Problem Set #6

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

Fig. 1 shows the pole-zero plots for four different LTI systems. Based on these plots, state whether or not each system is an all-pass system.

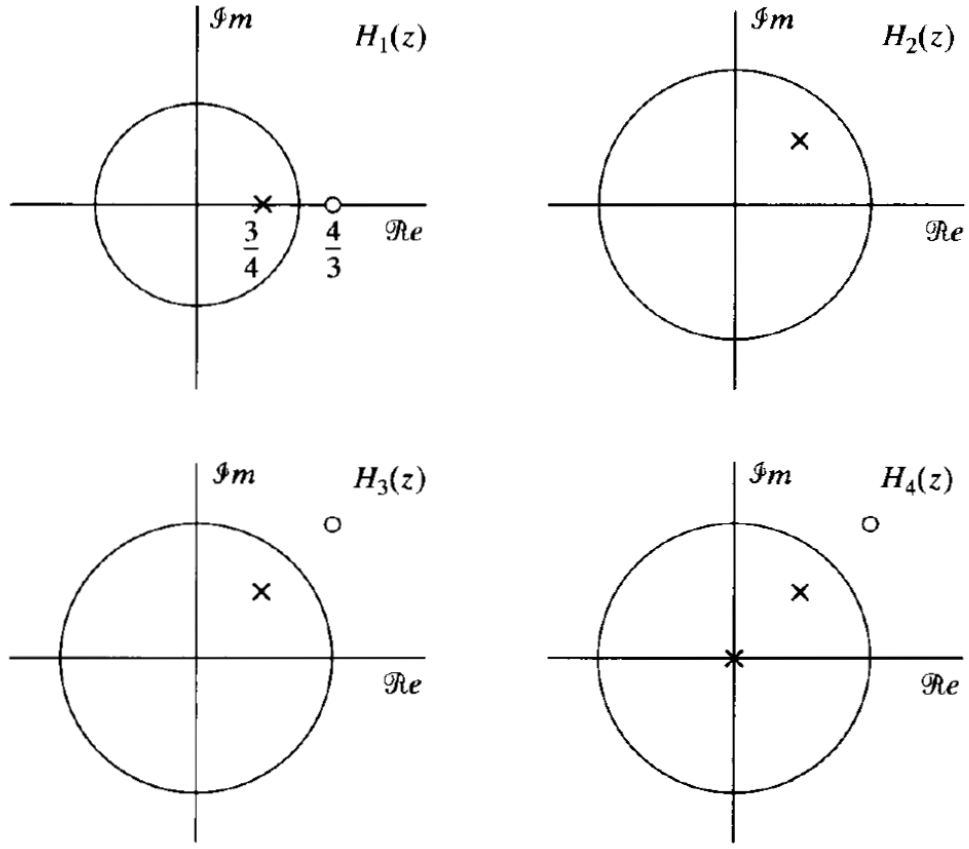


Figure 1

Problem 2

The system function of a linear time-invariant system is given by

$$H(z) = \frac{21}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})(1 - 4z^{-1})}.$$

It is known that the system is not stable and that the impulse response is two sided.

(a) Determine the impulse response $h[n]$ of the system.

(b) The impulse response found in Part (a) can be expressed as the sum of a causal impulse response $h_1[n]$ and an anticausal impulse response $h_2[n]$. Determine the corresponding system function $H_1(z)$ and $H_2(z)$.

Problem 3 I

In many practical situations, we are faced with the problem of recovering a signal that has been "blurred" by a convolution process. We can model this blurring process as a linear filtering operation, as depicted in Fig. 2, where the blurring impulse response is as shown in Fig. 3. This problem will consider ways to recover $x[n]$ from $y[n]$.

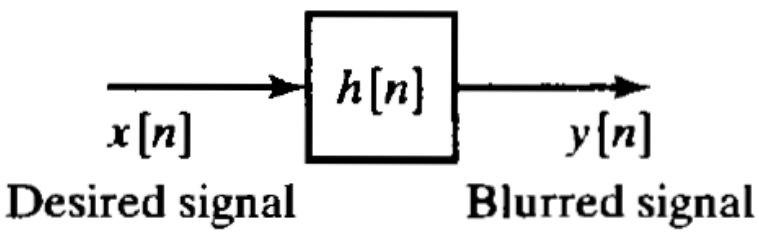


Figure 2

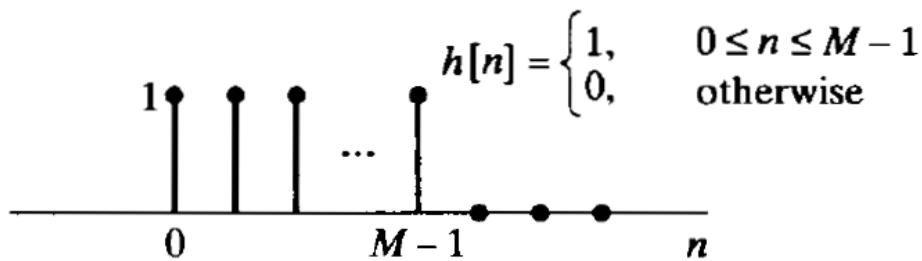


Figure 3

Problem 3 II

(a) One approach to recovering $x[n]$ from $y[n]$ is to use an inverse filter;i.e., $y[n]$ is filtered by a system whose frequency response is

$$H_i(e^{j\omega}) = \frac{1}{H(e^{j\omega})},$$

where $H(e^{j\omega})$ is the Fourier transform of $h[n]$. For the impulse response $h[n]$ shown in Fig. 3, discuss the practical problem involved in implementing the inverse filtering approach. Be complete, but also be brief and to the point.

Problem 3 III

(b) Because of the difficulties involved in inverse filtering, the following approach is suggested for recovering $x[n]$ from $y[n]$:The blurred signal $y[n]$ is processed by the system shown in Fig. 4, which produces an output $w[n]$ from which we can extract an improved replica of $x[n]$. The impulse response $h_1[n]$ and $h_2[n]$ are shown in Fig. 5. Explain in detail the working of this system. In particular, state precisely the conditions under which we can recover $x[n]$ exactly from $w[n]$. Hint:Consider the impulse response of the overall system from $x[n]$ and $w[n]$. To make the problem easier, let's assume we only include the results of $h_1[n] * h_2[n]$ up to and including qM, and not beyond that. So think about what happens when $q = 1$.

Problem 3 IV

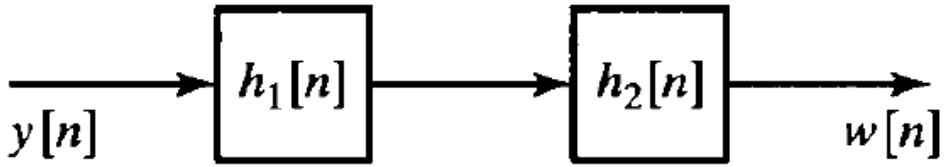
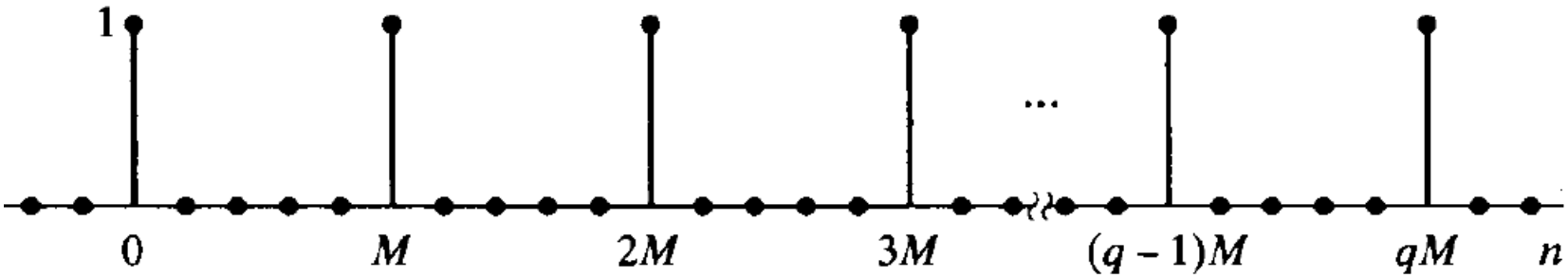


Figure 4

Problem 3 V

$$h_1[n] = \sum_{k=0}^{q} \delta[n - kM]$$



$$h_2[n] = \delta[n] - \delta[n-1]$$

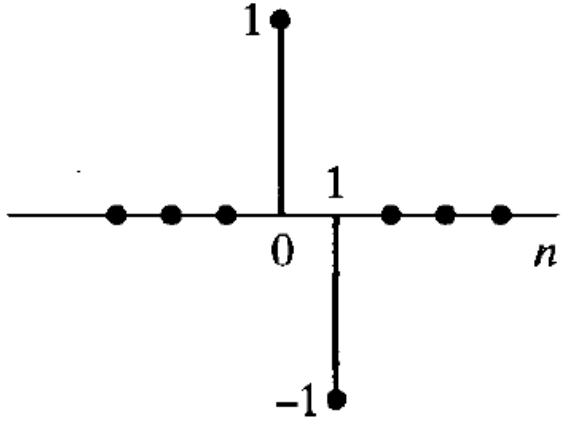


Figure 5