Problem Set #4

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

The continuous-time signal

$$x_c(t) = sin(20\pi t) + cos(40\pi t)$$

is sampled with a sampling period T to obtain the discrete-time signal

$$x[n] = sin\left(\frac{\pi n}{5}\right) + cos\left(\frac{2\pi n}{5}\right).$$

Determine a choice for T consistent with this information.

Problem 2 I

A complex-valued continuous-time signal $x_c(t)$ has the Fourier transform shown in Fig. 1, where $(\Omega_2 - \Omega_1) = \triangle\Omega$. This signal is sampled to produce the sequence $s[n] = x_c(nT)$.

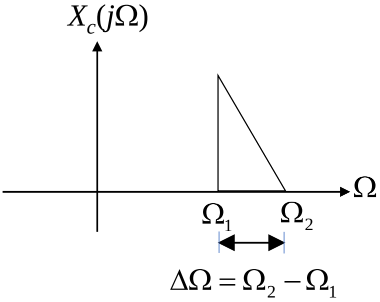

Figure 1: Prob. 2

Problem 2 II

(a) Sketch the Fourier transform $X(e^{j\omega})$ of the sequence $x[n]$ for $T = \pi/\Omega_2$.

(b) What is the lowest sampling frequency that can be used without incurring any aliasing distortion, i.e., so that $x_c(t)$ can be recovered from $x[n]$?

(c) Draw the block diagram of a system that can be used to recover $x_c(t)$ from $x[n]$ if the sampling rate is greater than or equal to the rate determined in Part (b). Assume that (complex) ideal filter are available.

Problem 3

In the system of Fig. 2,

$$X_c(j\Omega) = 0, \quad |\Omega| \geq \pi/T$$

and

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega} & , \quad |\omega| < \pi/L, \\ 0 & , \quad \pi/L < |\omega| \leq \pi. \end{cases}$$

How is $y[n]$ related to the input signal $x_c(t)$?

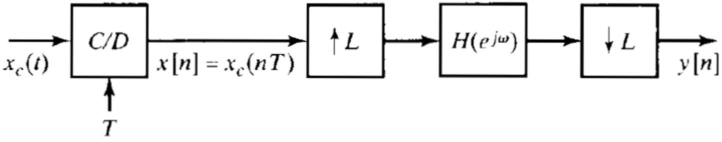

Figure 2

Problem 4

Let $x_c(t)$ be a real-valued continuous-time signal with highest frequency $2\pi(250)$ radians/second. Furthermore, let $y_c(t) = x_c(t - 1/1000)$.

(a) If $x[n] = x_c(n/500)$, is it theoretically possible to recover $x_c(t)$ from $x[n]$? Justify your answer.

(b) If $y[n] = y_c(n/500)$, is it theoretically possible to recover $y_c(t)$ from $y[n]$? Justify your answer.

(c) Is it possible to obtain $y[n]$ from $x[n]$ using the system in Fig. 3? If so, determine $H_1(e^{j\omega})$.

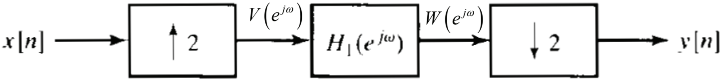

Figure 3: Only for part (c).