Problem Set #10

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

Suppose that we are given a continuous-time lowpass filter with frequency response $H_c(j\omega)$ such that

$$\begin{aligned} 1-\delta_1 \le |H_c(j\Omega)| \le 1+\delta_1, &\quad |\Omega| \le \Omega_p, \\ |H_c(j\Omega)| \le \delta_2, &\quad |\Omega| \ge \Omega_s. \end{aligned}$$

A set of discrete-time lowpass filter can be obtained from $H_c(s)$ by using the bilinear transformation, i.e.,

$$H(z) = H_c(s)\big|_{s=\left(\frac{2}{T_d}\right)\frac{1-z^{-1}}{1+z^{-1}}},$$

with T_d variable.

(a) Assuming that Ω_p is fixed, find the value of T_d such that the corresponding passband cutoff frequency for the discrete-time system is $\omega_p = \pi/2$.

(b) With Ω_p fixed, sketch ω_p as a function of $0 < T_d < \infty$.

(c) With both Ω_p and Ω_s fixed, find the expression for the transition region $\Delta\omega = (\omega_s - \omega_p)$ and sketch the transition region as a function of $0 < T_d < \infty$.

Problem 2 I

A continuous-time filter with impulse response $h_c(t)$ and frequency-response magnitude

$$|H_c(j\Omega)| = \begin{cases} |\Omega|, & |\Omega| < 10\pi, \\ 0, & |\Omega| > 10\pi, \end{cases}$$

is to be used as the prototype for the design of a discrete-time filter. The resulting discrete-time system is to be used in the configuration of Fig. 3 to filter the continuous-time signal $x_c(t)$.

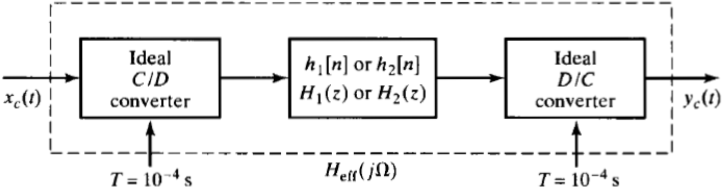

Figure 1

Problem 2 II

(a) A discrete-time system with impulse response $h_1[n]$ and system function $H_1(z)$ is obtained from the prototype continuous-time system by impulse invariance with $T_d = 0.01$; i.e., $h_1[n] = 0.01 h_c(0.01n)$. Plot the magnitude of the overall effective frequency response $H_{eff}(j\Omega) = Y_c(j\Omega)/X_c(j\Omega)$ when this discrete-time system is used in Fig. 3.

(b) Alternatively, suppose that a discrete-time system with impulse response $h_2[n]$ and system function $H_2(z)$ is obtained from the prototype continuous-time system by the bilinear transformation with $T_d = 2$; i.e.,

$$H_2(z) = H_c(s)|_{s=(1-z^{-1})/(1+z^{-1})},$$

Plot the magnitude of the overall effective frequency response $H_{eff}(j\Omega)$ when this discrete-time system is used in Fig. 3.