Various Digital to Digital Transformations

Filter Type	Transformation	Associated Design Formulas
Lowpass	$z^{-1} = \dfrac{\hat{z}^{-1} - a}{1 - a\hat{z}^{-1}}$	$a = \dfrac{\sin\left(\dfrac{\omega_p - \hat{\omega}_p}{2}\right)}{\sin\left(\dfrac{\omega_p + \hat{\omega}_p}{2}\right)}$ $\hat{\omega}_p = desired\ \ cutoff\ \ freq.$
Highpass	$z^{-1} = -\dfrac{\hat{z}^{-1} + a}{1 + a\hat{z}^{-1}}$	$a = -\dfrac{\cos\left(\dfrac{\omega_p - \hat{\omega}_p}{2}\right)}{\cos\left(\dfrac{\omega_p + \hat{\omega}_p}{2}\right)}$ $\hat{\omega}_p = desired\ \ cutoff\ \ freq.$

Various Digital to Digital Transformations (cont'd)

Filter Type	Transformation	Associated Design Formulas
Bandpass	$z^{-1} = -\dfrac{\hat{z}^{-2} - a_1\hat{z}^{-1} + a_2}{a_2\hat{z}^{-2} - a_1\hat{z}^{-1} + 1}$	$a_1 = 2\alpha K/(K+1)$ $a_2 = (K-1)/(K+1)$ $\alpha = \dfrac{\cos\left(\dfrac{\hat{\omega}_{c2}+\hat{\omega}_{c1}}{2}\right)}{\cos\left(\dfrac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)}$ $K = \cot\left(\dfrac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)\tan\left(\dfrac{\omega_c}{2}\right)$ $\hat{\omega}_{c1} = \textit{desired lower cutoff freq.}$ $\hat{\omega}_{c2} = \textit{desired upper cutoff freq.}$
Bandstop	$z^{-1} = \dfrac{\hat{z}^{-2} - a_1\hat{z}^{-1} + a_2}{a_2\hat{z}^{-2} - a_1\hat{z}^{-1} + 1}$	$a_1 = 2\alpha K/(K+1)$ $a_2 = (1-K)/(1+K)$ $\alpha = \dfrac{\cos\left(\dfrac{\hat{\omega}_{c2}+\hat{\omega}_{c1}}{2}\right)}{\cos\left(\dfrac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)}$ $K = \tan\left(\dfrac{\hat{\omega}_{c2}-\hat{\omega}_{c1}}{2}\right)\tan\left(\dfrac{\omega_c}{2}\right)$ $\hat{\omega}_{c1} = \textit{desired lower cutoff freq.}$ $\hat{\omega}_{c2} = \textit{desired upper cutoff freq.}$

Commonly Used Windows

Bartlett (triangular window):

$$w[n] = \begin{cases} \dfrac{2n}{M}, & 0 \le n \le \dfrac{M}{2} \\ 2 - \dfrac{2n}{M}, & \dfrac{M}{2} < n \le M \\ 0, & \text{otherwise} \end{cases}$$

Hanning window:

$$w[n] = \begin{cases} 0.5 - 0.5\cos\left(\dfrac{2\pi n}{M}\right), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Hamming window:

$$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{M}\right), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Blackman window:

$$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\dfrac{2\pi n}{M}\right) + 0.08\cos\left(\dfrac{4\pi n}{M}\right), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Comparison of Commonly Used Windows

Window Type	Peak sidelobe amplitude (relative)	Approximate width of mainlobe	Peak approx. error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser window β	Transition width of equivalent Kaiser window
Rectangular	-13	$4\pi/(M+1)$	-21	0	$1.81\pi/M$
Bartlett	-25	$8\pi/M$	-25	1.33	$2.37\pi/M$
Hanning	-31	$8\pi/M$	-44	3.86	$5.01\pi/M$
Hamming	-41	$8\pi/M$	-53	4.86	$6.27\pi/M$
Blackman	-57	$12\pi/M$	-74	7.04	$9.19\pi/M$