Problem Set #9

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

Consider a causal continuous-time system with impulse response $h_c(t)$ and system function

$$H_c(s) = \frac{s+a}{(s+a)^2 + b^2}.$$

(a) Use impulse invariance to determine $H_1(z)$ for a discrete-time system such that $h_1[n] = h_c(nT)$.

(b) Use step invariance to determine $H_2(z)$ for a discrete-time system such that $s_2[n] = s_c(nT)$,where

$$s_2[n] = \sum_{k=-\infty}^{n} h_2[k] \quad and \quad s_c(t) = \int_{-\infty}^{t} h_c(\tau) d\tau.$$

Problem 2

We are interested in implementing a continuous-time LTI lowpass filter $H(j\Omega)$ using the system shown in Fig. 1 when the discrete-time system has frequency response $H_d(e^{j\omega})$. The sampling time $T = 10^{-4}$ second and the input signal $x_c(t)$ is appropriately bandlimited with $X_c(j\Omega) = 0$ for $|\Omega| \geq 2\pi(5000)$. Let the specifications on $|H(j\Omega)|$ be

$$\begin{aligned} 0.99 \leq |H_e(j\Omega)| \leq 1.01, &\quad |\Omega| \leq 2\pi(1000), \\ |H_e(j\Omega)| \leq 0.01, &\quad |\Omega| \geq 2\pi(1100). \end{aligned}$$

Determine the corresponding specifications on the discrete-time frequency response $H_d(e^{j\omega})$.

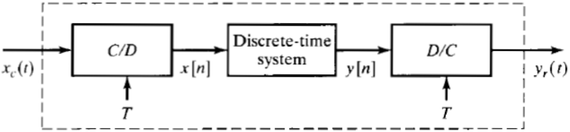

Figure 1

Problem 3

Suppose that we have an ideal discrete-time lowpass filter with cutoff frequency $\omega_c = \pi/4$. In addition, we are told that this filter resulted from applying impulse invariance to a continuous-time prototype lowpass filter using $T = 0.1$ ms. What was the cutoff frequency Ω_c for the prototype continuous-time filter?