Digital Signal Processing

Final Exam, June 2006

- 别忘寫姓名學號
- 請依序答題
- You may be graded on *method* and *ideas*, and the *clarity* with which you organize them, but accurate numbers are needed for full credits.

Good Luck!

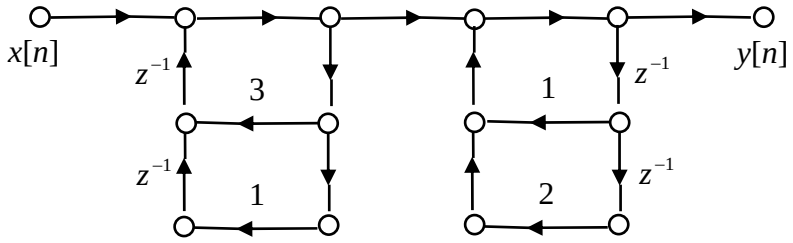
[18%] (1) The input ($x[n]$) and the output ($y[n]$) relationship of three systems are given below. For each of them, determine whether the system is (i) causal, (ii) linear, and (iii) time invariant. (*Note:* You receive 2% for each correct answer, 0 point for no answer, **-1% penalty** for each *incorrect* answer. You don't need to give reasons)

(a) $y[n] = \sin\left(\frac{2\pi n}{N}\right)x[n]$, for all n, and N is an integer constant.

(b) $y[n] = x[n] + \frac{1}{2}y[n-1]$, for $n>0$ and $y[0]=1$.

(c) $y[n] = y[n+1] + x[n]$, for all n.

[12%] (2) A linear time-invariant system is realized by the flow graph shown below.



(a) Write the difference equation relating $x[n]$ and $y[n]$ for this flow graph. [3%]

(b) Draw the transpose of this system. [3%]

(c) Show that the transpose system has the same system function as the original system. [3%]

(d) The above realization requires four storage registers (delay elements). Is it possible to reduce the number of storage registers by using a different structure and *why*? If so, draw its flow graph. [3%]

[6%] (3) A finite impulse response $h[n]$ has only 5 nonzero values; that is, $h[n] = 0$ for $n < 0$ and $n > 4$. An input sequence $x[n]$ is to be convolved with $h[n]$.

(a) Let $x[n]$ be nonzero for $0 \le n \le 19$ and $x[n] = 0$ outside the interval $0 \le n \le 19$. What is the maximum possible number of nonzero values in the output of linear convolution of $x[n]$ and $h[n]$. [3%]

(b) If a 20-point circular convolution of $x[n]$ and $h[n]$ is used, how many output points, in general, of this circular convolution have the same values as the

outputs of the linear convolution in (a)? What is the portion that they (linear and circular convolutions) differ if any? Use a simple illustration (plot) to show your reasoning. [3%]

[14%] (4) We like to use DFT to perform interpolation of finite length sequences. Let $x[n]$ be a sequence of duration $2N$ with support $[0, 2N-1]$ and its DFT equals to zero for $(3N/2) > k \geq (N/2)$, i.e., $X[k] = 0$, for $(3N/2) > k \geq (N/2)$.

We now decimate $x[n]$ by taking every other sample to generate an N-point sequence $x_1[n]$, i.e.,

$x_1[n] = x[2n]$, $n=0,\ldots, N-1$.

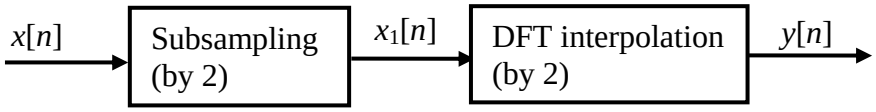
Let $X_1[k]$ be the N-point DFT of $x_1[n]$.

Set

$$Y[k] \equiv \begin{cases} 2X_1[k], & 0 \leq k \leq N/2 - 1 \\ 0, & N/2 \leq k \leq 3N/2 - 1 \\ 2X_1[k-N], & 3N/2 \leq k \leq 2N-1 \end{cases}$$

Let $y[n]$ be the $2N$-point IDFT of $Y[k]$.

This process is shown below.



Note that the sub-sampling and interpolation are both by a factor of 2.

(a) Show that $y[2n]=x_1[n]$. (You must show your derivation.) [7%]

(b) Show that $y[n]=x[n]$. (You must show your derivation.) [7%]

[24%] (5) Consider two BIBO stable LTI systems, whose frequency responses are

$$H_1(e^{jw}) = 2 + \cos(w) \text{ with } |w| < \pi$$

and

$$H_2(e^{jw}) = \frac{1}{2+\cos(w)} \text{ with } |w| < \pi, \text{ respectively,}$$

(a) Find the system function and ROC for each of these two systems. [6%]

(b) Find the impulse response for each of these two systems. [6%]

(c) Which systems are causal? [2%]

(d) Which systems are generalized linear-phase? [2%]

(e) If we define $\tilde{H}_1[k] = H_1(e^{jw})\,|_{\,w\,=\,0.5\pi k}$, for $\forall$ k, **find** and **plot** the inverse DFS of $\tilde{H}_1[k]$. [4%]

(f) If we define $\tilde{H}_2[k] = H_2(e^{jw})\,|_{\,w\,=\,0.5\pi k}$, for $\forall$ k, **find** and **plot** the inverse DFS of $\tilde{H}_2[k]$. [4%]

(Hint: $\cos w = \frac{e^{jw} + e^{-jw}}{2}$)

[12%] (6) Assume both System-A and System-B are **generalized linear-phase** systems. Specify whether the following statements are true or false. (*Note:* You receive 3% for each correct answer, 0 point for no answer, **-2% penalty** for each *incorrect* answer. You don't need to give reasons)

(a) The cascade of these two systems is also generalized linear-phase.
(b) Both systems are FIR systems.
(c) Both systems are BIBO stable.
(d) Both systems are causal.

[14%] (7) Assume the impulse response of a causal system is

$$h[n] = 2(0.2)^n u[n] - (-2)^n u[n].$$

(a) If the input is $x[n] = (0.3)^n u[n]$, find the corresponding output. [3%]
(b) Write the linear, constant-coefficient difference equation to describe the input-output relationship of this system. [4%]
(c) Unfortunately, this system is not BIBO stable. Please find another causal system that is BIBO stable and has the same magnitude response as this system. [7%]

=== The END ===