Problem Set #2

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

Which of the following discrete-time signals could be eigenfunctions of any stable LTI system?

(a) $5^n u[n]$

(b) $e^{j2\omega n}$

(c) $e^{j\omega n} + e^{j2\omega n}$

(d) 5^n

(e) $5^n e^{j2\omega n}$

Problem 2

Consider the difference equation

$$y[n] + \frac{1}{15} y[n-1] - \frac{2}{15} y[n-2] = x[n].$$

(a) Determine the general form of the homogeneous solution to this equation.

(b) Both a causal and an anticausal LTI system are characterized by the given difference equation. Using the z-Transform, find the impulse response of the two systems.

(c) Show that the causal LTI system is stable and the anticausal LTI system is unstable.

(d) Using the z-Transform, find a particular solution to the difference equation when $x[n] = \left(\frac{3}{5}\right)^n u[n]$.

Problem 3

Consider an LTI system with $\left|H\left(e^{j\omega}\right)\right| = 1$, and let $\arg\left[H\left(e^{j\omega}\right)\right]$ be as shown in Fig. 1. If the input is

$$x[n] = \cos\left(\frac{3\pi}{2}n + \frac{\pi}{4}\right),$$

determine the output $y[n]$. Note that the angle $\frac{3\pi}{2}$ is out of range in the figure.

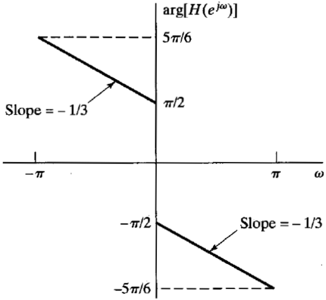

Figure 1: Phase of $H\left(e^{j\omega}\right)$.

Problem 4

Consider an LTI system with frequency response

$$H(e^{j\omega}) = e^{-j[(\omega/2)+(\pi/4)]}, \qquad -\pi < \omega \leq \pi$$

Determine $y[n]$, the output of this system, if the input is

$$x[n] = cos(\frac{15\pi n}{4} - \frac{\pi}{3})$$

for all n.

Problem 5

Consider an LTI system that is stable and for which $H(z)$, the z-transform of the impulse response, is given by

$$H(z) = \frac{3}{1 + \frac{1}{3}z^{-1}}$$

Suppose $x[n]$, the input to the system, is a unit step sequence.

(a) Find the output $y[n]$ by evaluating the discrete convolution of $x[n]$ and $h[n]$.

(b) Find the output $y[n]$ by computing the inverse z-transform of $Y(z)$.