Baseband Communication Systems In the Absence of Noise

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Digital Baseband Modulation

- aka line coding
  - Transfer digital bit stream (e.g. after pulse modulation) over an analog baseband channel (e.g. serial bus in PCs)
  - Includes pulse shaping, synchronization, bandwidth reduction
- Pulse shaping (filtering)
  - Avoidance of intersymbol interference (ISI)
    - Convolutive noise
- Synchronization
  - Carrier sync, **symbol sync**, frame (groups of symbols) synchronization

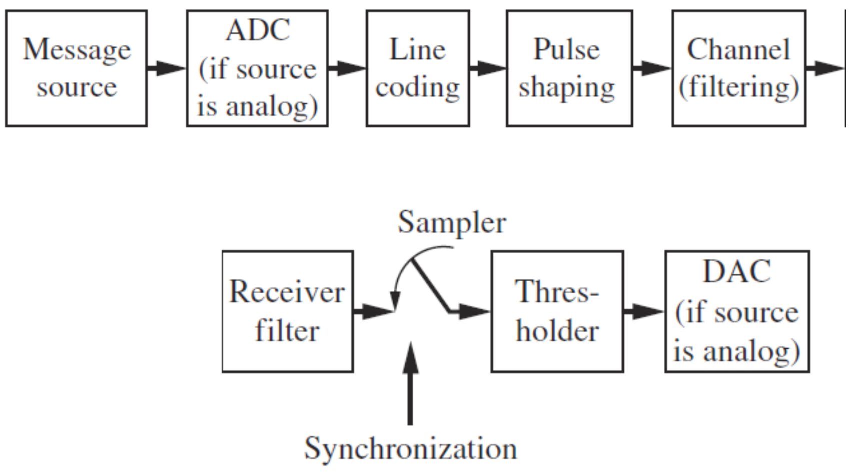


Figure 5.1
Block diagram of a baseband digital data transmission system.

Line Code

- Baseband data format can influence digital modulated signal. Several formats are available
- Nonreturn-to-zero (NRZ)
  - 1: positive level, *A*
  - 0: negative level, –*A*
- NRZ mark
  - 1: a change in level
  - 0: no change in level
- Unipolar RZ
  - 1: ½-width pulse (pulse that returns to 0)
  - 0: no pulse
- Polar RZ
  - 1: positive RZ pulse
  - 0: negative RZ pulse
- Bipolar RZ
  - 1: alternating RZ pulses
  - 0: 0 level
- Split phase (Manchester)
  - 1: *A* switching to –*A* at ½ symbol period
  - 0: -*A* switching to *A* at ½ symbol period
  - Transition occurs at low frequency
  - Can be obtained from NRZ by multiplying a square-wave clock waveform with a period equal to the symbol duration

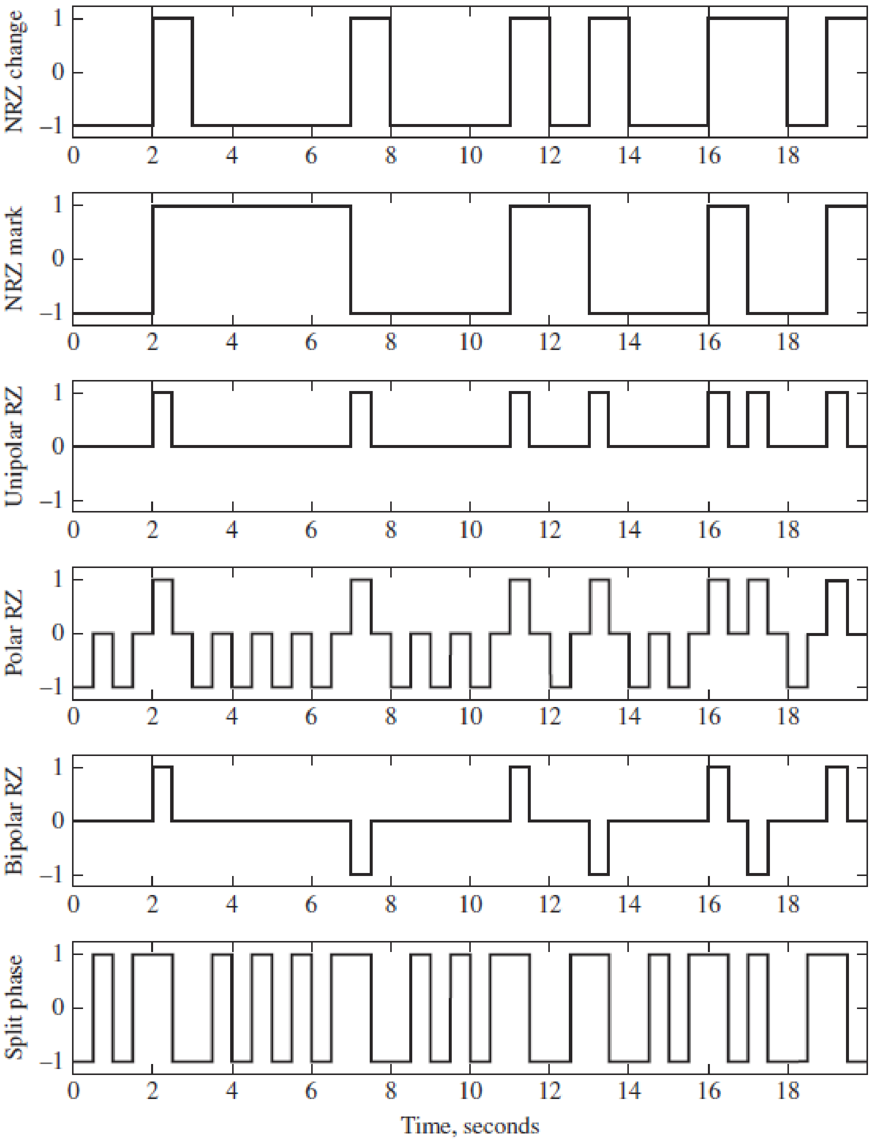


Figure 5.2
Abbreviated list of binary data formats.[1]

Issues Concerning Choice of Data Format

- Self-synchronization (symbol detection)
  - Can synchronizers be easily designed to extract timing clock from the code?
- Power spectrum
  - Is power spectrum of the code suitable for particular channel spectrum under consideration?
- Transmission BW
  - Which code occupies the least amount of BW? May conflict with other issues
  - Investigates its PSD
- Transparency
  - Every possible data sequence should be received faithfully and transparently
- Error detection capability
  - Some data format offers inherent data correction ability
- Good bit error probability performance
  - Easy to implement minimum error probability receivers using the chosen data format

Power Spectra of Line Coded Data

Bandwidth requirement of line-coded data can be computed by looking at its PSD

$$x(t) \triangleq \sum_k a_k p(t - kT - \Delta)$$

Let $\ldots, a_{-1}, a_0, a_1, \ldots, a_k, \ldots$ be a sequence of RVs, indep with Δ, with correlation

$$E[a_k a_{k+m}] = \int_a a_k a_{k+m} p_{A_k}(a_k)\, da_k = R_m,\ m = 0, \pm 1, \pm 2, \ldots$$

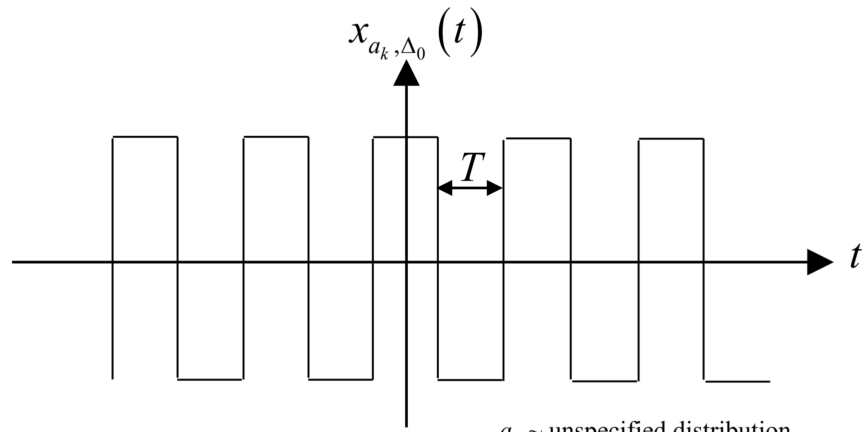
$$\Rightarrow R_{xx}(\tau) \triangleq E[x(t)x(t+\tau)]$$

$$= E\left[\sum_k \sum_m a_k a_{k+m} p(t - kT - \Delta)\, p(t + \tau - (k+m)T - \Delta)\right]$$

Indep. assumption →

$$= \sum_k \sum_m E[a_k a_{k+m}]\, E\begin{bmatrix} p(t - kT - \Delta) \\ p(t + \tau - (k+m)T - \Delta) \end{bmatrix}$$

$$= \sum_m R_m \sum_k \frac{1}{T} \int_{\Delta=-T/2}^{T/2} p(t - kT - \Delta)\, p(t + \tau - (k+m)T - \Delta)\, d\Delta$$

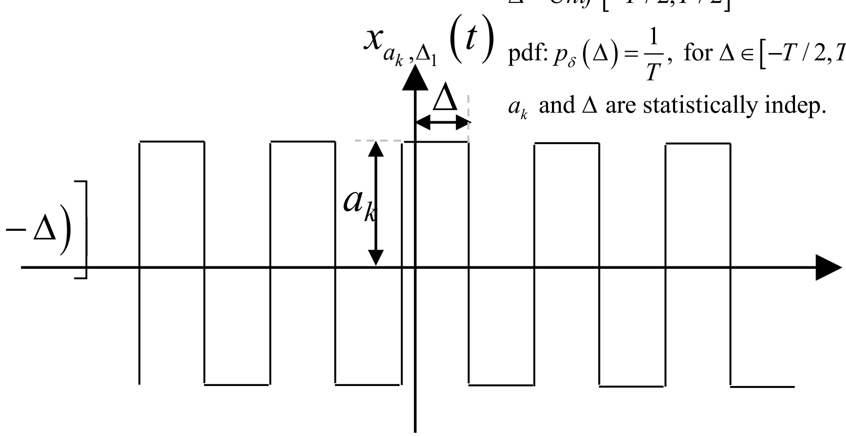


$a_k \sim$ unspecified distribution

$\Delta \sim Unif[-T/2, T/2]$

pdf: $p_\delta(\Delta) = \frac{1}{T}$, for $\Delta \in [-T/2, T/2]$

a_k and Δ are statistically indep.



$$x_{a_k,\Delta_1}(t) \triangleq \sum_k a_k p(t - kT - \Delta_1)$$

Power Spectra of Line Coded Data

$$R_{xx}(\tau)=\sum_m R_m \sum_k \frac{1}{T}\int_{\Delta=-T/2}^{T/2} p(t-kT-\Delta)\,p(t+\tau-(k+m)T-\Delta)\,d\Delta$$

Let $u=t-kT-\Delta$

$$\Rightarrow R_{xx}(\tau)=\sum_m R_m \sum_k \frac{1}{T}\int_{u=t-(k+1/2)T}^{t-(k-1/2)T} p(u)\,p(u+\tau-mT)\,du$$

$$=\sum_m R_m \left[\frac{1}{T}\int_u p(u)\,p(u+\tau-mT)\,du\right]$$

$$=\sum_m R_m r(\tau-mT),$$

where $r(\tau)\triangleq\frac{1}{T}\int_u p(t)\,p(t+\tau)\,dt=\frac{1}{T}p(t)*p(-t)$

Power Spectra of Line Coded Data

$$R_{xx}(\tau) = \sum_m R_m r(\tau - mT)$$

$$\Rightarrow S_{xx}(f) = F\{R_{xx}(\tau)\} = F\left\{\sum_m R_m r(\tau - mT)\right\}$$

$$= \sum_m R_m F\{r(\tau - mT)\}$$

$$= \sum_m R_m S_{rr}(f) e^{-j2\pi mTf}$$

$$= S_{rr}(f) \sum_m R_m e^{-j2\pi mTf}$$

Since $r(\tau) \triangleq \dfrac{1}{T}\int_u p(t)p(t+\tau)\,dt = \dfrac{1}{T} p(t) * p(-t)$

$$\Rightarrow \quad S_{rr}(f) = \frac{1}{T}\left|P(f)\right|^2, \text{ where } P(f) = F\{p(t)\}$$

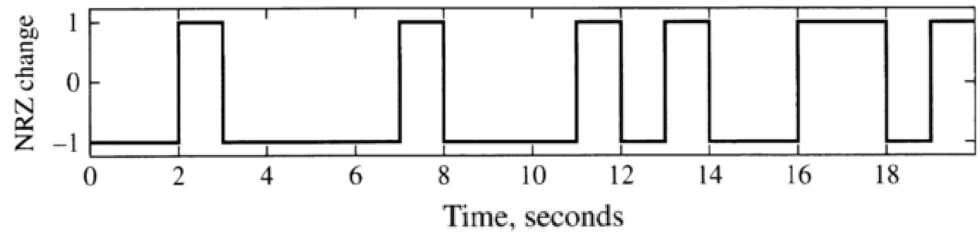
$$\Rightarrow \quad S_{xx}(f) = \frac{1}{T}\left|P(f)\right|^2 \sum_m R_m e^{-j2\pi mTf}$$

Example 4.1 – PSD of NRZ

For NRZ, pulse shape function $p(t) = \Pi(t/T)$

$\Rightarrow P(f) = T\text{sinc}(Tf)$

$$\Rightarrow S_{rr}(f) = \frac{|T\text{sinc}(Tf)|^2}{T} = T\text{sinc}^2(Tf)$$



For $R_m = E[a_k a_{k+m}]$, note that the amplitude $= +A$ half the time, and equals $-A$ half the time.

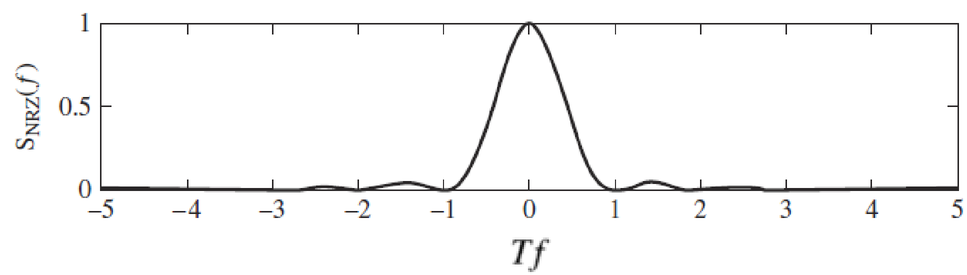
$$S_{xx}(f) = \frac{1}{T}|P(f)|^2 \sum_m R_m e^{-j2\pi mTf}$$

For a sequence of two pulses with a given sign on the first pulse, the second pulse is $+A$ half the time and $-A$ half the time

$$\Rightarrow R_m = \begin{cases} \frac{1}{2}A^2 + \frac{1}{2}(-A)^2 = A^2, & m = 0 \\ \frac{1}{4}A(A) + \frac{1}{4}A(-A) + \frac{1}{4}(-A)A + \frac{1}{4}(-A)(-A) = 0, & m \neq 0. \end{cases}$$

$\Rightarrow S_{NRZ}(f) = A^2 T\text{sinc}^2(Tf).$

First null @$1/T$ Hz



Example 4.2 – PSD of Split Phase

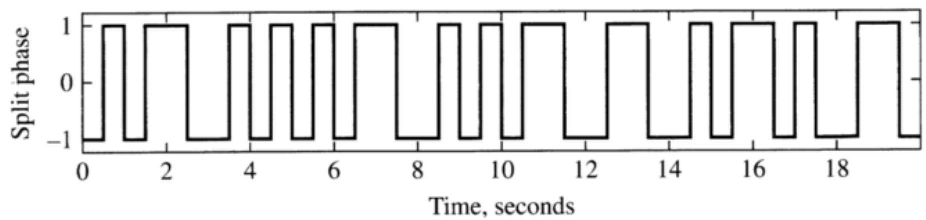
For split phase, pulse shape function $p(t)=\Pi\left(\frac{t+T/4}{T/2}\right)-\Pi\left(\frac{t-T/4}{T/2}\right)$

$$S_{xx}(f)=\frac{1}{T}\left|P(f)\right|^2\sum_m R_m e^{-j2\pi mTf}$$

$$\Rightarrow P(f)=\frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)e^{j2\pi(T/4)f}-\frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)e^{-j2\pi(T/4)f}$$

$$=\frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)\left(e^{j2\pi(T/2)f}-e^{-j2\pi(T/2)f}\right)$$

$$=jT\text{sinc}\left(\frac{T}{2}f\right)\sin\left(\frac{\pi T}{2}f\right)$$



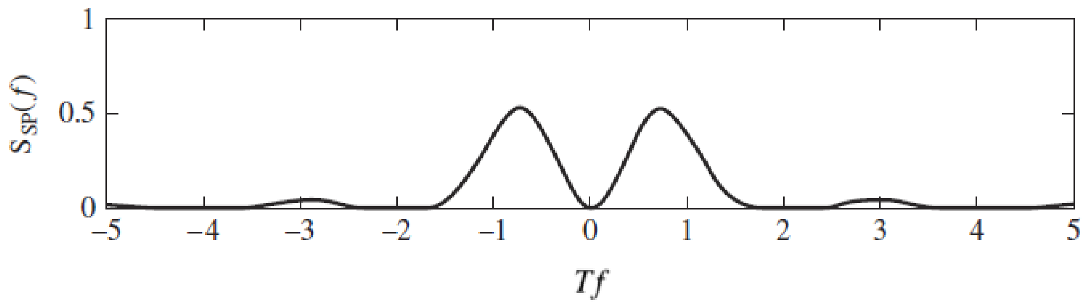
Note that the R_m is the same as that in NRZ

$$\Rightarrow S_{rr}(f)=\frac{1}{T}\left|jT\text{sinc}\left(\frac{T}{2}f\right)\sin\left(\frac{\pi T}{2}f\right)\right|^2=T\text{sinc}^2\left(\frac{T}{2}f\right)\sin^2\left(\frac{\pi T}{2}f\right)$$

$$\Rightarrow S_{SP}(f)=A^2T\text{sinc}^2\left(\frac{T}{2}f\right)\sin^2\left(\frac{\pi T}{2}f\right).$$

Split phase requires twice the BW as NRZ

1. First null @$2/T$ Hz
2. $S_{SP}(f)=0 \Rightarrow$ good if channel does not pass DC value



Example 4.3 – PSD of Unipolar RZ

For $R_m = E\left[a_k a_{k+m}\right]$, note that the amplitude $= +A$ half the time, and equals 0 half the time.

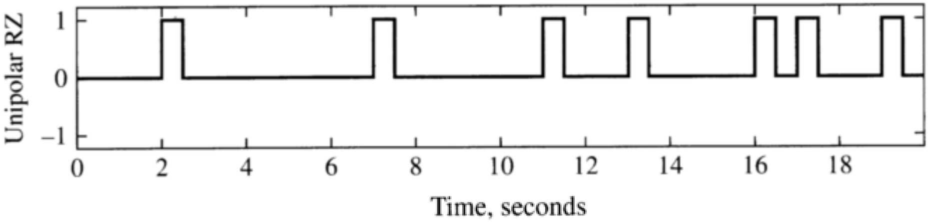
For a sequence of two pulses with a given sign on the first pulse, the second pulse is $+A$ half the time and 0 half the time

$$R_m = \begin{cases} \frac{1}{2}A^2 + \frac{1}{2}(0)^2 = \frac{1}{2}A^2, & m = 0 \\ \frac{1}{4}A(A) + \frac{1}{4}A(0) + \frac{1}{4}(0)A + \frac{1}{4}(0)(0) = \frac{1}{4}A^2, & m \neq 0. \end{cases}$$

For unipolar RZ, pulse shape function $p(t) = \prod(2t/T)$

$$\Rightarrow P(f) = \frac{T}{2}\operatorname{sinc}\left(\frac{T}{2}f\right)$$

$$\Rightarrow S_{rr}(f) = \frac{1}{T}\left|\frac{T}{2}\operatorname{sinc}\left(\frac{T}{2}f\right)\right|^2 = \frac{T}{4}\operatorname{sinc}^2\left(\frac{T}{2}f\right)$$



Example 4.3 – PSD of Unipolar RZ

$$\Rightarrow S_{URZ}(f)=\frac{T}{4}\mathrm{sinc}^2\left(\frac{T}{2}f\right)\left(\frac{1}{2}A^2+\frac{1}{4}A^2\sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty}e^{-j2\pi mTf}\right)$$

$$=\frac{T}{4}\mathrm{sinc}^2\left(\frac{T}{2}f\right)\left(\frac{1}{4}A^2+\frac{1}{4}A^2\sum_{m=-\infty}^{\infty}e^{-j2\pi mTf}\right)$$

$$S_{xx}(f)=\frac{1}{T}\left|P(f)\right|^2\sum_{m}R_m e^{-j2\pi mTf}$$

Since $\displaystyle\sum_{m=-\infty}^{\infty}e^{-j2\pi mTf}=\sum_{m=-\infty}^{\infty}e^{j2\pi mTf}=\frac{1}{T}\sum_{n}\delta\left(f-\frac{n}{T}\right)$

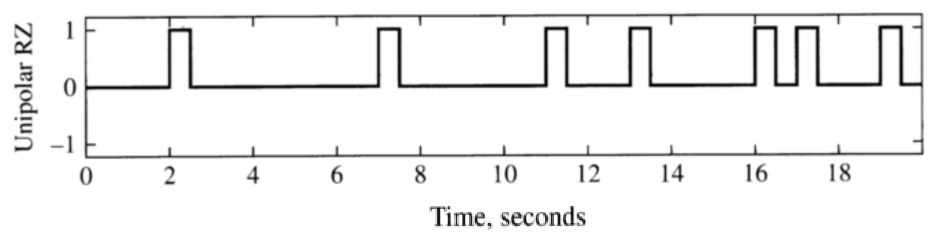
$$\Rightarrow S_{URZ}(f)=\frac{T}{4}\mathrm{sinc}^2\left(\frac{T}{2}f\right)\left[\frac{1}{4}A^2+\frac{1}{4}\frac{A^2}{T}\sum_{n}\delta\left(f-\frac{n}{T}\right)\right]$$

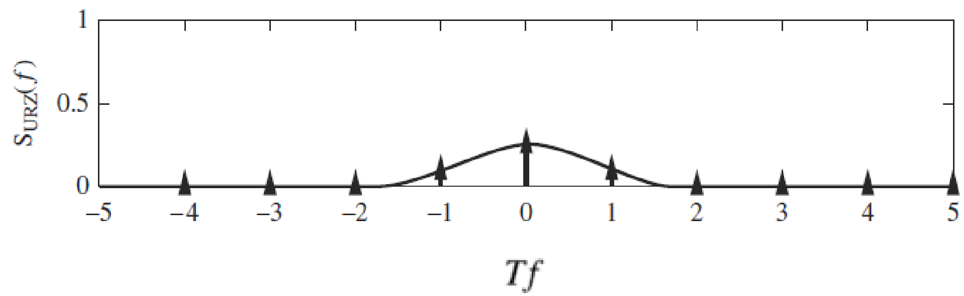
$$=\frac{A^2T}{16}\mathrm{sinc}^2\left(\frac{T}{2}f\right)+\frac{A^2}{16}\delta(f)$$

$$+\frac{A^2}{16}sinc^2\left(\frac{1}{2}\right)\left[\delta\left(f-\frac{1}{T}\right)+\delta\left(f+\frac{1}{T}\right)\right]$$

$$+\frac{A^2}{16}\mathrm{sinc}^2\left(\frac{3}{2}\right)\left[\delta\left(f-\frac{3}{T}\right)+\delta\left(f+\frac{3}{T}\right)\right]+\cdots$$

First null @$2/T$ Hz





Example 4.4 – PSD of Polar RZ

$$S_{xx}(f) = \frac{1}{T}\left|P(f)\right|^2 \sum_m R_m e^{-j2\pi mTf}$$

$R_m = E\left[a_k a_{k+m}\right]$ is same as NRZ

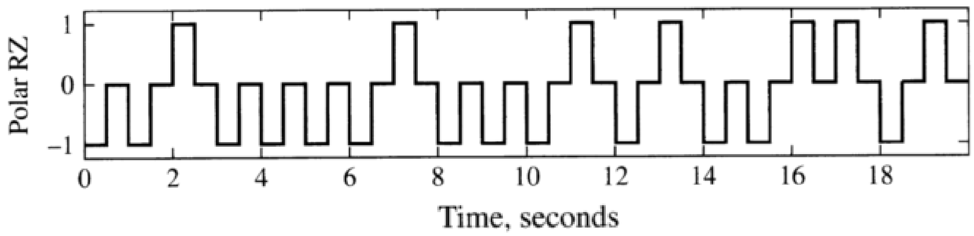
pulse shape function $p(t) = \Pi(2t/T)$ (same as URZ)

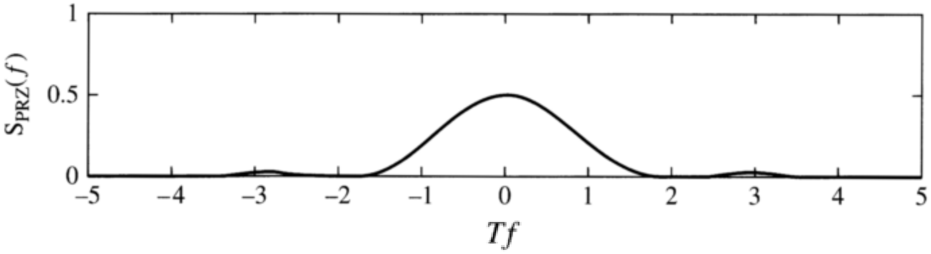
$$\Rightarrow P(f) = \frac{T}{2}\operatorname{sinc}\left(\frac{T}{2}f\right)$$

$$\Rightarrow S_{rr}(f) = \frac{\left|\frac{T}{2}\operatorname{sinc}\left(\frac{T}{2}f\right)\right|^2}{T} = \frac{T}{4}\operatorname{sinc}^2\left(\frac{T}{2}f\right)$$

$$\Rightarrow S_{PRZ}(f) = \frac{A^2T}{4}\operatorname{sinc}^2\left(\frac{T}{2}f\right).$$

First null $@2/T$ Hz





Example 4.5 – PSD of Bipolar RZ

For $m=0,\ R_0 = E\left[a_k a_k\right] = AA = (-A)(-A) = A^2$, each occurs at 1/4 the time, and 00 occurs 1/2 the time

For $m=\pm 1,\ R_{\pm 1} = E\left[a_k a_{k+1}\right] = -A^2, 0, 0$ and 0 for the data sequence (1,1),(1,0),(0,1) and (0,0), each occurs 1/4 the time

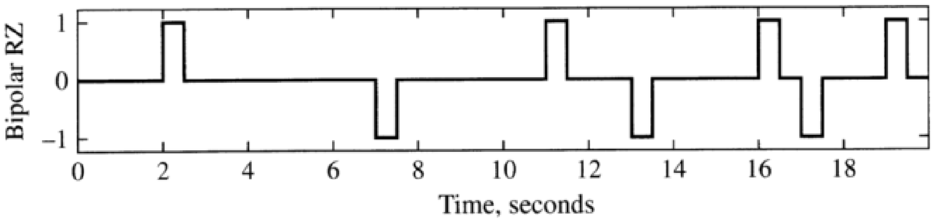
For $m>1$, possible products are A^2 and $-A^2$, each occurs 1/8 the time and $\pm A(0)$ and $(0)(0)$, each occurs 1/4 the time.

$$\Rightarrow R_m = \begin{cases} \frac{1}{4}A^2 + \frac{1}{4}(-A)^2 + \frac{1}{2}(0)^2 = \frac{1}{2}A^2, & m=0 \\ (-A)^2\frac{1}{4} + (A)(0)\frac{1}{4} + (0)(A)\frac{1}{4} + (0)(0)\frac{1}{4} = -\frac{A^2}{4}, & m=\pm 1 \\ A^2\frac{1}{8} + (-A)^2\frac{1}{8} + (A)(0)\frac{1}{4} + (-A)(0)\frac{1}{4} + (0)(0)\frac{1}{4} = 0, & |m|>1 \end{cases}$$

Pulse shape function $p(t) = \prod(2t/T)$

$$\Rightarrow P(f) = \frac{T}{2}\operatorname{sinc}\left(\frac{T}{2}f\right)$$

$$\Rightarrow S_{rr}(f) = \frac{1}{T}\left|\frac{T}{2}\operatorname{sinc}\left(\frac{T}{2}f\right)\right|^2 = \frac{T}{4}\operatorname{sinc}^2\left(\frac{T}{2}f\right)$$



Example 4.5 – PSD of Bipolar RZ

Pulse shape function $p(t)=\Pi(2t/T)$

$$\Rightarrow P(f)=\frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)$$

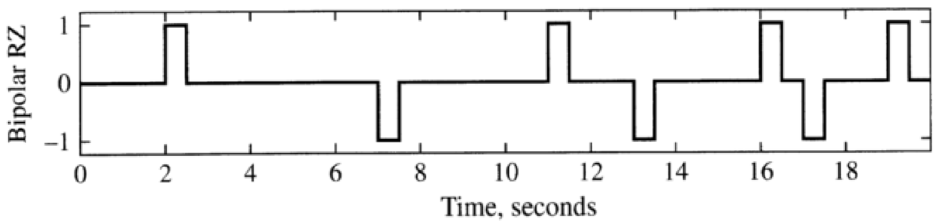
$$\Rightarrow S_{rr}(f)=\frac{1}{T}\left|\frac{T}{2}\text{sinc}\left(\frac{T}{2}f\right)\right|^2=\frac{T}{4}\text{sinc}^2\left(\frac{T}{2}f\right)$$

$$\Rightarrow S_{BRZ}(f)=S_{rr}(f)\sum_m R_m e^{-j2\pi mTf}$$

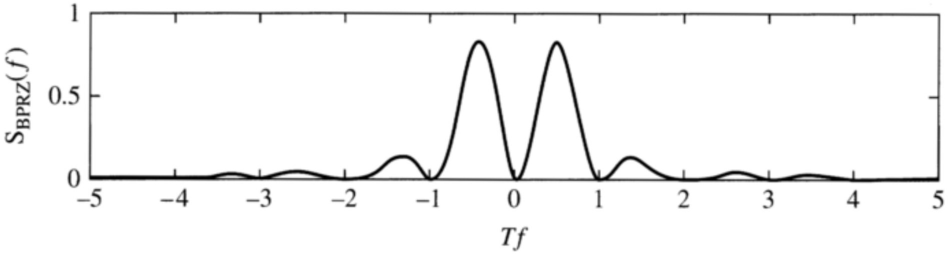
$$=\frac{A^2T}{8}\text{sinc}^2\left(\frac{T}{2}f\right)\left(1-\frac{1}{2}e^{j2\pi Tf}-\frac{1}{2}e^{-j2\pi Tf}\right)$$

$$=\frac{A^2T}{8}\text{sinc}^2\left(\frac{T}{2}f\right)\left[1-\cos(2\pi Tf)\right]$$

$$=\frac{A^2T}{8}\text{sinc}^2\left(\frac{T}{2}f\right)\sin^2(\pi Tf)$$



$$S_{xx}(f)=\frac{1}{T}\left|P(f)\right|^2\sum_m R_m e^{-j2\pi mTf}$$



Summary

- None of the above formats possesses all the desired properties below, tradeoff is needed
  - Self-synchronization
  - Power spectrum
  - Transmission BW
  - Transparency
  - Error detection capability
  - Good bit error probability performance

Intersymbol Interference (ISI)

- Convolutive noise
  - System will smear input signal during convolution
    - If system can be modeled as FIR filter
      - ISI will occur when length > 1
  - Hard to deal with as increasing SNR does not help
- Example: Covolving NRZ pulse with lowpass filter

$$h(t) = (1/\alpha)e^{-t/\alpha}u(t)$$

α can be controlled by values of RC in RC circuit
  - Different time constants have different smearing effect
    - Assume FIR model: one has longer impulse response than other

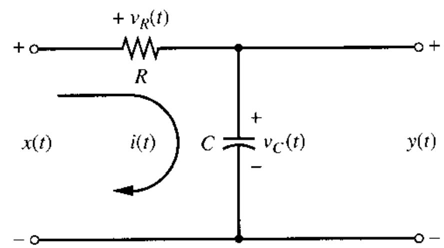


Figure 2.15
An RC lowpass filter.

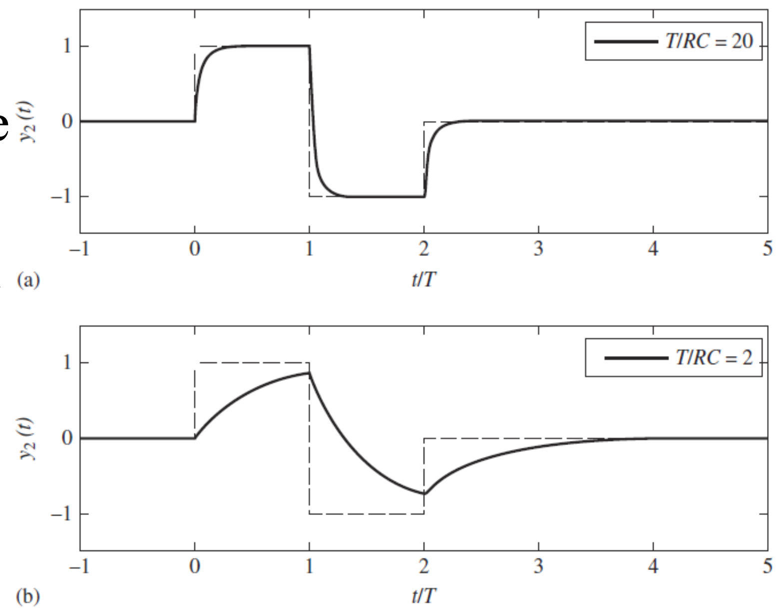


Figure 5.4
Response of a lowpass RC filter to a positive rectangular pulse followed by a negative rectangular pulse to illustrate the concept of ISI: (a) $T/RC = 20$; (b) $T/RC = 2$.

Effect of Different Degrees of ISI on Different Line Codes

System with shorter length

System with longer length

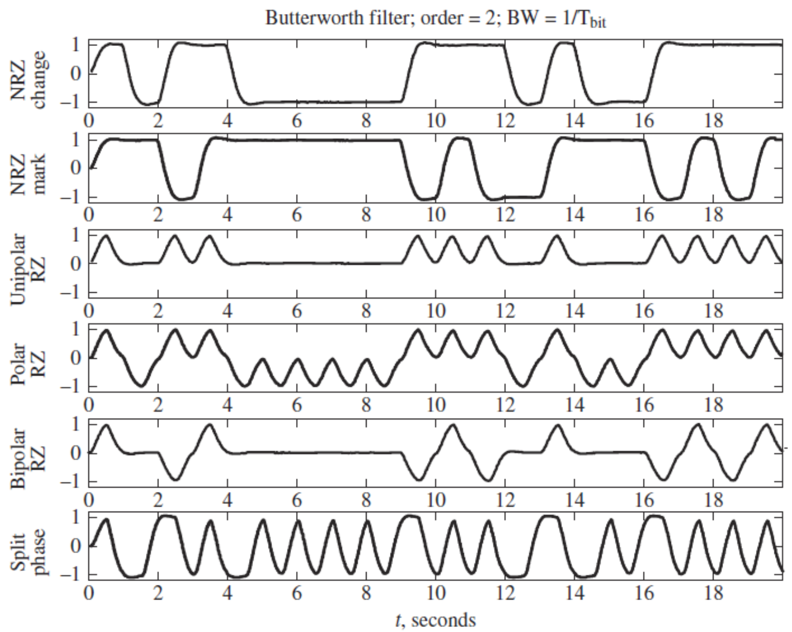


Figure 5.5
Data sequences formatted with various line codes passed through a channel represented by a second-order lowpass Butterworth filter of bandwidth one bit rate.

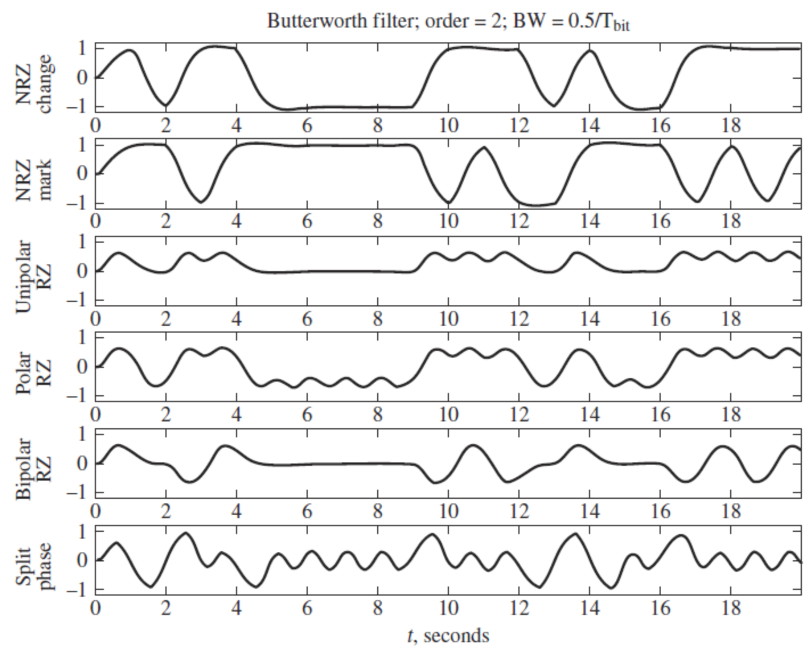


Figure 5.6
Data sequences formatted with various line codes passed through a channel represented by a second-order lowpass Butterworth filter of bandwidth one-half bit rate.

Split phase is more susceptible to ISI than NRZ (Why?)

Techniques Combat Against ISI

- Zero-ISI property (ideal – assuming no synchronization error)
  - Transmit and receive filters used to achieve zero-ISI condition
- Linear Receiver techniques
  - Deterministic (parameters are modeled as deterministic variables)
    - Zero-forcing equalizer
    - Least-squares equalizer
  - Bayesian/Stochastic (parameters are modeled as RVs)
    - Linear Minimum Mean-Squared Error (LMMSE) equalizer
- Nonlinear receiver techniques
  - Decision feedback equalizer (DFE)
    - ZF
    - LMMSE
  - Better performance than linear receivers
    - Increase computational complexity compared to linear receivers

Techniques Combat Against ISI

- Nonlinear transceiver techniques
  - Tomlinson-Harashima precoding
    - Move feedback portion of DFE to Tx
      - Presubtract interference
    - Modulo operation to prevent significant increase in input signal energy
    - Feedforward @Rx mitigates remaining interference
- Use different transmission methods
  - Block based transmission (DSP: Overlap-Save)
    - Discrete MultiTone (DMT)
      - Cable modem, ADSL
    - Orthogonal Frequency Division Multiplexing (OFDM), Orthogonal Frequency Division Multiple Access (OFDMA)
      - IEEE 802.11ax (Wi-Fi 6 standard), 5G NR downlink and uplink (frequency range 1: sub 6 GHz, frequency range 2: $\geq$ 6 GHz (likely 28, 38 GHz))
    - Single-carrier modulation (SC) (called DFT-precoded OFDM)
      - 3GPP LTE (uplink) and 5G NR (uplink)

ISI-free Transmission

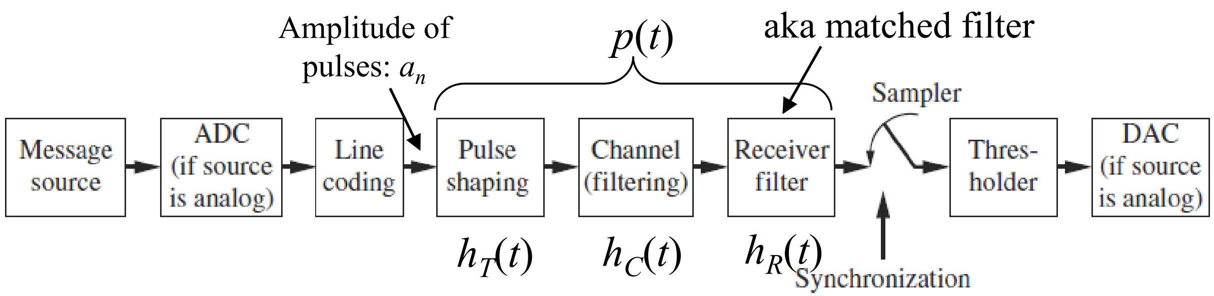


Figure 5.1
Block diagram of a baseband digital data transmission system.

Suppose noise-free received signal:

$$y(t) = \mu \sum_n a_n p(t - nT), \quad \text{where } \mu p(t) = h_T(t) * h_C(t) * h_R(t).$$

Assume $p(0) = 1$ ($p(t)$ is normalized).

Receive signal is sampled at $t_i = iT$, for $i \in \mathbb{Z}$

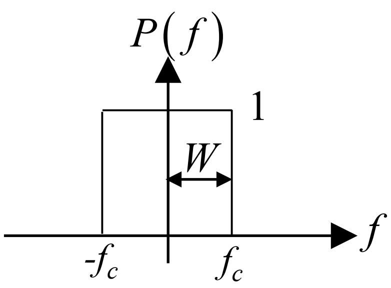
$$y(t_i) = \mu \sum_n a_n p\left[(i-n)T\right]$$

$$= \mu a_i + \mu \sum_{n \neq i} a_n p\left[(i-n)T\right].$$

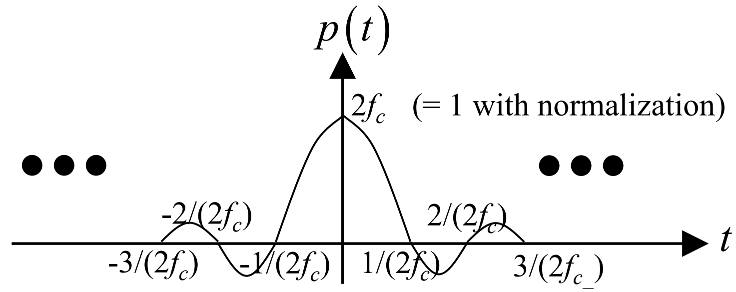
ISI-free transmission: $y(t_i) = \mu a_i$

Condition on pulse for ISI-free transmission: $p\left[(i-n)T\right] = \begin{cases} 1, & i = n, \\ 0, & i \neq n. \end{cases}$

Zero-ISI Property



$\Leftrightarrow$



Suppose we pass a train of pulses with amplitude a_n and spacing of $nT = n/2W$, for $n = \ldots,-2,-1,0,1,2,\ldots$ to an ideal lowpass LPF with $P(f) = \Pi\left(\frac{f}{2W}\right)$, with $W = f_c$. So the impulse response is

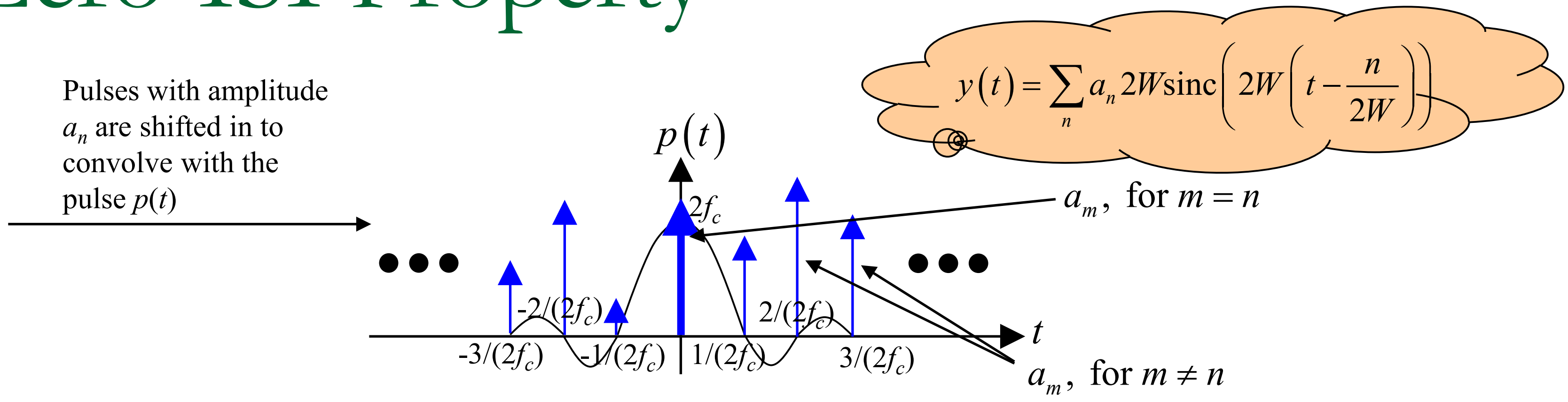
$$p(t) = \int_{-f_c}^{f_c} 1 \bullet e^{j2\pi f t}\, df = \frac{1}{\pi t}\sin(2\pi f_c t) = 2 f_c \mathrm{sinc}(2 f_c t)$$
$$= 2W\mathrm{sinc}(2Wt)$$

So, output is $y(t) = \sum_n a_n \delta(t - nT) * 2W\mathrm{sinc}(2Wt)$

$$= \sum_n a_n 2W\mathrm{sinc}(2W(t - nT))$$

$$= \sum_n a_n 2W\mathrm{sinc}\left(2W\left(t - \frac{n}{2W}\right)\right)$$

Zero-ISI Property



If we sample $y(t)$ at $t_m = m/2W$, the sample is a_m because

$$\mathrm{sinc}(m-n) = \begin{cases} 1, & m = n, \\ 0, & m \neq n. \end{cases}$$

Hence, output is not affected by preceding and succeeding samples.

However, we cannot realize such an ideal filter (infinite in time)

$\Rightarrow$ find a BW limited pulse (other than $\mathrm{sinc}(2Wt)$) which have zero crossings at

$$T = \frac{1}{2W}$$

$\Rightarrow$ Raised cosine: $p_{RC}(t) = \dfrac{\cos(\pi\beta t/T)}{1-(2\beta t/T)^2}\mathrm{sinc}\left(\dfrac{t}{T}\right)$

Zero-ISI Property: Raised cosine

Raised cosine: $p_{RC}(t) = \frac{\cos(\pi\beta t/T)}{1-(2\beta t/T)^2}\operatorname{sinc}\left(\frac{t}{T}\right) \Leftrightarrow$

$$P_{RC}(f) = \begin{cases} T, & |f| \le \frac{1-\beta}{2T}, \\ \frac{T}{2}\left\{1+\cos\left[\frac{\pi T}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right]\right\}, & \frac{1-\beta}{2T} < |f| \le \frac{1+\beta}{2T}, \\ 0, & |f| > \frac{1+\beta}{2T}. \end{cases}$$

β : roll-off factor

- If $\beta = 1$, single-side BW $= 1/T$ Hz
- $\beta = 1$ has narrow mainlobe width and low sidelobe, so error is minimal in case there is sampling offset.

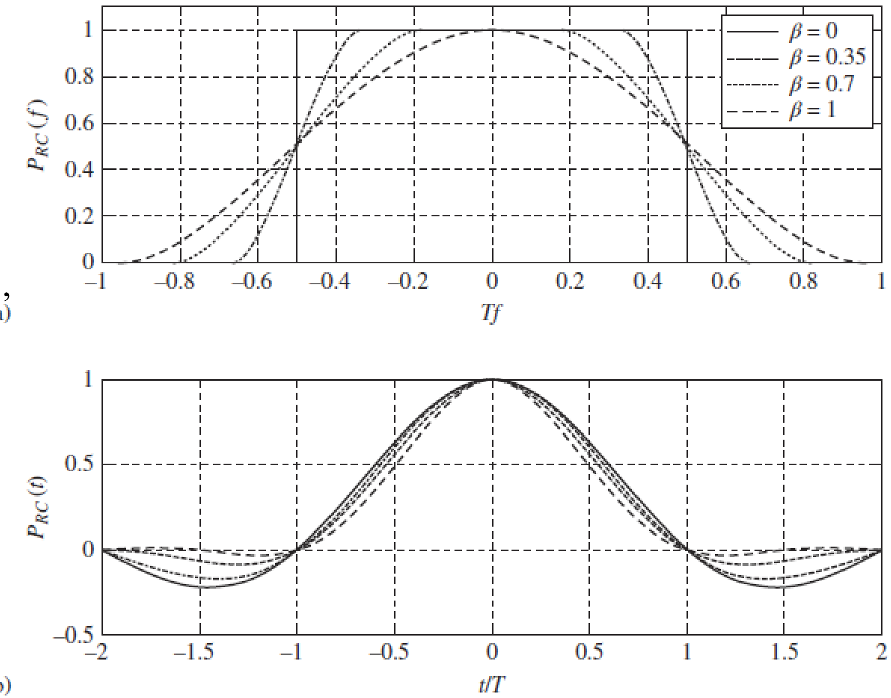


Figure 5.7
(a) Raised cosine spectra and (b) corresponding pulse responses.

Nyquist’s Pulse Shaping Criterion

Nyquist's Pulse Shaping Criterion:

The frequency function $P(f)$ eliminates intersymbol interference for samples taken at intervals T provided that it satisfies the condition[1]

$$\sum_k P\left(f - \frac{k}{T}\right) = T$$

Proof:

Recall: $y(t) = \mu \sum_n a_n p(t - nT)$

$$y(t_i) = \mu a_i + \mu \sum_{n \neq i} a_n p\left[(i-n)T\right].$$

$\Rightarrow$ Condition on pulse for ISI-free transmission: $p\left[(i-n)T\right] = \begin{cases} 1, & i = n, \\ 0, & i \neq n, \end{cases}$

and $p(0) = 1$ by normalization.

Nyquist's Pulse Shaping Criterion

Let $m = i - n \Rightarrow p\left[(i-n)T\right] = p(mT)$

From sampling theorem:

$$p(mT) = p_s(t) = p(t)\sum_m \delta(t - mT)$$

$$= \sum_m p(mT)\delta(t - mT)$$

Since $\sum_m \delta(t - mT) \Leftrightarrow \frac{1}{T}\sum_k \delta\left(f - \frac{k}{T}\right)$

$$\Rightarrow P\left(e^{j2\pi f}\right) = F\left\{p(t)\sum_m \delta(t - mT)\right\}$$

$$= P(f) * F\left\{\sum_m \delta(t - mT)\right\}$$

$$= P(f) * \frac{1}{T}\sum_k \delta\left(f - \frac{k}{T}\right)$$

$$= \frac{1}{T}\sum_k P\left(f - \frac{k}{T}\right)$$

However, by definition

$$P\left(e^{j2\pi f}\right) = \int_t \sum_m p(mT)\delta(t - mT)e^{-j2\pi ft}dt$$

$$= \sum_m p(mT)\int_t e^{-j2\pi ft}\delta(t - mT)dt$$

$$= \sum_m p(mT)e^{-j2\pi fmT}$$

Recall that $m = 0$ corresponds to ISI-free transmission:

$$P\left(e^{j2\pi f}\right)\Big|_{m=0} = p(0) = 1,$$

where the last equality comes from the assumption that $p(t)$ is normalized. Since $P\left(e^{j2\pi f}\right)\Big|_{m=0} = 1 = \frac{1}{T}\sum_k P\left(f - \frac{k}{T}\right)$

$$\therefore \sum_k P\left(f - \frac{k}{T}\right) = T$$

must satisfy the ISI-free transmission criterion.

Example 4.6: Other Zero-ISI Pulse

Triangular spectrum:

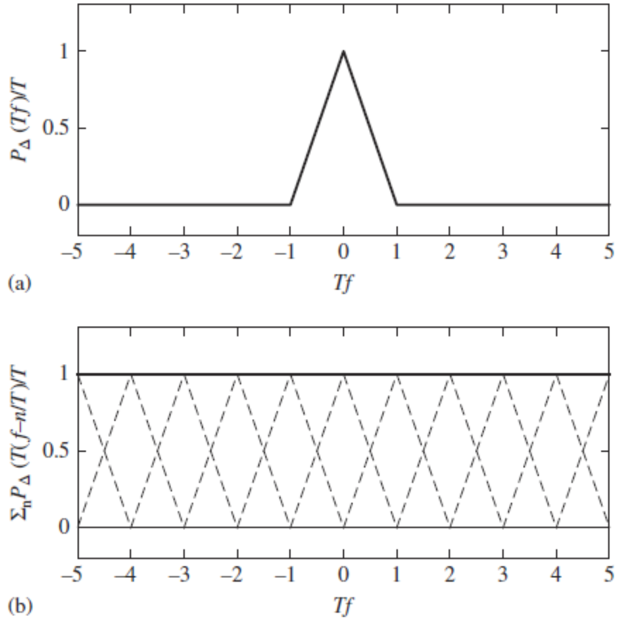
$$P_\Delta(f) = T\Lambda(Tf) \Leftrightarrow p_\Delta(t) = \text{sinc}^2(t/T)$$

Recall: $\Lambda\left(\frac{t}{B}\right) \Leftrightarrow B\text{sinc}^2(Bf)$

Zero-ISI condition is satisfied because

$$p_\Delta(nT) = \text{sinc}^2(n) = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases}$$

First condition is equivalent to $P_\Delta\left(e^{j2\pi f}\right)\Big|_{n=0} = p_\Delta(0) = 1.$



$$\sum_k P_\Delta\left(f - \frac{k}{T}\right)$$

Figure 5.8
Illustration that a triangular spectrum (a), satisfies Nyquist's zero-ISI criterion (b).

Transmit and Receive Filters

Suppose a_k is a sequence of sample values, e.g. 2-, 3-bit/sample. The k^{th} sample value multiplies a unit impulse occuring at time kT. The output of the transmit filter is

$$x(t) = \sum_k a_k \delta(t - kT) * h_T(t)$$

$$= \sum_k a_k h_T(t - kT).$$

aka matched filter

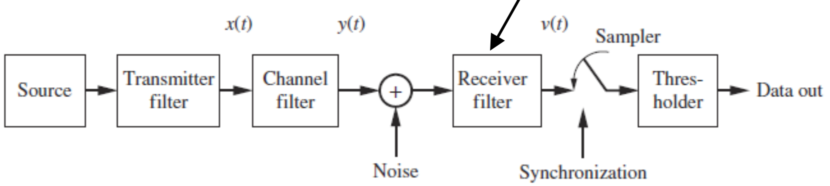


Figure 5.9
Transmitter, channel, and receiver cascade illustrating the implementation of a zero-ISI communication system.

Output of channel: $y(t) = x(t) * h_C(t)$

Output of receive filter: $v(t) = y(t) * h_R(t)$

Assume $p_{RC}(t)$ is used.

$$\Rightarrow v(t) = \sum_k a_k A p_{RC}(t - kT - t_d).$$

t_d : delay introduced by $h_T(t), h_C(t), h_R(t)$.

$$\Rightarrow A p_{RC}(t - t_d) = h_T(t) * h_C(t) * h_R(t) \quad \Leftrightarrow \quad A P_{\mathrm{RC}}(f) e^{-j2\pi f t_d} = H_T(f) H_C(f) H_R(f)$$

$$\Rightarrow A P_{RC}(f) = |H_T(f)||H_C(f)||H_R(f)| \quad \text{(i.e. in terms of magnitude)}$$

Assume the channel impulse response is known, and assume $h_T(t) = h_R(t)$

$$\Rightarrow |H_T(f)| = |H_R(f)| = \frac{A P_{RC}^{1/2}(f)}{|H_C(f)|^{1/2}}$$

What happens if this equality is not satisfied? E.g. $H_C(f)$ not available, but an estimate of $H_C(f)$?
➔ Solution: Equalization

Zero-Forcing Equalization (ZFE)

$p_{eq}(t)$: output of a non-causal equalizer (FIR filter)

$p_c(t)$: pulse response of the channel output

$$p_{eq}(t) = \sum_{n=-N}^{N} \alpha_n p_c(t - n\Delta),$$

Δ: tap spacing

$2N+1$: length of equalizer

Suppose we sample the output of the equalizer every T sec. Assume $\Delta = T$ ($\Delta = 0.5T$ for fractionally spaced equalizer). Recall the condition on pulse for ISI-free transmission:

$$p_{eq}(mT) = \sum_{n=-N}^{N} \alpha_n p_c[(m-n)T]$$

$$= \begin{cases} 1, & m = n, \\ 0, & m \neq n, \end{cases} \quad \text{for } m = 0, \pm 1, \ldots, N.$$

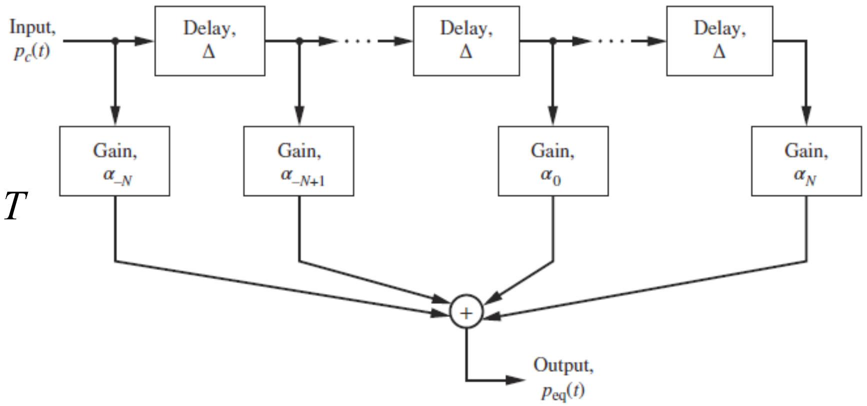


Figure 5.11
A transversal filter implementation for equalization of intersymbol interference.

Zero-Forcing Equalization

Note:

1. ISI-free condition can be satisfied at only $2N$ time instants since there are only $2N+1$.

$\Rightarrow$ length of channel greatly affects the length of (time-domain equalizers).

I.e. Longer the length of the channel, longer the length of the equalizer

Solution: frequency-domain equalizers

2. Output of the filter at $t=0$ is forced to be 1.

Zero-Forcing Equalization

> Recall the output signal of a causal FIR filter with impulse response $h[n]$ and input $x[n]$ can be written as
>
> $$\mathbf{y}=\begin{bmatrix} y[0] \\ \\ y[L_y-1]\end{bmatrix}=\begin{bmatrix} h[0] & 0 & \cdots & 0 \\ \vdots & h[0] & \ddots & \vdots \\ h[L_h-1] & \vdots & \ddots & 0 \\ 0 & h[L_h-1] & \ddots & h[0] \\ \vdots & 0 & \ddots & \vdots \\ 0 & \vdots & \ddots & h[L_h-1]\end{bmatrix}\begin{bmatrix} x[0] \\ \\ x[L_x-1]\end{bmatrix}$$
> $$=\mathbf{Hx}$$

Define the out signal vectors and matrix:

$$\mathbf{p}_{eq} \triangleq \begin{bmatrix}\mathbf{0}_{1\times N} & 1 & \mathbf{0}_{1\times N}\end{bmatrix}^T \in \mathbb{R}^{2N+1}$$

$$\mathbf{a} \triangleq \begin{bmatrix}\alpha_{-N} & \alpha_{-N+1} & \cdots & \alpha_N\end{bmatrix}^T \in \mathbb{C}^{2N+1}$$

$$\mathbf{P}_c \triangleq \begin{bmatrix} p_c(0) & p_c(-T) & \cdots & p_c(-2NT) \\ p_c(T) & p_c(0) & \cdots & p_c\left[(-2N+1)T\right] \\ \vdots & \vdots & \ddots & \vdots \\ p_c(2NT) & p_c(2NT-1) & \cdots & p_c(0)\end{bmatrix} \in \mathbb{C}^{(2N+1)\times(2N+1)}$$

The I/O response equation is

$$\mathbf{p}_{eq} = \mathbf{P}_c\mathbf{a}.$$

So the coefficents for the (FIR) equalizer are computed as

$$\mathbf{P}_c^{-1}\mathbf{p}_{eq} = \mathbf{a}$$

$$\Rightarrow \mathbf{a} = \mathbf{P}_c^{-1}\begin{bmatrix}\mathbf{0}_{N\times 1} \\ 1 \\ \mathbf{0}_{N\times 1}\end{bmatrix} = \text{middle column of } \mathbf{P}_c^{-1}$$

Example 4.7: ZFE Example

Consider a channel for which the following sample values of the channel pulse response are obtained:

$$p_c(-3T) = 0.02,\ \ p_c(-2T) = -0.05,\ \ p_c(-T) = 0.2,\ \ p_c(0) = 1.0$$

$$p_c(T) = 0.3,\ \ p_c(2T) = -0.07,\ \ p_c(3T) = 0.03$$

Suppose we use a 3-tap FIR equalizer, i.e. $N = 1$

$$\mathbf{P}_c = \begin{bmatrix} 1.0 & 0.2 & -0.05 \\ 0.3 & 1.0 & 0.2 \\ -0.07 & 0.3 & 1.0 \end{bmatrix}.$$

$$\Rightarrow \mathbf{a} = \mathbf{P}_c^{-1} \begin{bmatrix} \mathbf{0}_{N\times 1} \\ 1 \\ \mathbf{0}_{N\times 1} \end{bmatrix} = \begin{bmatrix} 1.0815 & -0.2474 & 0.1035 \\ -0.3613 & 1.1465 & -0.2474 \\ 0.1841 & -0.3613 & 1.0815 \end{bmatrix} \begin{bmatrix} \mathbf{0}_{N\times 1} \\ 1 \\ \mathbf{0}_{N\times 1} \end{bmatrix}$$

$$= \begin{bmatrix} -0.2474 \\ 1.1465 \\ -0.3613 \end{bmatrix}$$

$$\Rightarrow p_{eq}(m) = -0.2474\, p_c\big((m+1)T\big) + 1.1465\, p_c(mT) - 0.3613\, p_c\big((m-1)T\big),\ \text{for } m = \ldots,-1,0,1,\ldots$$

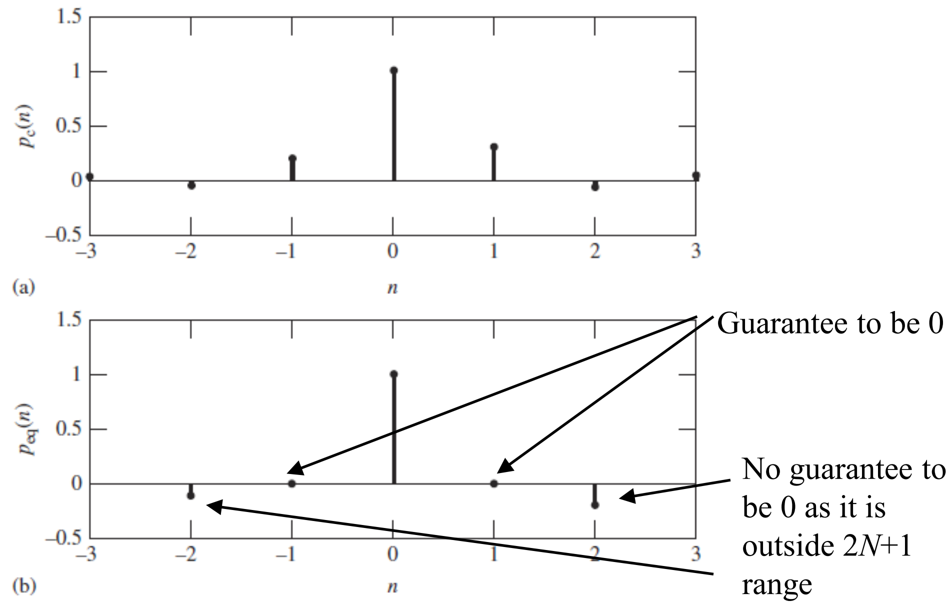


Figure 5.12
Samples for (a) an assumed channel response and (b) the output of a zero-forcing equalizer of length 3.

Eye Diagram

- Gives qualitative measure of system performance in terms of ISI
- Constructed by plotting overlapping *k*-symbol segments of a baseband signal
  - Can be displayed on an oscilloscope by triggering the time sweep of the oscilloscope at $t=nkT_s$
    - T_s: symbol period
    - kT_s: eye period
    - n: integer
- NRZ wave input into 3rd order Butterworth filter
  - 4 symbols are shown
  - BW is normalized to symbol rate
    - E.g. if symbol rate = 1000 sym/s, BW of filter = 600Hz ➔ normalized BW = 0.6
    - Graph spanned 200 samples
    - Given 50 samples/symbol ➔ k = 200/50 = 4 symbols are shown
  - Note: as BW↑, length of filter↓

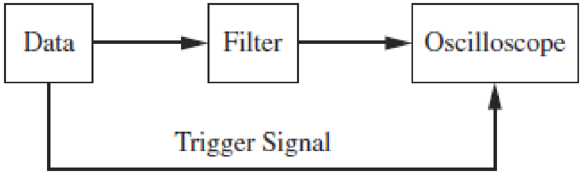


Figure 5.13
Simple technique for generating an eye diagram for a bandlimited signal.

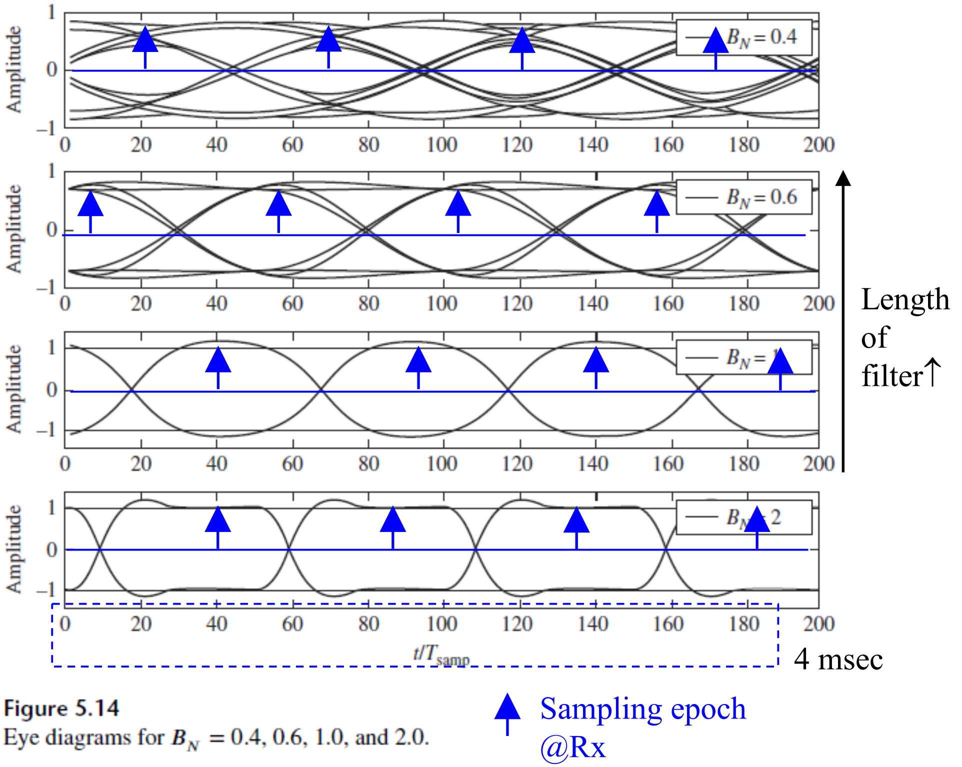


Figure 5.14
Eye diagrams for B_N = 0.4, 0.6, 1.0, and 2.0.

Characteristics of Eye Diagram

- Two symbols are shown instead of 4
- Optimal sampling is when eye is most open
- Significant bandlimiting (increase in filter length) closes the eye
  - Causes amplitude jitter A_j
- Filter length↑ ➔ timing jitter T_j↑ (perturbation of zero-crossings) ➔ more difficult for synchronization
- BW of channel (filter)↓ ➔ additive noise↑ ➔ A_j↑ and T_j↑

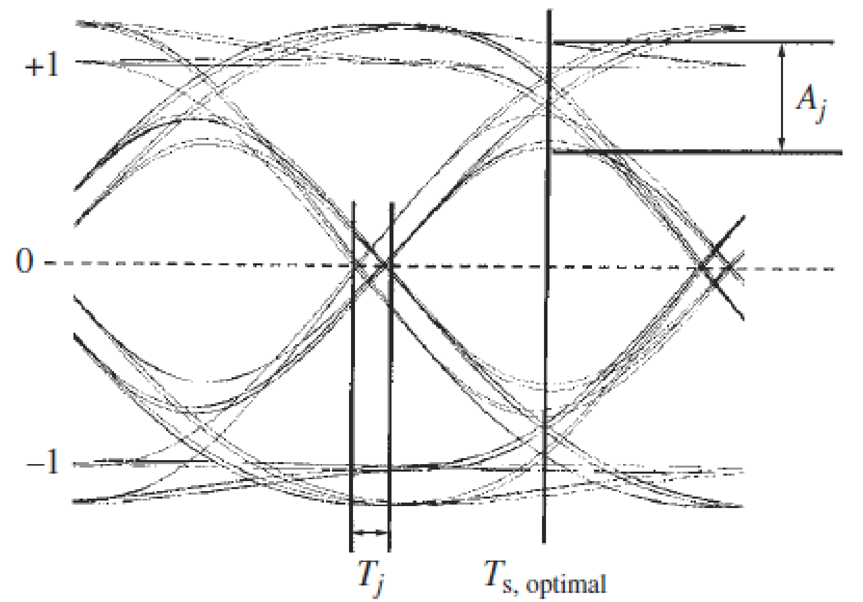


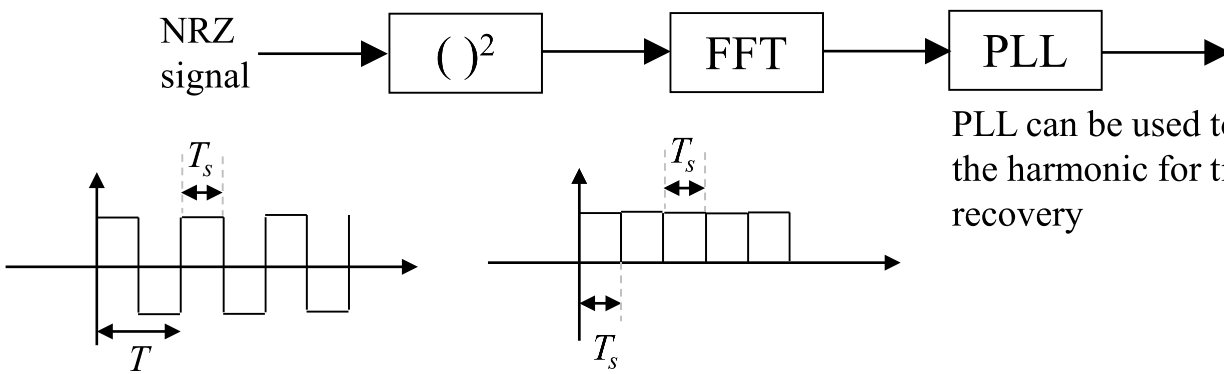
Figure 5.15
Two-symbol eye diagrams for $B_N = 0.4$.

Synchronization

- A detection problem
  - Making decision (binary problem: 1/0, signal present/not present, …)
- Carrier synchronization
  - Coherent detection
- Word synchronization
  - Detection of initial symbol in codewords (from channel coding) in digital communication
- Frame synchronization
  - Symbols group together to form frames
  - Detection of starting and ending of frame
- Consider symbol synchronization
  - Derivation from a primary or secondary standard
    - E.g. Tx and Rx slaved to a master timing source
  - Use of separate synchronization signal (pilot clock)
  - Derivation from the modulation itself
    - Known as self-synchronization

Self Synchronization Method 1

Idealized system, i.e. no channel



PLL can be used to track the harmonic for timing recovery

$T_s = T/2$: symbol period

T: period of NRZ wave

FFT will contain harmonic component at $1/T_s$, the actual symbol rate

Self-Synchronization Method 1

Contains 600 samples → 600 samples/ 50 samples/sym = 12 sym

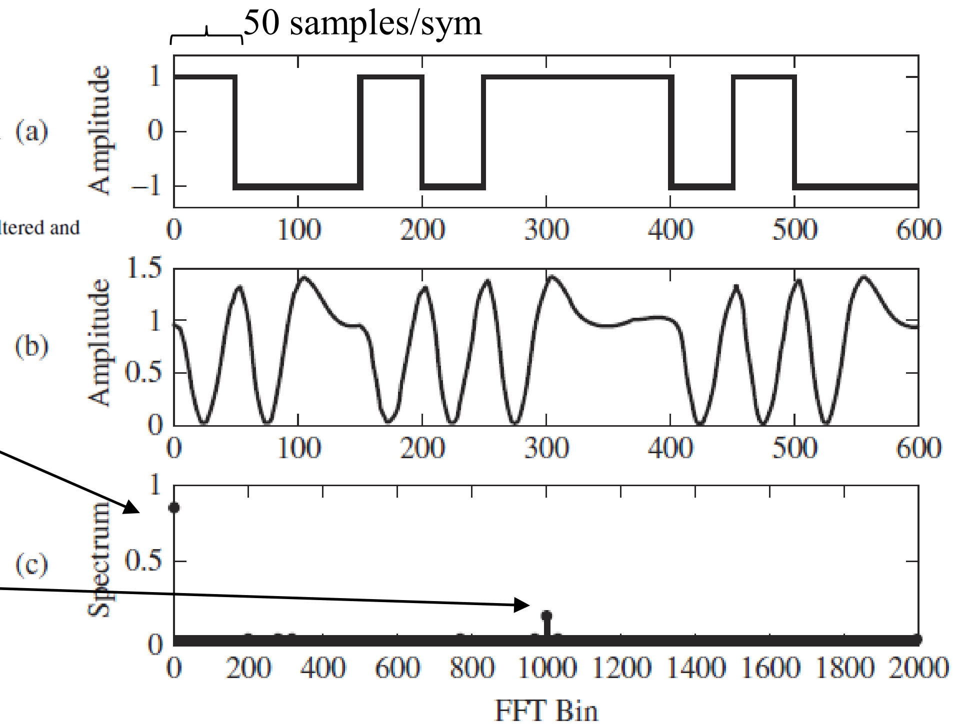


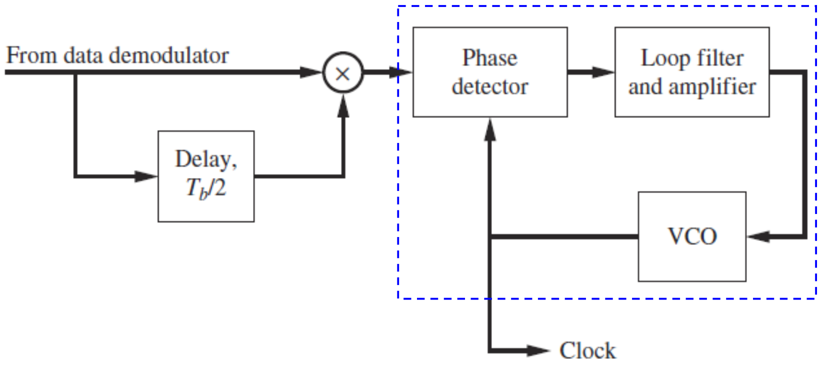
Figure 5.16
Simulation results for Computer Example 5.2: (a) NRZ waveform; (b) NRZ waveform filtered and squared; (c) FFT of squared NRZ waveform.

Peak created by squaring operation (ideally, the signal will have constant amplitude)sym/s

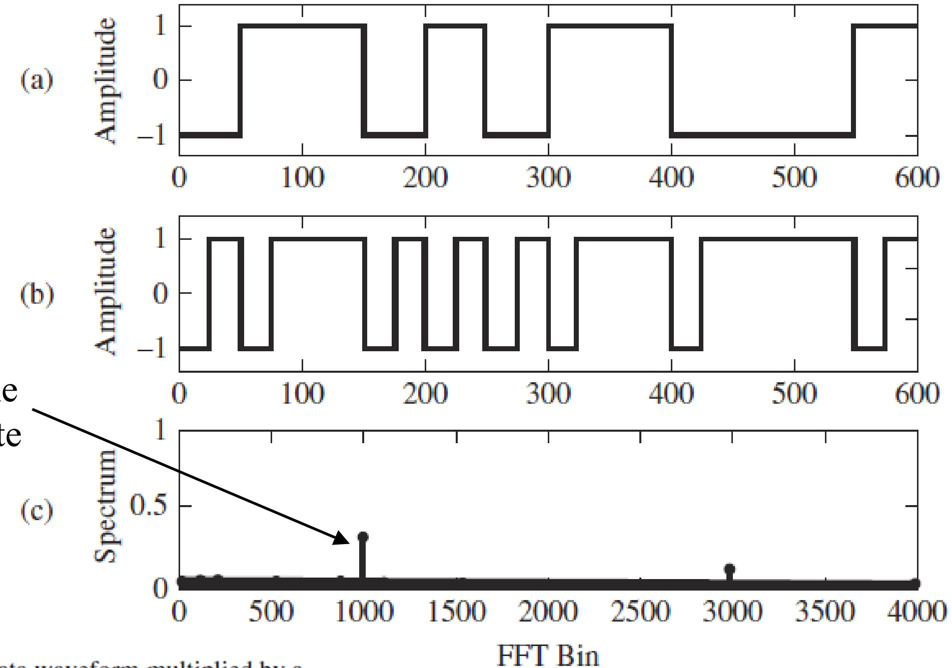
2nd peak at symbol rate 1000 sym/s

is 1000 symbols/s and, since the NRZ signal is sampled at 50 samples/symbol, the sampling frequency is 50,000 samples/second. Figure 5.16(a) illustrates 600 samples of the NRZ signal. Filtering by a third-order Butterworth filter having a bandwidth of twice the symbol rate and squaring this signal results in the signal shown in Figure 5.16(b). The second-order harmonic created by the squaring operation can clearly be seen by observing a data segment consisting of alternating data symbols. The spectrum, generated using the FFT algorithm, is illustrated in Figure 5.16(c). Two spectral components can clearly be seen; a component at DC (0 Hz), which results from the squaring operation, and a component at 1000 Hz, which represents the component at the symbol rate. This component is tracked by a PLL to establish symbol timing.

Self-Synchronization Method 2



PLL is used to track the harmonic for timing recovery



Peak indicates the actual symbol rate located @1000 sym/s

Figure 5.18
Simulation results for Computer Example 5.4: (a) data waveform; (b) data waveform multiplied by a half-bit delayed version of itself; (c) FFT spectrum of (b).

Digital RF Modulation

ASK:

$$x_{ASK}(t) = A_c\left[1 + d(t)\right]\cos\left(2\pi f_c t\right)$$

Similar to AM, except $d(t)$ is a line code, e.g. NRZ

PSK:

$$x_{PSK}(t) = A_c \cos\left(2\pi f_c t + \frac{\pi}{2} d(t)\right)$$

FSK:

$$x_{FSK}(t) = A_c \cos\left(2\pi f_c t + k_f \int^{t} d(\alpha)\, d\alpha\right)$$

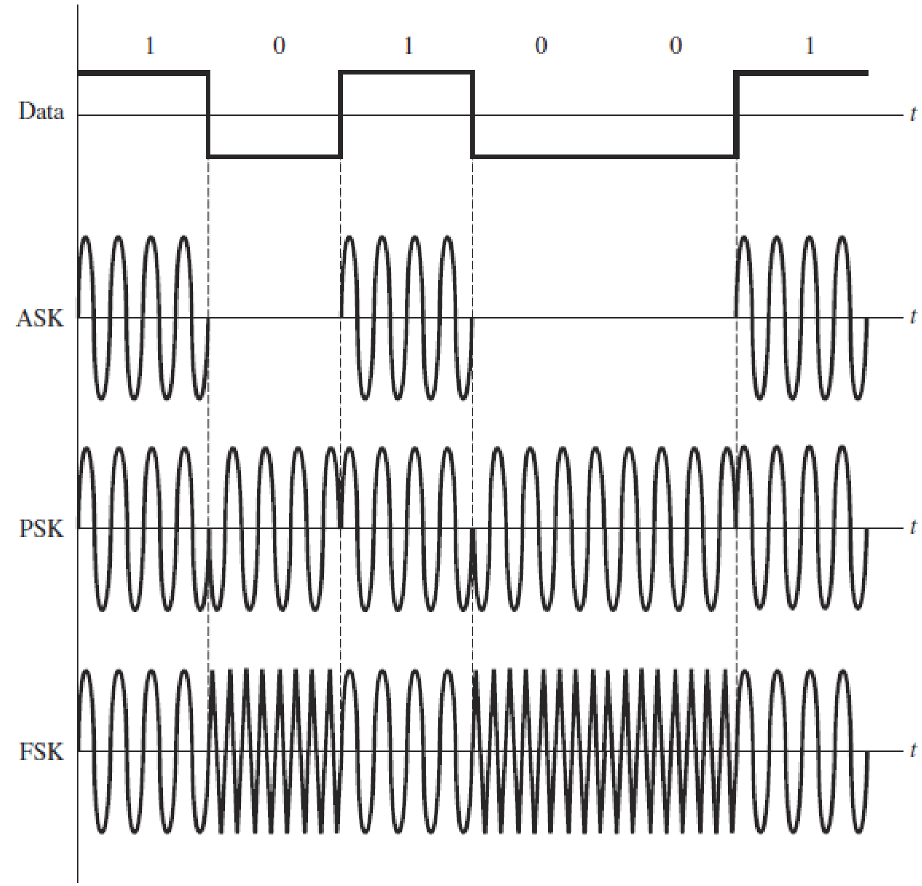


Figure 5.19
Examples of digital modulation schemes.