Problem Set #13

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

Suppose $x_c(t)$ is a periodic continuous-time signal with period 1 ms and for which the continuous-time Fourier series (CTFS) is

$$x_c(t) = \sum_{k=-9}^{9} a_k e^{j(2\pi kt/10^{-3})}.$$

The Fourier series coefficients a_k are zero for $|k| > 9$. $x_c(t)$ is sampled with a sample spacing $T = \frac{1}{6} \times 10^{-3} s$ to form $x[n]$. That is,

$$x[n] = x_c\left(\frac{n10^{-3}}{6}\right).$$

(a) Is $x[n]$ periodic and, if so, with what period?

(b) Is the sampling rate above the Nyquist rate? That is, is T sufficiently small to avoid aliasing?

(c) Find the discrete Fourier series (DFS) coefficients of $x[n]$ in terms of a_k. (Using the orthogonality relationship in Ex. 8.1)

Problem 2

Suppose $\widetilde{x}[n]$ is a periodic sequence with period N. Then $\widetilde{x}[n]$ is also periodic with period $3N$. Let $\widetilde{X}[k]$ denote the DFS coefficients of $\widetilde{x}[n]$ considered as a periodic sequence with period N, and let $\widetilde{X}_3[k]$ denote the DFS coefficients of $\widetilde{x}[n]$ considered as a periodic sequence with period $3N$.

(a) Express $\widetilde{X}_3[k]$ in terms of $\widetilde{X}[k]$.

(b) By explicitly calculating $\widetilde{X}[k]$ and $\widetilde{X}_3[k]$, verify your result in Part (a) when $\widetilde{x}[n]$ is as given in Fig. 1.

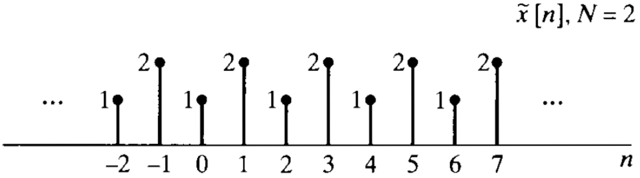

Figure 1

Problem 3 I

Consider a finite-length sequence $x[n]$ of length N; i.e.,

$$x[n] = 0 \quad \text{outside } 0 \leq n \leq N-1.$$

$X(e^{j\omega})$ denotes the Fourier transform of $x[n]$. $\widetilde{X}[k]$ denotes the sequence of 64 equally spaced samples of $X(e^{j\omega})$, i.e.,

$$\widetilde{X}[k] = X(e^{j\omega})\Big|_{\omega=2\pi k/64}.$$

It is known that in the range $0 \leq k \leq 63$, $\widetilde{X}[32] = 1$ and all the other values of $\widetilde{X}[k]$ are zero.
If the sequence length is $N = 64$, determine one sequence $x[n]$ consistent with the given information. Indicate whether the answer is unique. If it is, clearly explain why. If it is not, give a second distinct choice.