta(s) = \frac{s}{K_v}\frac{Ak_f}{s^2}\frac{K_t}{s+K_t} = \frac{Ak_f}{K_v}\frac{K_t}{s(s+K_t)}$$

$$= \frac{Ak_f}{K_v}\left(\frac{C_1}{s} + \frac{C_2}{s+K_t}\right)$$

$$C_1 = s\frac{K_t}{s(s+K_t)}\bigg|_{s=0} = 1; \quad C_2 = (s+K_t)\frac{K_t}{s(s+K_t)}\bigg|_{s=-K_t} = -1$$

$$\Rightarrow E_v(s) = \frac{Ak_f}{K_v}\left(\frac{1}{s} - \frac{1}{s+K_t}\right)$$

$$\Rightarrow e_v(t) = \frac{Ak_f}{K_v}\left(1-e^{-K_t t}\right)u(t)$$

Unfortunately, K_t cannot be made arbitrarily big without increasing the bandwidth of $H(s) = \frac{K_t}{s+K_t}$

For $t >> 1/K_t$ and $k_f = K_v$, $e_v(t) = Au(t) = m(t)$. Note that the transient time is set by the total loop gain K_t and k_f / K_v is simply an amplitude scaling of the demodulated output.

1st order PLL Tracking Mode: Linear Model Example

Steady-state phase error

$$\psi_{ss} = \lim_{s\to 0} s\left[\Phi(s) - \Theta(s)\right]$$

$$= \lim_{s\to 0} s\left[\frac{Ak_f}{s^2} - \frac{Ak_f}{s^2}\frac{K_t}{s+K_t}\right]$$

$$= \lim_{s\to 0} s\left[\frac{Ak_f}{s^2}\left(\frac{s}{s+K_t}\right)\right]$$

$$= \frac{Ak_f}{K_t}$$

Summary of Steady-State Errors

$$2\pi f_\Delta = Ak_f$$

Table 3.5 Steady-State Errors

PLL order	$\boldsymbol{\theta_0 \neq 0}$, $\boldsymbol{f_\Delta = 0}$, $\boldsymbol{R = 0}$	$\boldsymbol{\theta_0 \neq 0}$, $\boldsymbol{f_\Delta \neq 0}$, $\boldsymbol{R = 0}$	$\boldsymbol{\theta_0 \neq 0}$, $\boldsymbol{f_\Delta \neq 0}$, $\boldsymbol{R \neq 0}$
1 $(a = 0,\ b = 0)$	0	$2\pi f_\Delta / K_t$	∞
2 $(a \neq 0,\ b = 0)$	0	0	$2\pi R / K_t$
3 $(a \neq 0,\ b \neq 0)$	0	0	0

Summary of 1st order PLL

- Nonzero steady state error $\psi_{ss} \propto \dfrac{Ak_f}{K_t}$ $(2\pi f_\Delta = Ak_f \Rightarrow f_\Delta \neq 0)$
- The complete system loop gain parameter is

  $$K_t = \frac{1}{2}\mu A_c A_v K_d K_v$$

  (discussion above assumed $\mu = 1$)

  System loop gain is a function of the amplitude of the input signal.
- K_t also controls the BW of the system $H(s) = \dfrac{K_t}{s + K_t}$

  K_t is the 3dB point
- A large K_t is impractical

  (a) Hardware design

  (b) Noise increases due to the wide bandwidth

  Solution: 2^{nd} order PLL

2nd order PLL Tracking Mode: Linear Model

Loop filter: $F(s) = \frac{s+a}{s}$

Recall $\Theta(s) = K_t\left[\Phi(s) - \Theta(s)\right]\frac{F(s)}{s}$

$$= K_t\left[\Phi(s) - \Theta(s)\right]\frac{s+a}{s^2}$$

$$\Rightarrow \Theta(s)\left[1 + K_t\frac{s+a}{s^2}\right] = K_t\frac{s+a}{s^2}\Phi(s)$$

$$\Rightarrow H(s) \triangleq \frac{\Theta(s)}{\Phi(s)} = \frac{K_t\frac{s+a}{s^2}}{1 + K_t\frac{s+a}{s^2}} = \frac{K_t(s+a)}{s^2 + K_t s + K_t a}$$

Also, $G(s) \triangleq \frac{\Psi(s)}{\Phi(s)} = 1 - H(s) = \frac{s^2}{s^2 + K_t as + K_t a}$

In terms of fundamental parameters: natural frequency and damping factor

$$\frac{\Psi(s)}{\Phi(s)} = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

ζ : damping factor

ω_n : natural frequency

$$\Rightarrow \zeta = \frac{1}{2}\sqrt{\frac{K_t}{a}}, \quad \omega_n = \sqrt{K_t a}$$

Solving for the physical parameters

$$\Rightarrow a = \frac{\omega_n}{2\zeta} = \frac{\pi f_n}{\zeta}, \quad K_t = 4\pi\zeta f_n$$

2^{nd} order PLL Tracking Mode: Linear Model Example

Similar to previous example, we let input to PLL be $m(t) = u(t)$

Recall $\phi(t) = 2\pi\Delta f \int^t u(\alpha)\,d\alpha \quad \Leftrightarrow \quad \Phi(s) = \dfrac{2\pi\Delta f}{s^2}$

(for some reason, your text changes from f_d to Δf)

Assuming small Δf to ensure linear model is valid.

Since $\dfrac{\Psi(s)}{\Phi(s)} = \dfrac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\Rightarrow \Psi(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\Phi(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\frac{2\pi\Delta f}{s^2}$$

$$= \frac{\Delta\omega}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For $\zeta < 1$, inverse transforming:

$$\psi(t) = \frac{\Delta f}{f_n\sqrt{1-\zeta^2}} e^{-2\pi\zeta f_n t}\left[\sin\left(2\pi f_n\sqrt{1-\zeta^2}\,t\right)\right]u(t)$$

As $t \to \infty$, $\psi(t) \to 0$. Hence steady state error becomes 0.

Summary of Steady-State Errors

$$2\pi f_\Delta = Ak_f$$

Table 3.5 Steady-State Errors

PLL order	$\theta_0 \neq 0$, $f_\Delta = 0$, $R = 0$	$\theta_0 \neq 0$, $f_\Delta \neq 0$, $R = 0$	$\theta_0 \neq 0$, $f_\Delta \neq 0$, $R \neq 0$
1 ($a = 0$, $b = 0$)	0	$2\pi f_\Delta / K_t$	∞
2 ($a \neq 0$, $b = 0$)	0	0	$2\pi R / K_t$
3($a \neq 0$, $b \neq 0$)	0	0	0

1^{st} PLL Acquisition Mode: 1^{st} PLL (Loop Filter)

Easy for analysis: let $F(s)=1 \Leftrightarrow f(t)=\delta(t)$

Recall $\theta(t)=K_t\int^t\int^\alpha \sin[\phi(\lambda)-\theta(\lambda)]f(\alpha-\lambda)d\lambda d\alpha$

$\Rightarrow \theta(t)=K_t\int^t \sin[\phi(\alpha)-\theta(\alpha)]d\alpha$

$\Rightarrow \dfrac{d\theta(t)}{dt}=K_t\sin[\phi(t)-\theta(t)]$

Let $m(t)=u(t)$, for FM: $\dfrac{d\theta(t)}{dt}=2\pi f_\Delta m(t)=2\pi f_\Delta$, for $t>0$

$\psi(t)=\phi(t)-\theta(t)\Rightarrow\theta(t)=\phi(t)-\psi(t)$ (phase error)

$\Rightarrow \dfrac{d\theta(t)}{dt}=\dfrac{d\phi(t)}{dt}-\dfrac{d\psi(t)}{dt}=2\pi f_\Delta-\dfrac{d\psi(t)}{dt}=\Delta\omega-\dfrac{d\psi(t)}{dt}$

$\Rightarrow \Delta\omega=\dfrac{d\psi(t)}{dt}+\dfrac{d\theta(t)}{dt}=\dfrac{d\psi(t)}{dt}+K_t\sin[\phi(t)-\theta(t)]$

Graphical representation (analysis): Phase-plane plot

Initial: zero phase and frequency errors (point B)

Step message applies and see trajectory

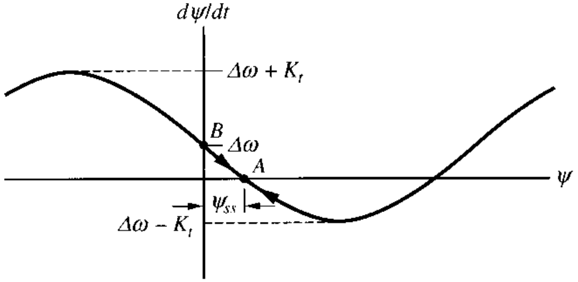


Figure 3.49
Phase-plane plot.

1st PLL Acquisition Mode: 1st PLL (Loop Filter)

$$\frac{d\psi(t)}{dt} = \Delta\omega - K_t \sin\psi(t)$$

Point B (when the step is applied): $\frac{d\psi(t)}{dt} \approx \Delta\omega$

Observation: $\frac{d\psi(t)}{dt} > 0$ if $\psi(t) > 0$ since dt always > 0, hence, we can start at point B

$\psi(t)$ increases $\rightarrow$ $\sin\psi(t)$ increases

$\rightarrow$ $\frac{d\psi(t)}{dt}$ decreases $\rightarrow$ $\frac{d\psi(t)}{dt}$ becomes negative

$\frac{d\psi(t)}{dt} = 0 \leftrightarrow$ point A

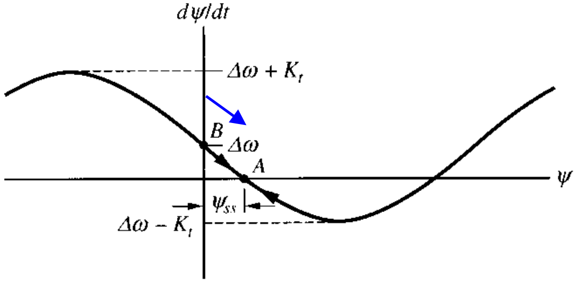


Figure 3.49
Phase-plane plot.

1^{st} PLL Acquisition Mode: 1^{st} PLL (Loop Filter)

$$\frac{d\psi(t)}{dt} = \Delta\omega - K_t \sin\psi(t)$$

Again, since $dt>0$, if $d\psi / dt < 0 \Rightarrow d\psi < 0$, hence, we can start at point C

@ C: $\dfrac{d\psi(t)}{dt} = \Delta\omega - K_t \qquad (\psi(t) = \pi/2)$

$$\frac{d\psi(t)}{dt} < 0 \rightarrow \psi(t) \text{ decreases} \rightarrow \sin\psi(t) \text{ decreases}$$

$$\rightarrow \frac{d\psi(t)}{dt} \text{ increases} \rightarrow \frac{d\psi(t)}{dt} \text{ becomes positive}$$

$\Rightarrow$ Point A is a locally stable point

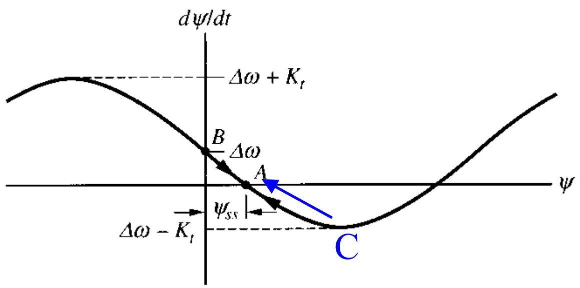


Figure 3.49
Phase-plane plot.

1^{st} PLL Acquisition Mode: 1^{st} PLL (Loop Filter)

$$\frac{d\psi(t)}{dt} = \Delta\omega - K_t \sin\psi(t)$$

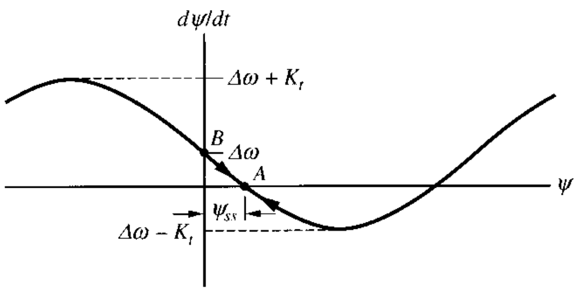


Figure 3.49
Phase-plane plot.

Remarks:

1) Steady-state error:

In this case, point A, $\frac{d\psi(t)}{dt} = 0$ (no frequency error)

As $t \to \infty$, $\psi(t) = \psi_{ss} \neq 0$ (phase error exists)

$K_t = 2\pi(50)$

$\Delta f = 12,\ 24,\ 48,\ 55\ \text{Hz}$

2) Lock range:

If the system is to converges to Point A, then $\Delta\omega < K_t$. So K_t is the lock range for the 1^{st} order PLL. If $\Delta\omega > K_t$, $\frac{d\psi(t)}{dt} = \Delta\omega - K_t > 0$. The phase-plane plot does not intersect with the $\frac{d\psi(t)}{dt} = 0$ axis.)

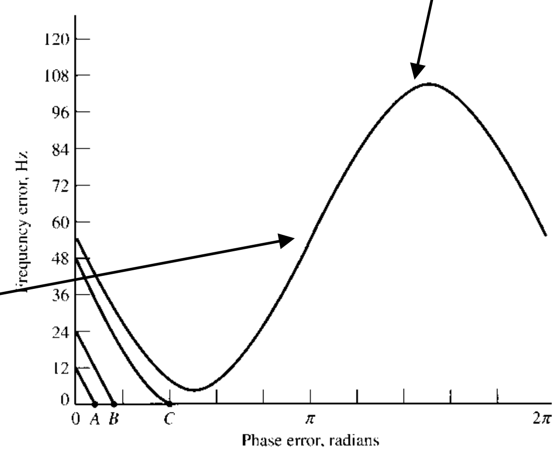


Figure 3.50
Phase-plane plot of first-order PLL for several initial frequency errors.

2nd order PLL: Transient and SS Responses

Remarks:

- 2nd order PLL has lock range $= \infty$ but has cycle-slipping
- cycle-slipping: the steady-state phase error is multiple of 2π rad
- E.g.

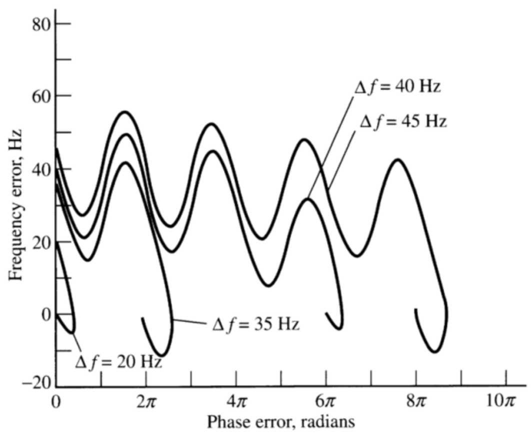
No cycle-slipping for $\Delta f = 20$ Hz

SS phase error $= 2\pi$ for $\Delta f = 35$ Hz (slipped one cycle)

SS phase error $= 6\pi$ for $\Delta f = 40$ Hz (slipped 3 cycles)

SS phase error $= 8\pi$ for $\Delta f = 45$ Hz (slipped 4 cycles)

Figure 3.51
Phase-plane plot for second-order PLL.



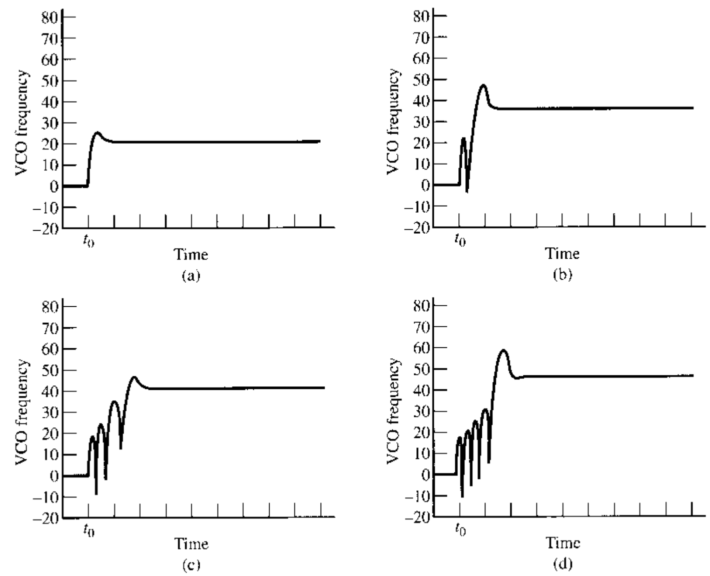


Figure 3.52
voltage-controlled oscillator frequency for four values of input frequency step. (a) VCO frequency for $\Delta f = 20$ Hz. (b) VCO frequency for $\Delta f = 35$ Hz. (c) VCO frequency for $\Delta f = 40$ Hz. (d) VCO frequency for $\Delta f = 45$ Hz.

Analog Pulse Modulation

- Samples from uniform sampling can have different representation
  - A sampled value can have 1-to-1 correspondence to some attribute of a pulse
    - If attribute changes continuously → analog pulse modulation
    - If attribute also takes on a certain value from a set of allowable values → digital pulse modulation
- Three attributes can be used
  - amplitude, width/duration, or position
- PAM – pulse amplitude (related to AM)
- PDM/PWM – pulse duration (related to angle mod)
- PPM – pulse position (related to angle mod)

Analog Pulse Modulation

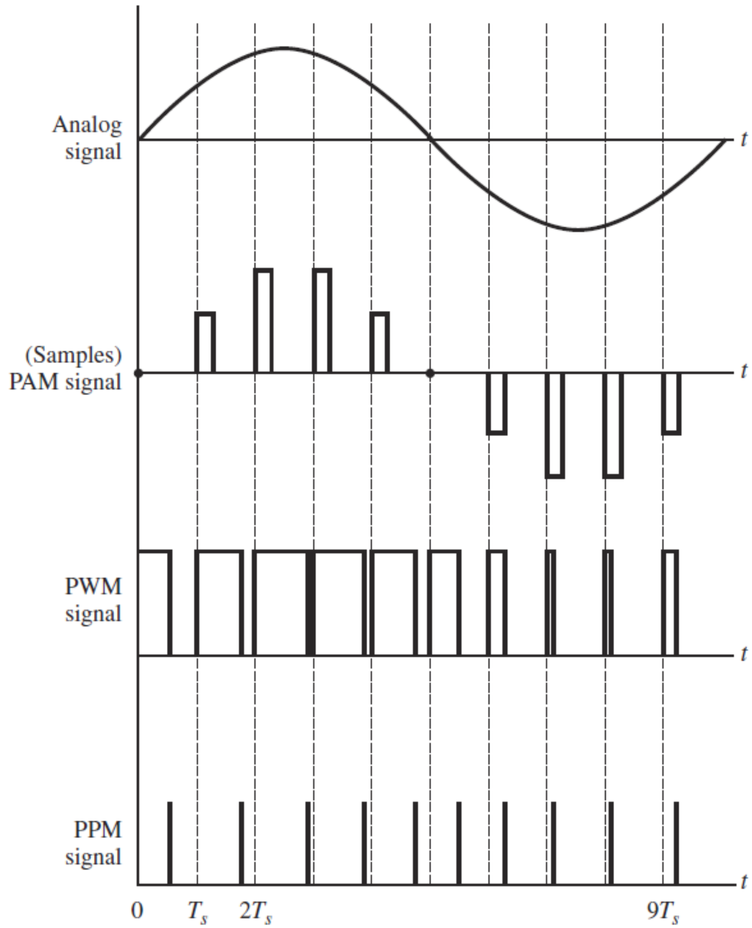


Figure 3.25
Illustration of PAM, PWM, and PPM.

PAM

Amplitude of each pulse corresponds to the value of the message signal $m(t)$ (at the leading edge of the pulse)

Different from sampling in previous chapter, PAM's sampling pulse has finite width

- can be generated using holding circuit
- Impulse and frequency response of holding circuit:

$$h(t) = \Pi\left(\frac{t - \frac{1}{2}\tau}{\tau}\right) \Leftrightarrow H(f) = \tau \operatorname{sinc}(f\tau) e^{-j\pi f\tau}.$$

It transforms the impulse function samples $m_\delta(t) = \sum_n m(nT_s)\delta(t - nT_s)$

to PAM waveform $$m_c(t) = \sum_n m(nT_s)\Pi\left(\frac{t - \left(nT_s + \frac{1}{2}\tau\right)}{\tau}\right)$$

PAM

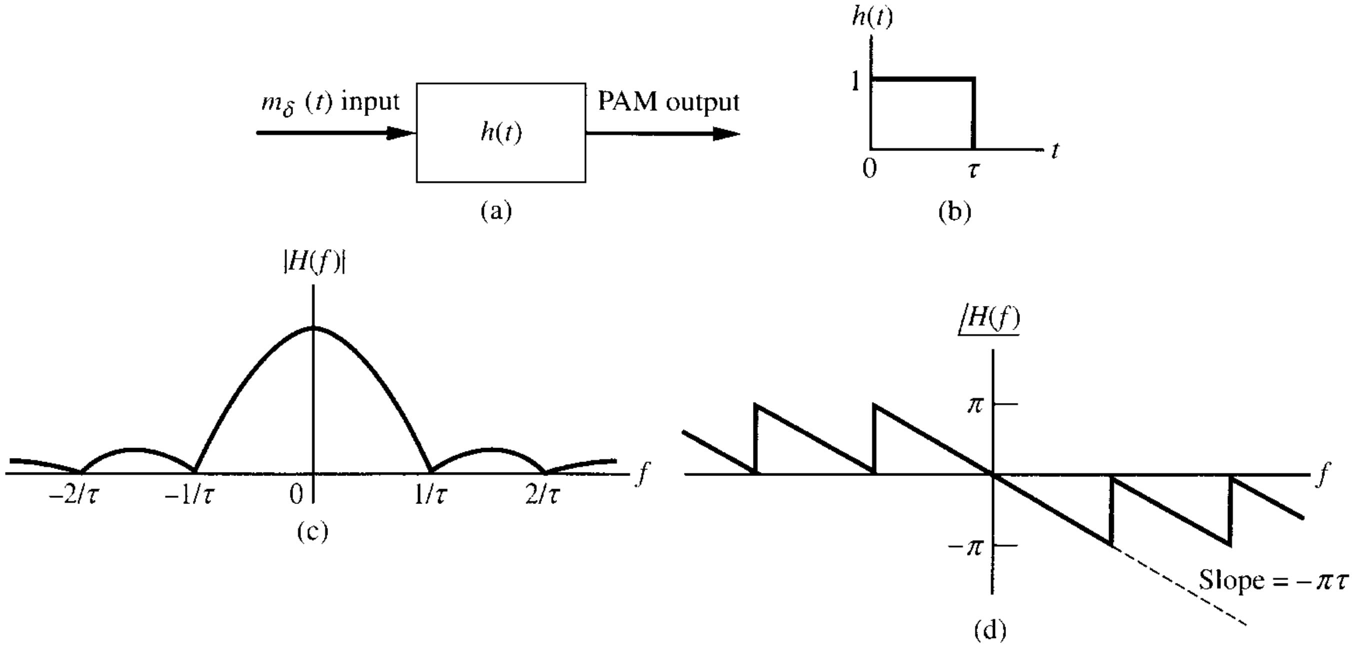


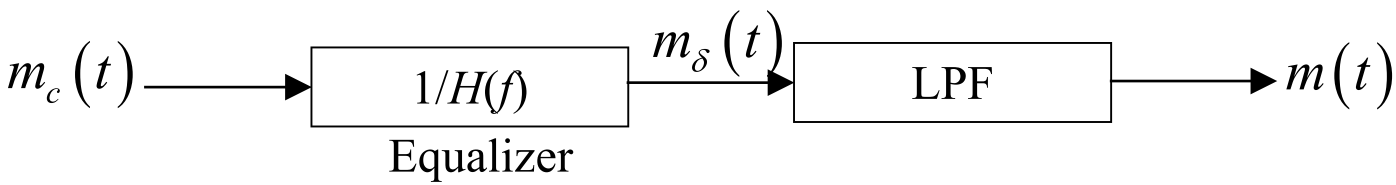
Figure 3.57

Generation of PAM. (a) Holding network. (b) Impulse response of holding network. (c) Amplitude response of holding network. (d) Phase response of holding network.

PAM

Unless pulse width τ is small, amplitude distortion in $m_c(t)$ can be significant.

- Solution: equalization, i.e. pass $m_c(t)$ through filter $\dfrac{1}{|H(f)|}$ prior to reconstruction



$$\therefore m_c(t) = m_\delta(t) * h(t) \Leftrightarrow M_c(f) = M_\delta(f) H(f)$$

Demodulation:

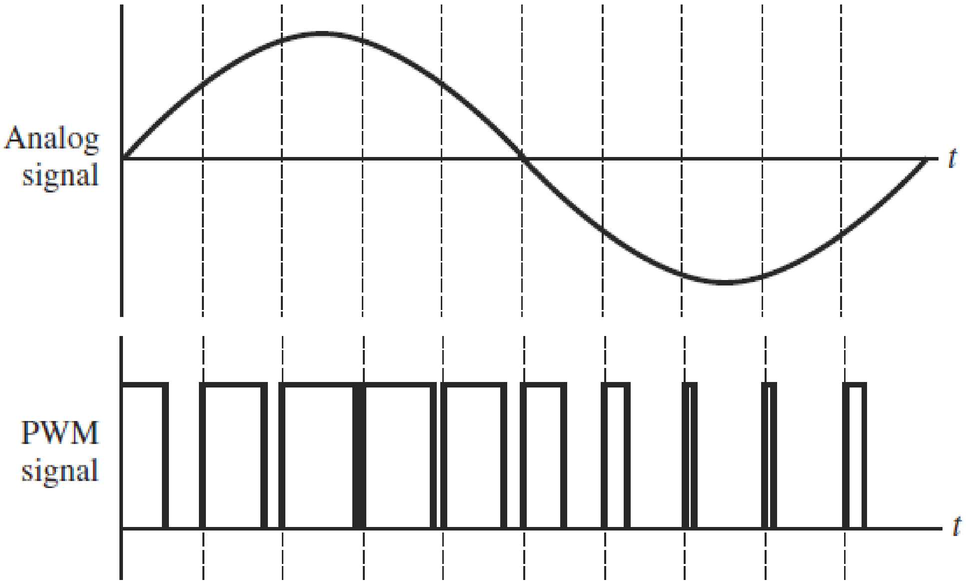
Recover $M_\delta(f) \Leftrightarrow m_\delta(t)$ samples

Recover $M(f) \Leftrightarrow m(t)$ message

$$M_\delta(f) = \frac{M_c(f)}{H(f)}$$

PWM

- Pulse width proportional to values of messasge
  - $0 = ½\ T_s$
  - Negative: $< ½\ T_s$
  - Positive: $> ½\ T_s$
  - Max. value $= 1/\ T_s$



PPM

- Pulse position is proportional to the values of the message
- PPM signal

$$x(t) = \sum_n g(t - t_n)$$

- $g(t)$ represents shape of the individual pulses
- t_n is the occurrence time – related to values of the message signal $m(t)$

Digital Pulse Modulation (related to AM)

- Messages are discrete-amplitude (finite levels) samples
  - DM – delta modulation
  - PCM – pulse-code modulation
- These methods fall into the category of *predictive coded modulation* where the difference between the current value of the input and predicted value are coded
  - Why? The difference contains less variance than coding the actual sample value, thus less bits need to be used to represent the coded value

DM

$m(t) \;\rightarrow\;$ samples (analog amplitude) $\rightarrow$ difference $\rightarrow$ binary

or

$m(t) \;\rightarrow\;$ difference $\rightarrow$ binary $\rightarrow$ samples

Operations:

Ms(t) is a reference signal

1) $d(t) = m(t) - m_s(t)$

2) $\Delta(t) = \text{threshold}(d(t)) \quad \begin{cases} \delta_0, & d(t) \geq 0, \\ -\delta_0, & d(t) < 0 \end{cases}, \quad \delta_0 = 1$ usually

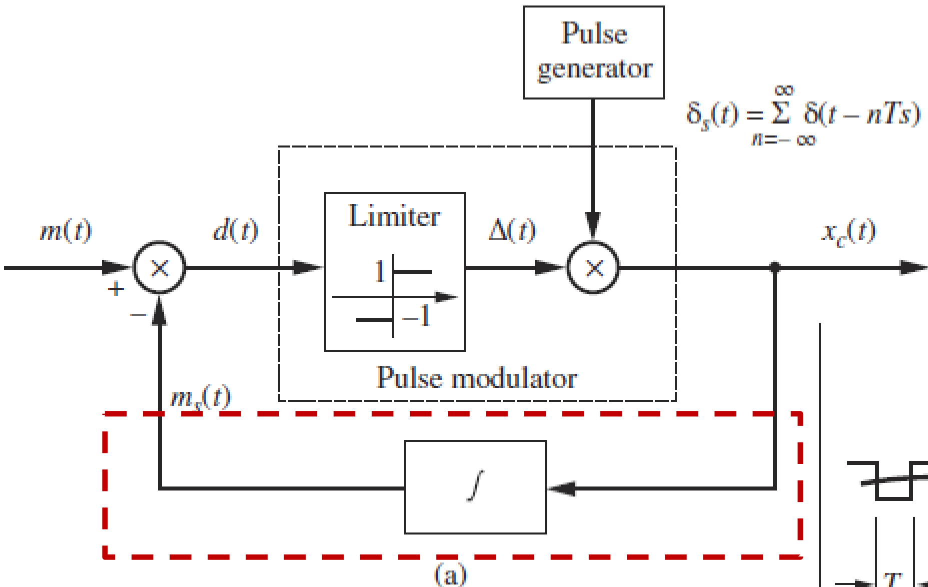
3) $x_c(t) =$ samples of $\Delta(t) = \Delta(t)\sum_n \delta(t - nT_s) = \sum_n \Delta(nT_s)\delta(t - nT_s)$

4) Prediction: $m_s(t) = \sum_n \Delta(nT_s)\int^t \delta(\alpha - nT_s)\,d\alpha$

DM

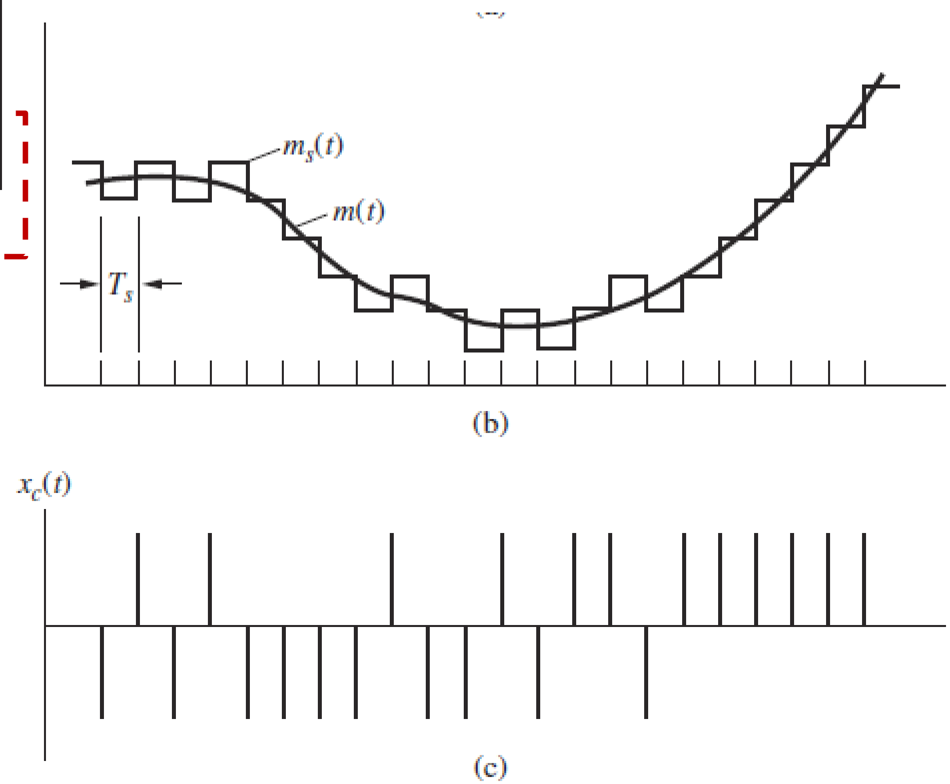
- Output of DM is a series of impulses, each having positive or negative polarity depending on the sign of $d(t)$ at the sampling instants
- In practice, pulse generator does not produce sequence of impulses functions
  - Output pulse of finite width
  - Impulses are assumed for analysis
- After integration, reference signal $m_s(t)$ is a stairstep approximation of $m(t)$

DM



Acts as a predictor that uses the previous sample in $x_c(t)$ and predict forward the next sample and compare to current "sample" in m(t).

- The integrator takes the value in $x_c(t)$ and holds in for T_s second. This value will be compare to the current value (next time instant) of $m(t)$



Delta modulation. (a) Delta modulator. (b) Modulation waveform and stairstep approximation. (c) Modulator output.

Demodulating DM

- Integrate $x_c(t)$ to form $m_s(t)$
- Then lowpass filter $m_s(t)$ to eliminate the jumps to get $m(t)$
- Note that the modulator/encoder contains part of the demodulator/decoder

Problem with DM: Slope Overload

- If message signal $m(t)$ has a slope greater than can be followed by the stair-step approximation $m_s(t)$
- Assume step-size = δ_0 → max. slope = δ_0/T_s can be used to follow $m(t)$
- Example (right)

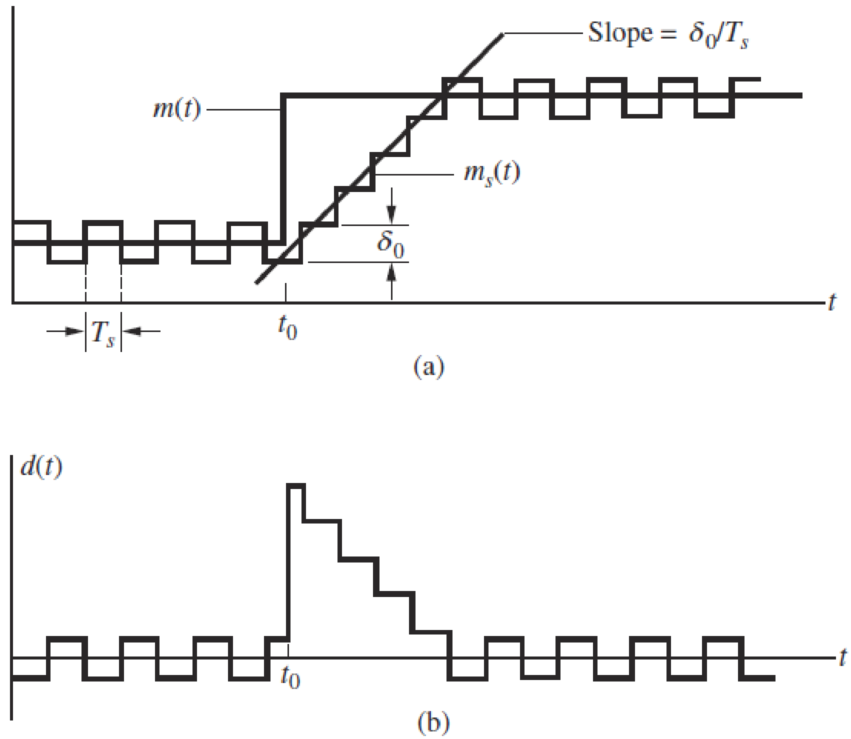


Illustration of slope overload. (a) Illustration of $m(t)$ and $m_s(t)$ for a step change in $m(t)$. (b) Error between $m(t)$ and $m_s(t)$ resulting from a step change in $m(t)$.

Mathematical Analysis of Slope Overload Problem

Assuming $m(t) = A\sin(2\pi f_1 t)$

Max. slope that $m_s(t)$ can follow is $S_m = \dfrac{\delta_0}{T_s}$

$$\frac{d}{dt}m(t) = 2\pi A f_1 \cos(2\pi f_1 t)$$

$\therefore\ m_s(t)$ can follow $m(t)$ without slope overload if

$$\frac{\delta_0}{T_s} \geq 2\pi A f.$$

So there is a BW constraint on $m(t)$ in order to avoid this problem.

Adaptive DM: Solution to Slope Overload

- Adjust the step-size δ_0 based on $x_c(t)$
- Idea:
  - If $m(t) \approx$ constant, $x_c(t)$ alternates in sign ➔ leads to small DC (close to zero) at output of LPF – this controls gain at variable gain amplifier ➔ $\delta_0\downarrow$ at the integrator input
  - If $m(t)\uparrow$ (or $\downarrow$) rapidly, $x_c(t)$ has the same polarity ➔ leads to big value of the magnitude of the output of LPF ➔ $\delta_0\uparrow$ at the integrator input ➔ reducing time-span of slope overload

Adjust step-size: if several output samples have same slope (sign), then increase step-size, else decrease

- Tradeoff smaller slope error with larger quantization error
- Solution: Use LPF to smooth the error

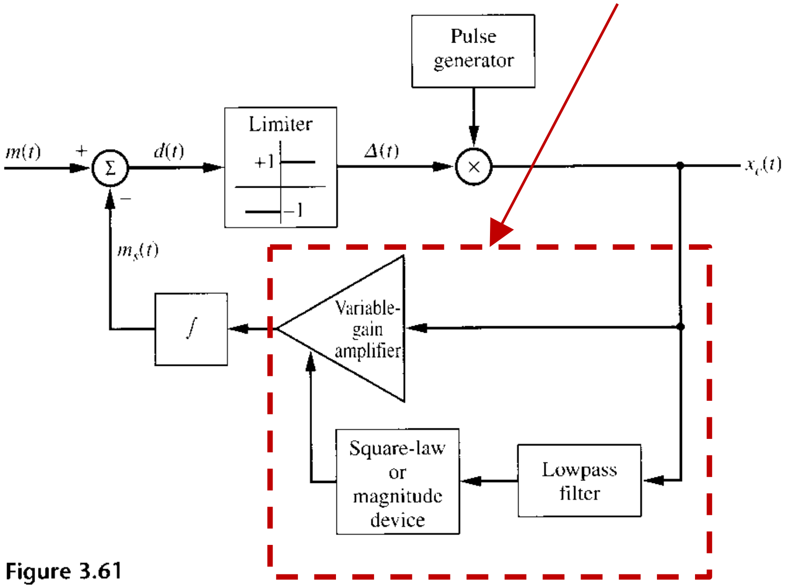


Figure 3.61
Adaptive delta modulator.

Adaptive Delta Demodulator

- Notice the demodulator is part of the modulator
  - The receiver is required to match changes in δ_0 that was made at the modulator
  - This is often used in waveform coders (speech and video)
    - Known as analysis-by-synthesis coding
      - Determine what parameters the coder should used by duplicating what the decoder does

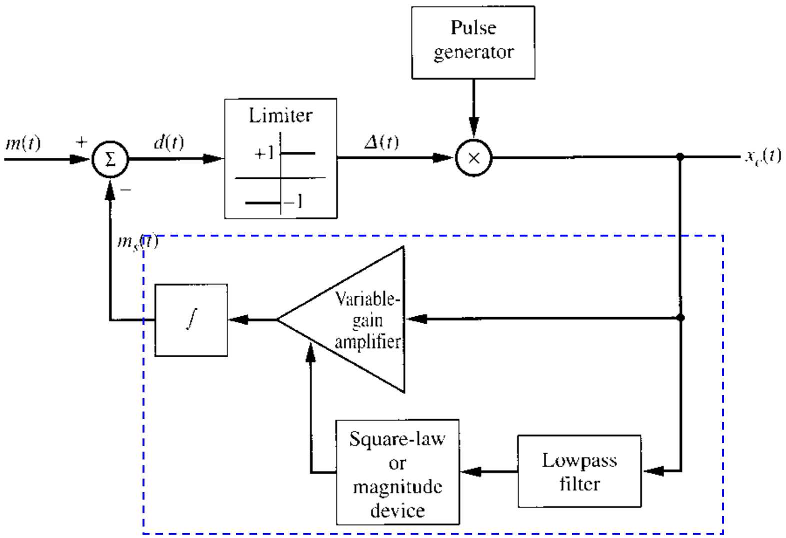


Figure 3.61
Adaptive delta modulator.

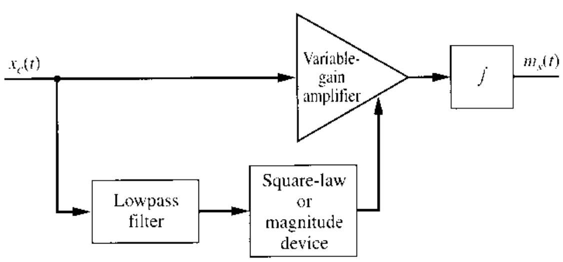


Figure 3.62
Adaptive DM receiver.

Pulse-Code Modulation (PCM)

- *m*(*t*) → samples (analog amplitude) → quantized samples → binary representation (e.g. 8 levels in Fig. 3.29(b)) → representation as pulses
- Pros
  - More reliable communication
- Cons
  - Wide BW (← reduced by "compression")
  - Complicated circuits (← cost reduced by VLSI)

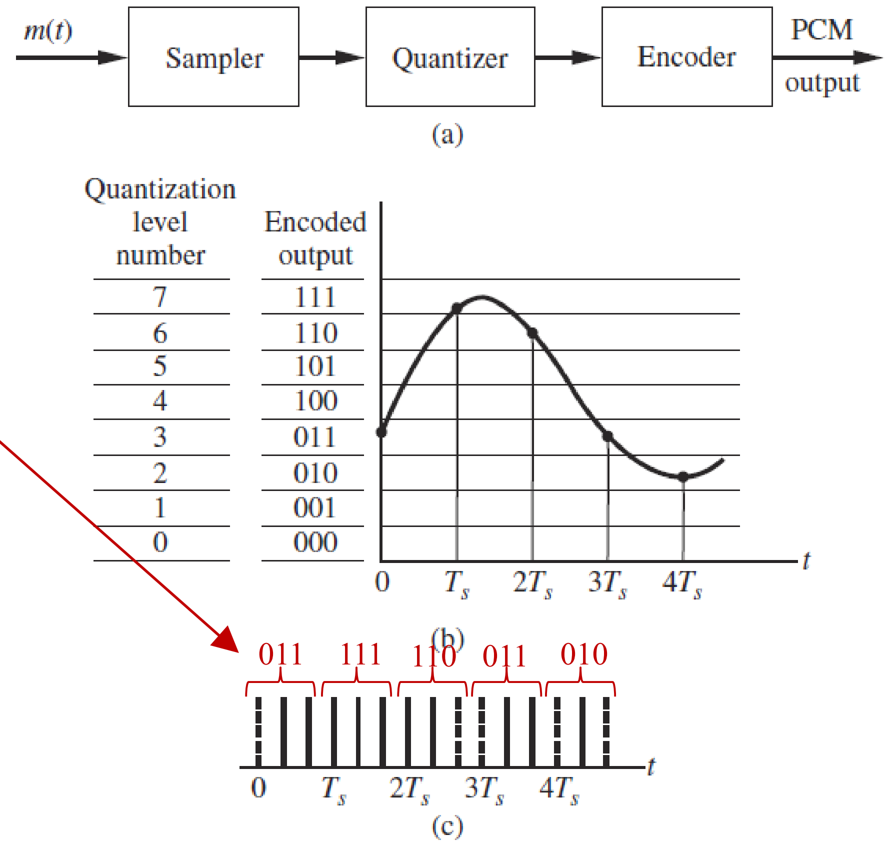


Fig. 3.29. Generation of PCM. (a) PCM modulator. (b) Quantizer and coder. (c) Representation of coder output.

BW of PCM

Assume the number of quantization levels $= q = 2^n$ (e.g. on last page: $q = 8,\ n = 3$)

$\Rightarrow n = \log_2 q$ binary pulses must be transmitted for each sample of the message signal.

Let: Message BW= W

Sampling rate $= 2W$

$\Rightarrow 2nW$ binary pulse/second

Thus, max. width of each binary pulse is

$$(\Delta\tau)_{\max} = \frac{1}{2nW}$$

$$\Rightarrow \text{transmission BW } \approx 2knW,$$

k is a proportionality constant

Hence, $B \approx 2Wk \log_2 q$.

Recovered message error is due mainly to quantization error

Thus, $q \uparrow \ \rightarrow$ error $\downarrow \ \rightarrow B \uparrow$

PCM Modulating RF Carrier

- PCM waveform can be transmitted on an RF carrier using amplitude, phase, or frequency modulation
- Figures shows data bits are represented by an non-return to zero (NRZ) waveform for **serial transmission** (hence, symbol sync is important)
  - 6 bits are shown (101001)
  - ASK
    - Carrier amplitude determined by data bit for that interval
  - PSK
    - Phase of carrier is established by the data bit
  - FSK
    - Carrier freq. is established by the data bit

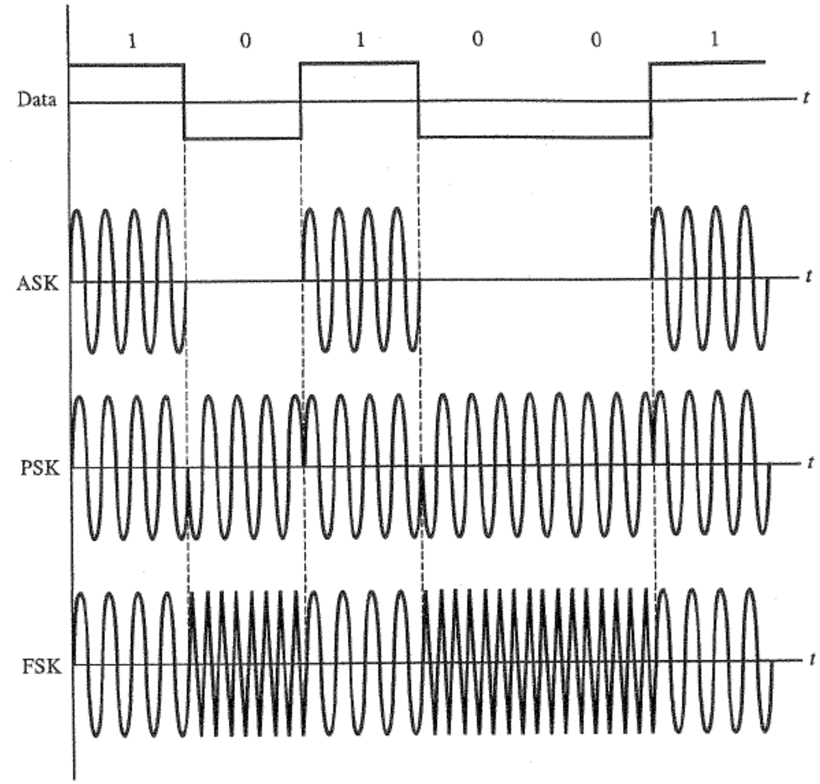


FIGURE 3.66 An example of digital modulation schemes.

Multiuser Communication Systems - Multiplexing

- A number of data sources share the same communication
- Used mainly to multiplex signals from different users onto the same channel for transmission
  - Can also be used for stereophonic FM transceiver to multiplex sum and differences of signals
- Different multiplexing techniques
  - Frequency-division multiplexing (FDM)
  - Quadrature multiplexing (QM)
  - Time-division multiplexing (TDM)

Frequency- Division Multiplexing (FDM)

- Signals from different sources can used different modulation
  - Source 1 uses DSB
  - Source 2 uses SSB
  - Source 3 uses FM
- BPF used at receiver to retrieve signal from different sources
  - Guard bands are injected between each source signal before transmission to realize non-ideal BP filtering at Rx
- BW is lower bounded by the sum of the BWs of the message signals:

$$B = \sum_{i=1}^{N} W_i$$

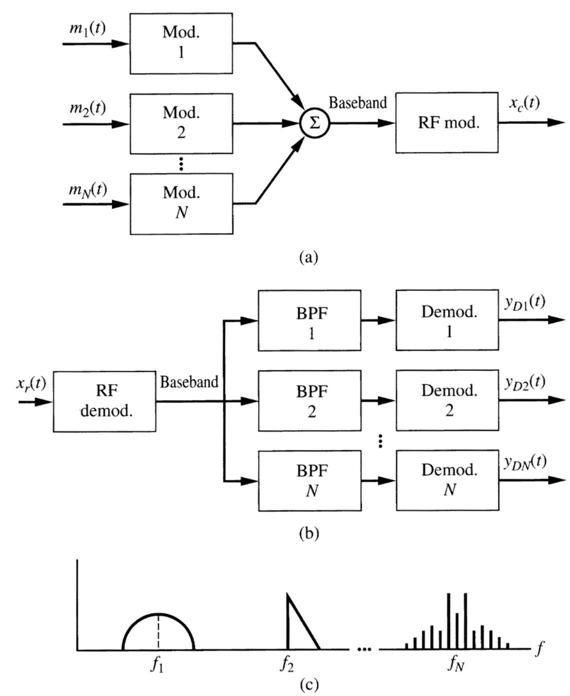


Figure 3.64 Frequency-division multiplexing. (a) FDM modulator. (b) FDM demodulator. (c) Baseband spectrum.

Example of FDM: Stereophonic FM Broadcasting

- Stereo signal is perceived by having speakers outputing sum and differences of the monotonically recorded signal
- Backward compatibility is required
  - Necessary for stereophonic FM receiver to demodulate monophonic FM signal
  - 0-15 kHz carries L+R (for monophonic receiver)
  - 24-53 kHz carries L-R (stereophonic receiver uses L+R and L-R)
    - Information about the carrier is inserted by the Tx for coherent demodulation at the Rx

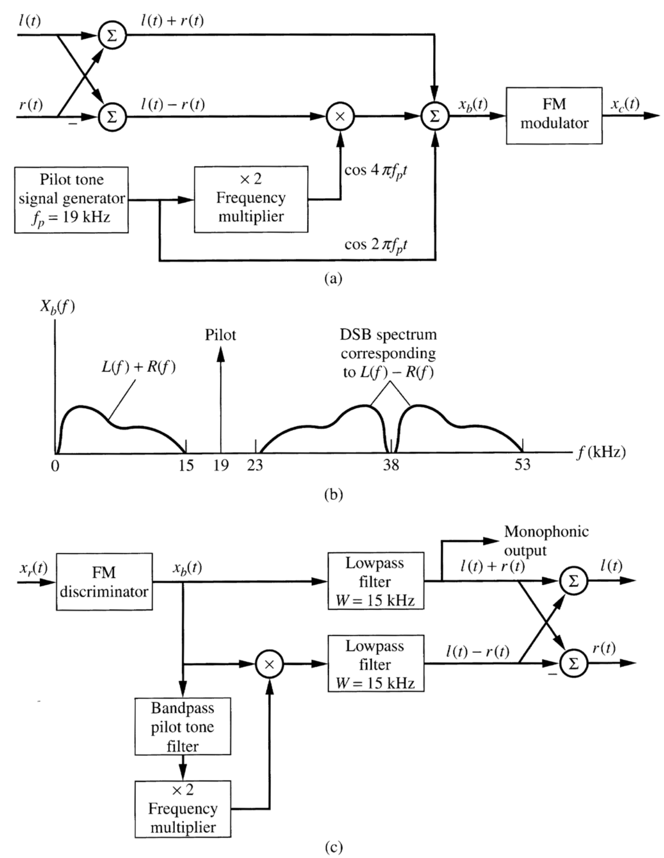


Figure 3.65
Stereophonic FM transmitter and receiver. (a) Stereophonic FM transmitter. (b) Single-sided spectrum of FM baseband signal. (c) Stereophonic FM receiver.

QM

- QM is not a FDM technique as spectra of $m_1(t)$ and $m_2(t)$ overlap in frequency
  - SSB is a QM signal with $m_1(t) = m(t)$ and $m_2(t) = \pm\hat{m}(t)$

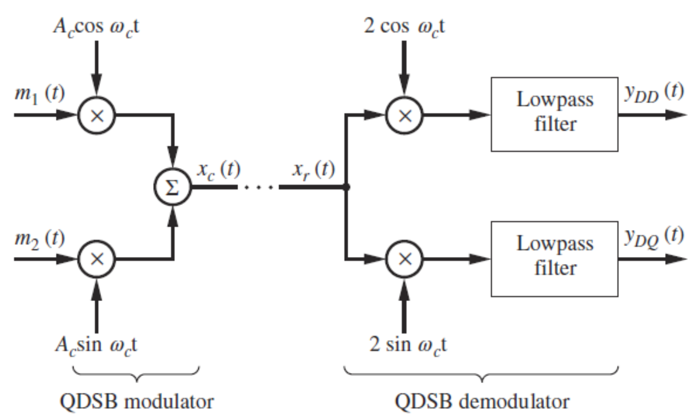


Figure 4.36
Quadrature multiplexing.

Modulation: $x_c(t) = A_c\left[m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)\right]$

Demodulation: If carrier phase is unknown, i.e.

$$x_r(t)\bullet 2\cos(2\pi f_c t+\theta) = A_c\left[m_1(t)\cos\theta - m_2(t)\sin\theta\right]$$
$$+A_c\left[m_1(t)\cos(4\pi f_c t+\theta) + m_2(t)\sin(4\pi f_c t+\theta)\right]$$

After LPF: output becomes

$$y_{DD}(t) = A_c\left[m_1(t)\cos\theta - m_2(t)\sin\theta\right] \quad \text{(ideal: } \theta \to 0)$$

Time-Division Multiplexing (TDM)

- Tx
  - Data sources are assumed to have been sampled at Nyquist rate or higher
  - Commutator interlaces the samples to form the baseband signal
- Rx
  - Baseband signal is demultiplexed by using a second commutator

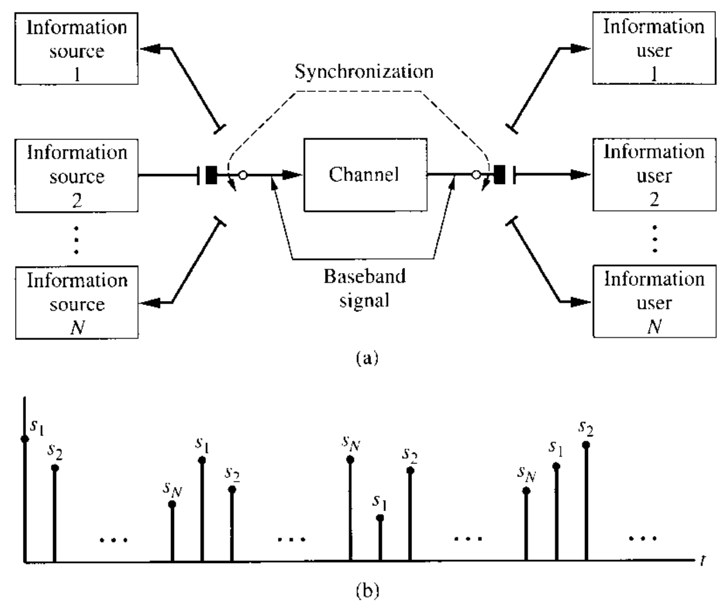


Figure 3.67
Time-division multiplexing. (a) TDM system. (b) Baseband signal.

BW of TDM

A "rough" estimate of BW

Let:

BW of i^{th} channel $= W_i$

Sampling period of baseband signal = T

$\Rightarrow$ Samples in every T second for the ith channel = $2W_iT$ samples

$\Rightarrow$ Total samples (for all channels) in every T second: $n_s = \sum_{i=1}^{N} 2W_iT$

or

Assuming baseband is lowpass signal with BW B, required sampling rate is $2B$.

In a T second interval, there are $2BT$ total samples.

$$\Rightarrow n_s = 2BT = \sum_{i=1}^{N} 2W_iT$$

so
$$B = \sum_{i=1}^{N} W_i$$

This is same as FDM.

Example of TDM: Digital Telephone System

- Voice signal sampling: 8,000 samp/s
  - Each sample is quantized to 7 + 1 bit
    - 1-bit for signaling
      - call establishment and synchronization
- Bit rate: 8 bit/samp * 8,000 samp/s= 64 kbps
- T1 line
  - Group of 24 8-bit voice channels
    - 24 voice ch * 8 bit/samp + 1 = 193 bit
      - Extra 1-bit for frame synchronization
  - Frame rate
    - 193 bit/frame * 8,000 frame/sec – 1.544 Mbps

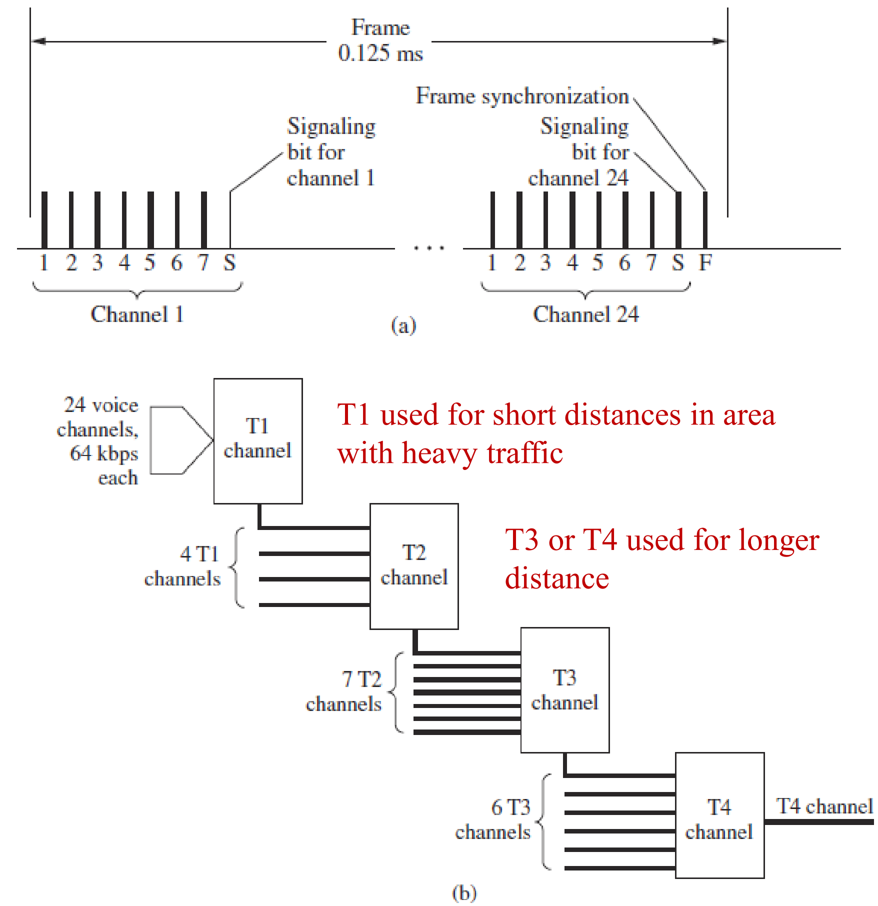


Figure 3.31
Digital multiplexing scheme for digital telephone. (a) T1 frame. (b) Digital multiplexing.

Comparison Between Different Mux Techniques

- FDM
  - Pros
    - Simple to implement
  - Cons
    - Intermodulation distortion (crosstalk) due to nonlinear channel
- TDM
  - Pros
    - Less crosstalk (assuming memoryless channel)
  - Cons
    - Difficult to keep synchronization (frame structure, header)
- QM
  - Pros
    - Efficient use of channel
  - Cons
    - Crosstalk between I and Q channels (needs coherent demodulation)