

# Final Exam Topics

## 1) Discrete-Time Signals: Sequence

- Memoryless, LTI, Causality, BIBO Stability
- Properties of LTI
- Linear difference equations
- Eigenfunctions for LTI Systems
- Transform domain representation: DTFT
- Properties of the DTFT: linearity, Time/Frequency shifting, time reversal, differentiation in frequency, Parseval's theorem, convolution theorem, modulation/windowing theorem
- Symmetry properties of DTFT
- Common DTFT pairs

## 2) $z$ -Transform

- Region of convergence
- Common  $z$ -transform pairs
- Properties of ROC for the  $z$ -transform
- Pole location and time-domain behavior for causal signals
- Inverse  $z$ -transform
  - table lookup or inspection
  - partial fraction expansion
  - power series expansion
- $z$ -transform properties: linearity, time shifting, multiplication by an exponential sequence, differentiation, conjugation of a complex sequence, time reversal, convolution, initial value theorem, final value theorem
- Solving linear difference equations

## 3) Sampling of Continuous-Time Signals

- Periodic sampling
- Frequency-domain representation of sampling
  - Nyquist sampling theorem:  $\Omega_s \geq 2\Omega_N$
- Reconstruction of bandlimited signal from its samples
- Discrete-time processing of continuous-time signals
- Continuous-time processing of discrete-time signals
- Change of sampling rate using discrete-time processing
  - Integer factor
  - Non-integer factor
- Digital processing of analog signal

## 4) Transform Analysis of LTI Systems

- Frequency response LTI systems
  - Magnitude, phase, and group delay
- System functions for systems characterized by linear constant-coefficient difference equations
- Frequency response for rational system functions
  - single zero or pole
  - second-order IIR
  - second-order FIR
- Relationship between magnitude and phase
  - If  $H(z)$  is causal and stable, then all its poles are inside the unit circle
- Allpass systems
  - phase of allpass systems is non-positive for  $0 \leq \omega < \pi$

- Poles and zeros form conjugate reciprocal pair
- Minimum phase system
  - system factorization:  $H(z) = H_{min}(z)H_{ap}(z)$
  - stable and causal inverse systems
  - minimum phase lag, minimum group delay, minimum energy-delay properties
  - maximum phase system - all zeros outside the unit circle
- Generalized linear phase
  - symmetry: even and odd order
  - antisymmetry: even and odd order
- Causal generalized linear phase FIR systems
  - even order:  $H(e^{j\omega}) = A_e(e^{j\omega})e^{\frac{-j\omega M}{2}}$ ,  $A_e(e^{j\omega})$  is even and real
  - odd order:  $H(e^{j\omega}) = A_o(e^{j\omega})e^{\frac{-j\omega M + j\pi}{2}}$ ,  $A_o(e^{j\omega})$  is odd and real
  - Type I - IV systems
  - zeros form reciprocal pair:  $z_0 = re^{j\theta} \implies z_0^{-1} = r^{-1}e^{-j\theta}$
  - for real  $h[n]$ , zeros come in conjugate pairs:  $z_0^* = re^{-j\theta} \implies (z_0^*)^{-1} = r^{-1}e^{j\theta}$
- Linear system factorization:  $H_{linp}(z) = H_{min}(z)H_{uc}(z)H_{max}(z)$

## 5) Structures For Discrete-Time Systems

- Basic Structures for IIR Systems
  - Direct Form I and II
  - Cascade Form
  - Parallel Form
- Transpose Form
- Basic Structure for FIR Systems
  - Direct Form
  - Cascade Form
- Linear Phase FIR Systems
  - Type I, II, III and IV
- Effects of Coefficient Quantization
  - effect of poles/zeros location
  - reducing quantization effect by using cascade and parallel structures

## 6) Filter Design Techniques

- Analog design techniques for IIR filter design
  - Butterworth, Chebyshev I and II, Elliptic
    - \* impulse Invariance
    - \* bilinear Transform
- FIR filter design using windowing techniques
  - Rectangular (boxcar), Barlett (triangular), Hanning, Hamming, Blackman
  - Kaiser
    - \* trade-off between mainlobe width and sidelobe attenuation
  - optimization technique
    - \* least-squares criterion
    - \* minimax criterion
    - \* Least-squares filter design
    - \* Filter design via linear programming
- comparison between IIR and FIR filters

## 7) Discrete Fourier Transform

- Discrete Fourier Series (DFS)
  - Sampling of the unit circle

- DFT
  - properties of DFS and DFT
- circular convolution
- linear convolution using DFT
- block convolution
  - overlap-add
  - overlap-save
    - \* vector-matrix equation
    - \* interblock interference
    - \* diagonalization of circulant matrices

#### 8) Computation of the DFT: Fast Fourier Transform (FFT)

- Decimation-in-time (radix-2)
- Decimation-in-frequency (radix-2)
- in-place computation, storage, access
- Cooley-Tukey algorithm (Composite  $N$  algorithm)
- inverse DFT