Problem Set #12

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1 I

Consider a continuous-time system with system function

$$H_c(s) = \frac{1}{s}.$$

This system is called an *integrator*, since the output $y_c(t)$ is related to the input $x_c(t)$ by

$$y_c(t) = \int_{-\infty}^{t} x_c(\tau) d\tau.$$

Suppose a discrete-time system is obtained by applying the bilinear transformation to $H_c(s)$.

(a) What is the system function $H(z)$ of the resulting discrete-time system? What is the impulse response $h[n]$?

Problem 1 II

(b) If $x[n]$ is the input and $y[n]$ is the output of the resulting discrete-time system, write the difference equation that is satisfied by the input and output. What problems do you anticipate in implementing the discrete-time system using this difference equation?

(c) Obtain an expression for the frequency response $H(e^{j\omega})$ of the system. Sketch the magnitude of the discrete-time system for $0 \leq |\omega| \leq \pi$. Compare them with the magnitude of the frequency response $H_c(j\Omega)$ of the continuous-time integrator. Under what conditions could the discrete-time "integrator" be considered a good approximation to the continuous-time integrator?

Problem 1 III

Now consider a continuous-time system with system function

$$G_c(s) = s.$$

This system is called a *differentiator*, since the output is the derivative of the input. Suppose a discrete-time system is obtained by applying the bilinear transformation to $G_c(s)$.

(d) What is the system function $G(z)$ of the resulting discrete-time system? What is the impulse response $g[n]$?

(e) Obtain an expression for the frequency response $G(e^{j\omega})$ of the system. Sketch the magnitude of the discrete-time system for $0 \leq |\omega| \leq \pi$. Compare them with the magnitude of the frequency response $G_c(j\Omega)$ of the continuous-time differentiator. Under what conditions could the discrete-time "differentiator" be considered a good approximation to the continuous-time differentiator?

Problem 1 IV

(f) The continuous-time integrator and differentiator are exact inverse of one another. Is the same true of their discrete-time approximations?

Problem 2 I

If we are given a basic filter module (a hardware or computer subroutine), it is sometimes possible to use it repetitively to implement a new filter with sharper frequency-response characteristics. One approach is to cascade the filter with itself two or more times, but it can easily be shown that, while stopband error are squared (thereby reducing them if they are less than 1), this approach will increase the passband approximation error. Another approach, suggested by Tukey (1977), is shown in Fig. 3. Tukey called this approach "twicing."

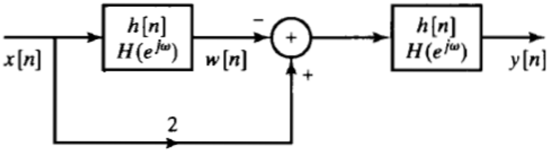

Figure 1

Problem 2 II

(a) Assume that the basic system has a symmetric finite-duration impulse response; i.e.,

$$h[n] = \begin{cases} h[-n] & , -L \leq n \leq L, \\ 0 & , otherwise \end{cases}$$

Determine whether the overall impulse response $g[n]$ is (i) an FIR and (ii) symmetric.

(b) Suppose that $H(e^{j\omega})$ satisfy the following approximation error specifications:

$$\begin{aligned} (1-\delta_1) \leq & H(e^{j\omega}) \leq (1+\delta_1), & 0 \leq \omega \leq \omega_p, \\ -\delta_2 \leq & H(e^{j\omega}) \leq \delta_2, & \omega_s \leq \omega \leq \pi. \end{aligned}$$

Problem 2 III

It can be shown that if the basic system has these specifications, the overall frequency response $G(e^{j\omega})$ satisfy specifications of the form

$$A \leq G(e^{j\omega}) \leq B, \quad 0 \leq \omega \leq \omega_p,$$
$$C \leq G(e^{j\omega}) \leq D, \quad \omega_s \leq \omega \leq \pi.$$

Determine A, B, C and D in terms of δ_1 and δ_2. If $\delta_1 \ll 1$ and $\delta_2 \ll 1$, what are the approximate maximum passband and stopband approximation errors for $G(e^{j\omega})$?

(c) As determined in Part(b), Tukey's twicing method improves the passband approximation error, but increase the stopband error. Kaiser and Hamming (1977) generalized the twicing method so as to improve both the passband and the stopband. They called their approach "sharpening." The simplest sharpening system that improves both passband and stopband is shown in Fig. 4. Assume again that the impulse response of

Problem 2 IV

the basic system is as given in Part (a). Repeat Part (b) for the system of Fig. 4.

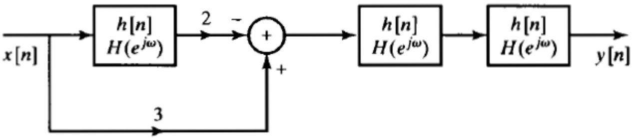

Figure 2