Problem Set #11

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

We wish to use the Kaiser window method to design a discrete-time filter with generalized linear phase that meets specifications of the following form:

$$
\begin{aligned}
|H(e^{j\omega})| &\leq 0.01, \quad 0 \leq |\omega| \leq 0.25\pi, \\
0.95 \leq |H(e^{j\omega})| &\leq 1.05, \quad 0.35\pi \leq |\omega| \leq 0.6\pi, \\
|H(e^{j\omega})| &\leq 0.01, \quad 0.65\pi \leq |\omega| \leq \pi.
\end{aligned}
$$

(a) Determine the minimum length $(M+1)$ of the impulse response and the value of the Kaiser window parameter β for a filter that meets the preceding specifications.

(b) What is the delay of the filter?

(c) Determine the ideal impulse response $h_d[n]$ to which the Kaiser window should be applied.

Problem 2 I

We wish to design an FIR a lowpass filter satisfying the specifications

$$0.95 < H(e^{j\omega}) < 1.05, \quad 0 \leq |\omega| \leq 0.25\pi,$$
$$-0.1 < H(e^{j\omega}) < 0.1, \quad 0.35\pi \leq |\omega| \leq \pi,$$

by applying a window $w[n]$ to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $\omega_c = 0.3\pi$. Which of the filters listed in Section 7.2.1 (Fig. 1) can be used to meet this specification? For each window that you claim will satisfy this specification, give the minimum length $M + 1$ required for the filter.

Problem 2 II

Rectangular

$$w[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47a}$$

Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \\ 2 - 2n/M, & M/2 < n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47b}$$

Hanning

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47c}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47d}$$

Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases} \tag{7.47e}$$

Figure 1

Problem 3 I

An ideal discrete-time Hilbert transformer is a system that introduces -90 degrees ($-\pi/2$ radians) of phase shift for $0 < \omega < \pi$ and $+90$ degrees ($+\pi/2 radians$) of phase shift for $-\pi < \omega < 0$. The magnitude of the frequency response is constant (unity) for $0 < \omega < \pi$ and for $-\pi < \omega < 0$. Such systems are also called ideal $90-$degree phase shifters.
Fig. 2 shows the ideal frequency response $H_d(e^{j\omega})$ of an ideal discrete-time Hilbert transformer that also includes nonzero group delay.

$$H_d(e^{j\omega}) = [1 - 2u(\omega)]e^{j(\pi/2 - \tau\omega)} \quad for - \pi < \omega < \pi$$

$$|H_d(e^{j\omega})| = 1, \quad \forall\omega$$

$$\angle H_d(e^{j\omega}) = \begin{cases} \frac{\pi}{2} - \tau\omega & \pi < \omega < 0 \\ -\frac{\pi}{2} - \tau\omega & 0 < \omega < \pi \end{cases}$$

Problem 3 II

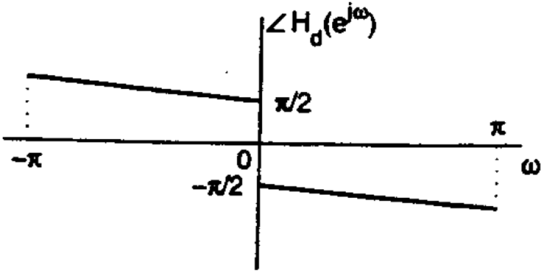

Figure 2

(a) What type(s) of FIR linear-phase systems(I, II, III, or IV) can be used to approximate the ideal Hilbert transformer $H_d(e^{j\omega})$?

Problem 3 III

(b) Suppose that we wish to use the window method to design a linear-phase approximation to the ideal Hilbert transformer. Use $H_d(e^{j\omega})$ to determine the ideal impulse response $h_d[n]$ if the FIR system is to be such that $h[n] = 0$ for $n < 0$ and $n > M$.

(c) What is the delay of the system if $M = 21$? Sketch the magnitude of the frequency response of the FIR approximation for this case, assuming a rectangular window.

(d) What is the delay of the system if $M = 20$? Sketch the magnitude of the frequency response of the FIR approximation for this case, assuming a rectangular window.