Midterm Examine: Digital Signal Processing (EE, April 2006)

- 別忘寫姓名學號
- 請依序答題

(20%) **1.** Assume a system is BIBO (Bounded-Input Bounded-Output) stable and its linear, constant-coefficient difference equation (LCCDE) is described as

$$y[n]-2.5y[n-1]+y[n-2]=x[n]-0.3x[n-1].$$

(a) Does this system have the frequency response? If yes, please find its frequency response; if no, please state your reasoning. (3 pts)

(b) Is this a causal system? (2 pts)

(c) Find the impulse response of the system. (5 pts)

(d) If $x[n]=(0.5)^n u[n]$, find the output y[n]. (5 pts)

(e) If $x[n]=(-1)^n$, find the output y[n]. (5 pts)

(20%) **2.** Consider three LTI systems, System-A, System-B, and System-C, with their impulse responses being $h_A[n]$, $h_B[n]$, and $h_C[n]$, respectively. Assume the linear, constant-coefficient difference equation (LCCDE) of System-A is described as

$$\sum_{k=0}^{N} a_k y[n-k]=\sum_{k=0}^{M} b_k x[n-k].$$

Moreover, assume

$$h_B[n]=\begin{cases} h_A[n/2] & n=0,\pm 2,\pm 4,.... \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h_C[n]=h_A[2n].$$

(a) Find the LCCDE equation for System-B. (4 pts)

(b) If System-A is BIBO stable, will System-B and System-C also be BIBO stable? Why? (4 pts)

(c) If System-A is a low-pass filter, will System-B and System-C also be low-pass filters? Why? (4 pts)

(d) If System-A is a high-pass filter, will System-B and System-C also be high-pass filters? Why? (4 pts)

(e) If System-A is an all-pass filter, will System-B and System-C also be all-pass filters? Why? (4 pts)

(10%) **3.** Assume x[n] and h[n] are defined as

$$x[n]=\begin{cases} 5 & n=0 \\ 3 & n=1 \\ 2 & n=2 \\ 2 & n=3 \\ 3 & n=4 \\ 5 & n=5 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h[n]=\begin{cases} 3 & n=-2 \\ 2 & n=-1 \\ 1 & n=0 \\ 2 & n=1 \\ 3 & n=2 \\ 0 & \text{otherwise} \end{cases}.$$

Find the value of x[n] ∗ h[n] for every n. Here, ∗ denotes the convolution operation.

(10%) **4.** Determine the inverse z-transform of

$$X(z)=\frac{z^{-1}(1-2z^{-1})}{(1-0.8z^{-1})(1+1.2z^{-1})}$$

for the following regions of convergence (ROC).

(a) $|z|>1.2$, (b) $|z|<0.8$, (c) $0.8<|z|<1.2$.

(10%) **5.** We want to design a causal discrete-time LTI system with the property that if the input is

$$x[n]=\left(\frac{1}{2}\right)^n u[n]-\frac{1}{4}\left(\frac{1}{2}\right)^{(n-1)} u[n-1]$$

then the output is

$$y[n]=\left(\frac{1}{3}\right)^n u[n],$$

where $u[n]$ is the unit-step function.

(a) Determine the impulse response $h[n]$ of a system that satisfies that foregoing conditions. (4 pts)

(b) What are the poles and zeros of the corresponding system function $H(z)$? (3 pts)

(c) How do you check the stability property of a discrete-time LTI pole-zero system? Is this system stable? (3 pts)

(15%) **6.** The frequency response of an ideal **bandpass** filter with real-valued coefficients is given by

$$H(e^{j\omega})=\begin{cases}0, & |\omega|<\pi/8\\ 1, & \pi/8\le|\omega|\le 3\pi/8\\ 0, & 3\pi/8<|\omega|\le\pi\end{cases}$$

(a) Determine its impulse response (9 pts)

(b) Show that this impulse response can be expressed as the product of $\cos(n\pi/4)$ and the impulse response of a **lowpass** filter. (6 pts)

(15%) **7.** Let $x[n]$ be a *real*, *causal*, and *stable* sequence with a Fourier transform $X(e^{j\omega})$ whose real part is given as:

$$\text{Re}\ X(e^{j\omega})=\frac{0.9+0.9\cos\omega}{1.01-0.2\cos\omega}.$$

What is $x[n]$ for all n? (Show your derivation in details.)

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