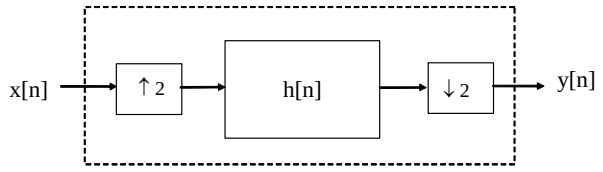
Digital Signal Processing

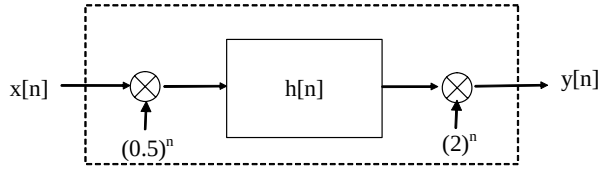
Midterm Exam #1

Spring, 2004

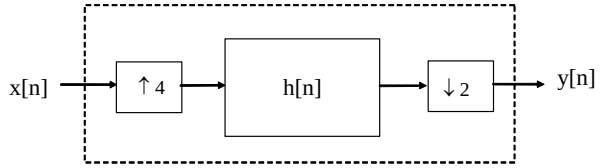
(30%) 1. Consider the following three systems.



System 1



System 2



System 3

Assume h[n] is the impulse response of an LTI system, which is causal and BIBO stable. For each of these three systems, answer the following questions.

(i) Is it linear, time-invariant? (2 pts × 3)

(ii) Is it causal? (1 pt × 3)

(iii) Find the impulse response. (2 pts × 3)

(iv) Is it BIBO stable? (1 pt × 3)

(v) Represent Y(z) in terms of X(z) and H(z). (2 pts × 3)

(vi) Find the system function. (2 pts × 3)

(Hint: $\frac{1}{2}\{\sum_{n=-\infty}^{\infty} x[n]z^{-n} + \sum_{n=-\infty}^{\infty}(-1)^n x[n]z^{-n}\} = \sum_{\substack{n=-\infty \\ n \text{ even}}}^{\infty} x[n]z^{-n}$)

(30 %) 2. When the input to an LTI system is $x[n] = -2(3)^n u[-n-1] + (0.5)^n u[n]$, the output is

$$y[n] = -\frac{54}{25}(3)^n u[-n-1] - \frac{8}{25}(0.5)^n u[n] + \frac{4}{25}(-2)^n u[n].$$

(a) Find the system function of this system and the region of convergence. (4 pts)

(b) Is this system causal? (4 pts)

(c) Is this system BIBO stable? (4 pts)

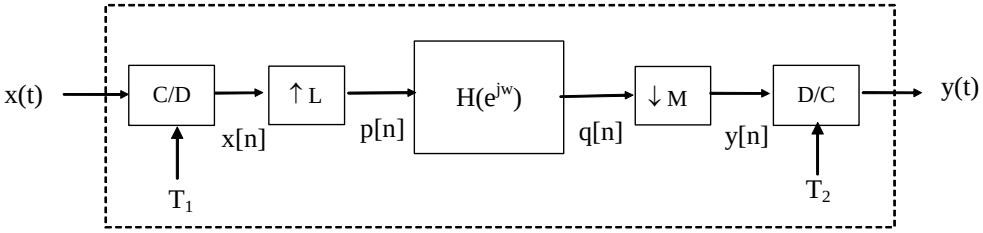
(d) Find the frequency response of this system. (4 pts)

(e) Find the impulse response of this system. (4 pts)

(f) If $x[n] = (5)^n \cos(0.5n\pi)$, find the corresponding output y[n]. (5 pts)

(g) Find a minimum-phase system $H_{min}(z)$ that has the same magnitude response as this system and has the same phase response at $\omega = 0$. (5 pts)

(40%) 3. Consider the following system.



Case 1: Assume $X(j\Omega) = \Lambda(\frac{\Omega}{40\pi})$ and $H(e^{jw}) = \begin{cases} 1 & 0 \le |\omega| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$.

(a) If $T_1 = 0.01$, $T_2 = 0.01$, L = 2, and M = 2, find the spectrums of x[n], p[n], q[n], y[n], and y(t). (15 pts)

(b) If $T_1 = 0.02$, $T_2 = 0.02$, L = 3, and M = 2, find the spectrums of x[n], p[n], q[n], y[n], and y(t). (15 pts)

Case 2: Assume the input signal x(t) is bandlimited with bandwidth W. The frequency response $H(e^{jw})$ is a low-pass filter with bandwidth B.

(c) What constraints shall we put over T_1, T_2, L, and M so that the overall system is guaranteed to be an LTI system? (10 pts)