Problem Set #3

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

Following are four z-transform. Determine which could be the z-transform of a causal sequence. Do not evaluate the inverse transform. You should be able to give the answer by inspection. Clearly state your reasons in each case.

(a) $\frac{(1-z^{-1})^2}{(1-\frac{1}{2}z^{-1})}$

(b) $\frac{(z-1)^2}{(z-\frac{1}{2})}$

(c) $\frac{(z-\frac{1}{4})^5}{(z-\frac{1}{2})^6}$

(d) $\frac{(z-\frac{1}{4})^6}{(z-\frac{1}{2})^5}$

Problem 2

Determine the inverse z-transform of each of the following. You should find the z-transform properties in Section 3.4 helpful.

(a) $X(z) = \frac{3z^{-3}}{(1-\frac{1}{4}z^{-1})^2} = 12z^{-2}\left[-z\frac{d}{dz}\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\right]$, $x[n]$ left sided. Use

$$nx[n] \Leftrightarrow -z\frac{d}{dz}X(z)$$
$$x[n-n_0] \Leftrightarrow z^{-n_0}X(z)$$

(b) $X(z) = sin(z)$, ROC includes $|z| = 1$

(c) $X(z) = \frac{z^7-2}{1-z^{-7}}$, $|z| > 1$

Problem 3

Let $x[n]$ be sequence with the pole-zero plot shown in Fig. 1. Sketch the pole-zero plot for:

(a) $y[n] = (\frac{1}{2})^n x[n]$

(b) $w[n] = cos(\frac{\pi n}{2}) x[n]$

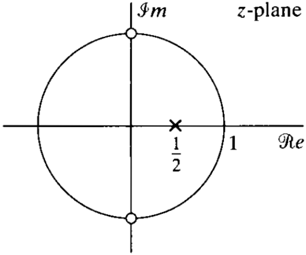

Figure 1

Problem 4 I

In Fig. 4, $h[n]$ is the impulse response of the LTI system within the inner box. The input to the system $h[n]$ is $v[n]$, and the output is $w[n]$. The z-transform of $h[n]$, $H(z)$, exists in the following region of convergence:

$$0 < r_{min} < |z| < r_{max} < \infty.$$

(a) Can the LTI system within impulse response $h[n]$ be BIBO stable? If so, determine inequality constraints on r_{min} and r_{max} such that it is stable. If not, briefly explain why.

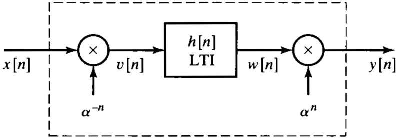

Figure 2

Problem 4 II

(b) Is the overall system (in the large box, with input $x[n]$ and output $y[n]$) LTI? If so, find its impulse response $g[n]$. If not, briefly explain why?

(c) Can the overall system be BIBO stable? If so, determine inequality constraints relating α, r_{min}, and r_{max} such that it is stable. If not, briefly explain why.