Problem Set #7

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

For each of the following system functions $H_k(z)$, specify a minimum-phase system function $H_{min}(z)$ such that the frequency-response magnitudes of the two systems are equal, i.e., $|H_k(e^{j\omega})| = |H_{min}(e^{j\omega})|$.

(a)

$$H_1(z) = \frac{1 - 2z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

(b)

$$H_2(z) = \frac{(1 + 3z^{-1})(1 - \frac{1}{2}z^{-1})}{z^{-1}(1 + \frac{1}{3}z^{-1})}$$

(c)

$$H_3(z) = \frac{(1 - 3z^{-1})(1 - \frac{1}{4}z^{-1})}{(1 - \frac{3}{4}z^{-1})(1 - \frac{4}{3}z^{-1})}$$

Problem 2

Consider a causal sequence $x[n]$ with the z-transform

$$X(z) = \frac{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{5}z)}{(1 - \frac{1}{6}z)}$$

For what value of α is $\alpha^n x[n]$ a real, minimum-phase sequence?

Problem 3 and Hints I

Each of the pole-zero plots in Fig. 1, together with the specification of the region of convergence, describes a linear time-invariant system with system function $H(z)$. In each case, determine whether any of the following statements are true. Justify your answer with a brief statement or a counterexample.

(a) The system is a zero-phase or a generalized linear-phase system.

(b) The system has a stable inverse $H_i(z)$.

Problem 3 and Hints II

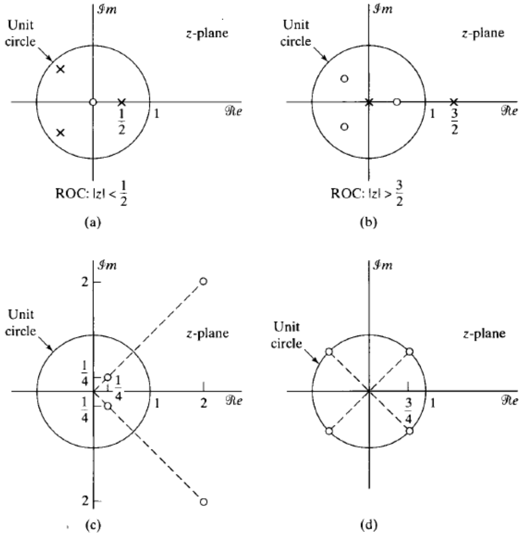

Figure 1

Problem 3 and Hints III

A generalized linear phase system is written as $H\left(e^{j\omega}\right) = A\left(e^{j\omega}\right) e^{-j\alpha\omega + j\beta}$, where $A\left(e^{j\omega}\right)$ is the zero phase response. If $\beta = 0$, then $H\left(e^{j\omega}\right)$ has linear phase. A zero phase sequence has all its poles and zeros in conjugate reciprocal pair because the sequence is real and symmetric around $n = 0$, so all the proofs (see ihw03) that are done for zeros can be equally applied to poles. In addition, generalized linear phase systems are zero phase systems with additional poles and zeros at $z = 0, \infty, 1$ or -1.