Problem Set #14

EEEC20034 - Introduction to Digital Signal Processing

NYCU

Problem 1

Compute the DFT of each of the following finite-length sequence considered to be of length N (where N is even) :

(a) $x[n] = \delta[n]$,

(b) $x[n] = \delta[n - n_0], \quad 0 \leq n_0 \leq N - 1$,

(c) $x[n] = \begin{cases} 1, & n \text{ even}, \quad 0 \leq n \leq N - 1, \\ 0, & n \text{ odd}, \quad 0 \leq n \leq N - 1, \end{cases}$

(d) $x[n] = \begin{cases} 1, & 0 \leq n \leq N/2 - 1, \\ 0, & N/2 \leq n \leq N - 1, \end{cases}$

(e) $x[n] = \begin{cases} a^n, & 0 \leq n \leq N - 1, \\ 0, & \text{otherwise}. \end{cases}$

Problem 2 I

Determine a sequence $x[n]$ that satisfies all of the following three conditions:

Condition 1: The Fourier transform of $x[n]$ has the form

$$X(e^{j\omega}) = 1 + A_1 \cos(\omega) + A_2 \cos(2\omega),$$

where A_1 and A_2 are some unknown constants.

Condition 2: The sequence $x[n] * \delta[n-3]$ evaluated at $n = 2$ is 5.

Condition 3: For the three-point sequence $w[n]$ shown in Fig. 1, the result of the eight-point circular convolution of $w[n]$ and $x[n-3]$ is 11 when $n = 2$; i.e.,

$$\sum_{m=0}^{7} w[m]x[((n-3-m))_8]|_{n=2} = 11.$$

Problem 2 II

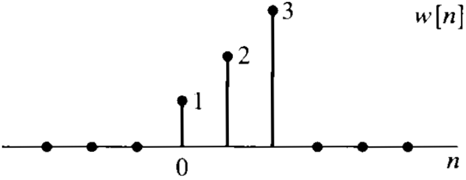

Figure 1

Problem 3

Two finite-length sequence $x_1[n]$ and $x_2[n]$, which are zero outside the interval $0 \leq n \leq 99$, are circularly convolved to form a new sequence $y[n]$; i.e.,

$$y[n] = x_1[n] \textcircled{\scriptsize 100} x_2[n] = \sum_{k=0}^{99} x_1[k] x_2[((n-k))_{100}], \quad 0 \leq n \leq 99.$$

If, in fact, $x_1[n]$ is nonzero only for $10 \leq n \leq 39$, determine the set of values of n for which $y[n]$ is guaranteed to be identical to the linear convolution of $x_1[n]$ and $x_2[n]$.