Problem Set #5

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1 I

To process sequences on a digital computer, we must quantize the amplitude of the sequence to a set of discrete levels. This quantization can be expressed in terms of passing the input sequence $x[n]$ through a quantizer $Q(x)$ that has an input-output relation as shown in Fig. 1.

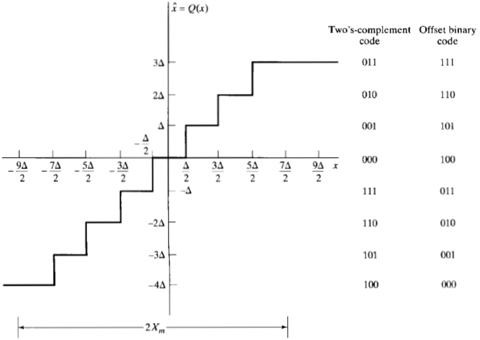


Figure 1: Typical uniform quantizer for A/D conversion

Problem 1 II

If the quantization interval $\triangle$ is small compared with changes in the level of the input sequences, we can assume that the output of the quantizer is of the form

$$y[n] = x[n] + e[n],$$

where $e[n] = Q(x[n]) - x[n]$ and $e[n]$ is a stationary random process with a first-order probability density uniformly distributed between $-\triangle/2$ and $\triangle/2$, uncorrelated from sample to sample and uncorrelated with $x[n]$, so that $E\left(e[n]x[m]\right) = 0$ for all m and n. Moreover, since $e[n]$ is a stationary random process, its autocorrelation equals $E\left(e[n]e[\ell]\right) = E\left(e[n-\ell]\right) = \sigma_e^2[n-\ell]$. That is, the autocorrelation of $e[n]$ only depends on the lag $n-\ell$ and does not depend on n and ℓ, individually. The variance of $e[n]$ equals to $\sigma_e[0]$, i.e. when $n = \ell$. Let $x[n]$ be a stationary white-noise process with zero mean and variance σ_x^2.

Problem 1 III

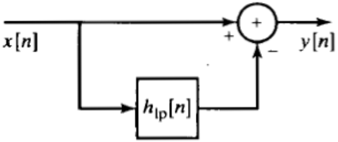
(a) Find the mean and variance of $e[n]$.

(b) What is the signal-to-quantizing-noise ratio σ_x^2/σ_e^2?

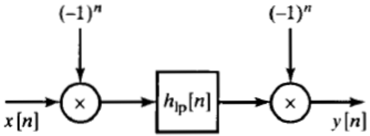
Problem 2 I

Let $h_{lp}[n]$ denote the impulse response of an ideal lowpass filter with unity passband gain and cutoff frequency $\omega_c = \pi/4$. Fig. 2 shows four systems, each of which is equivalent to an ideal LTI frequency-selective filter. For each system shown, sketch the equivalent frequency response, indicating explicitly the band-edge frequencies in terms of ω_c. In each case, specify whether the system is a lowpass, highpass, bandpass, bandstop, or multiband filter.

Problem 2 II



(a)



(b)

Problem 2 III

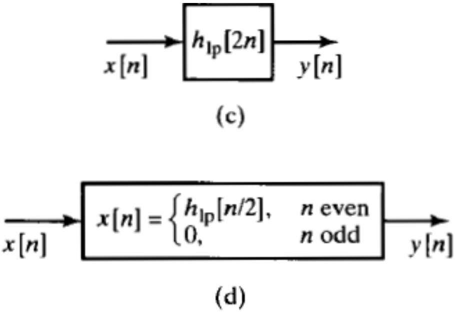


Figure 2