Digital Signal Processing

Final Exam

Spring, 2004

(27%) 1. Consider a system, whose system function is expressed as

$$H(z) = \frac{(1+0.5z^{-1})(1-0.5z^{-1})(1+5z^{-1})}{(1+4z^{-1})}$$

Problem Set I: Assume this is a **causal** system.

(a) Find the impulse response h[n] of this system. (3 pts)

(b) Find the frequency response $H(e^{jw})$ of this system. (3 pts)

(c) Draw a direct form II implementation of this system. (3 pts)

(d) Find the system function of a causal, BIBO-stable system, which has the same magnitude function as H(z). (3 pts)

Problem Set II: Assume this is a **BIBO stable** system.

(e) Find the impulse response h[n] of this system. (3 pts)

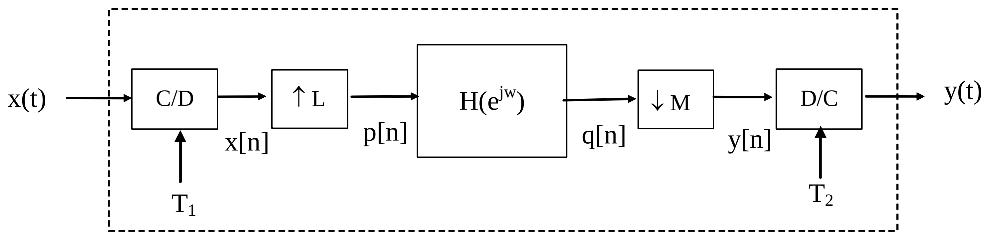
(f) Find the frequency response $H(e^{jw})$ of this system. (3 pts)

(g) Assume the input is $x[n] = (0.5)^n u[n]$. Find the output y[n]. (3 pts)

(h) Assume there is another system $H_1(z)$, whose impulse response is defined as $h_1[n] = \cos(\alpha n) h[n]$. Could you find a proper range of α such that $H_1(z)$ is a minimum-phase system? (3 pts)

(i) Assume there is another system $H_2(z)$, whose impulse response is defined as $h_2[n] = (\alpha)^n h[n]$. Could you find a proper range of α such that $H_2(z)$ is a minimum-phase system? (3 pts)

(24%) 2. Consider the following system.



Assume $X(j\Omega) = \Lambda(\frac{\Omega}{100\pi})$ and $H(e^{jw}) = \begin{cases} 1 & 0 \le |\omega| \le 0.5\pi \\ 0 & 0.5\pi \le |\omega| \le \pi \end{cases}$.

(a) If $T_1 = 0.01$, $T_2 = 0.01$, L = 2, and M = 3, find the spectrums of x[n], p[n], q[n], y[n], and y(t). (12 pts)

(b) If $T_1 = 0.015$, $T_2 = 0.015$, L = 3, and M = 2, find the spectrums of x[n], p[n], q[n], y[n], and y(t). (12 pts)

(24%) 3. Assume the 8-point DFT's of $x_1[n]$ and $x_2[n]$ are expressed as

$$X_1[k] = \begin{cases} 5-k & 0 \le k \le 5 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad X_2[k] = \begin{cases} k & 0 \le k \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Is $x_1[n]$ a real-valued sequence? Why? (6 pts)

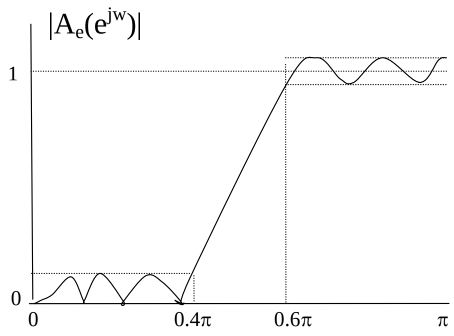
(b) Find the relationship between $x_1[n]$ and $x_2[n]$. (6 pts)

(c) If $x_3[n]$ is defined as $x_3[n] = x_1[n]x_2[n]$, find the 8-point DFT of $x_3[n]$. (6 pts)

(d) In terms of $x_1[n]$ and $x_2[n]$, can you find a sequence $x_4[n]$ whose 8-point DFT is

$$X_4[k] = \begin{cases} 5-k & 0 \le k \le 4 \\ k-3 & 5 \le k \le 7 \end{cases}.$$ (6 pts)

(25%) 4. An optimal equiripple FIR **Type-IV** linear-phase filter was designed by the Parks-McClellan algorithm. The magnitude response is shown as below.



(a) What is the length of the impulse response of the system? (6 pts)

(b) If this system is causal, what is the smallest delay that it can have? (6 pts)

(c) Draw roughly the pole-zero plot of this system. (6 pts)

(d) How can you convert the above system to a low-pass system whose magnitude response is as shown below? (7 pts)