TABLE I
CONTINUOUS-TIME FOURIER TRANSFORM/SERIES PAIRS

Time Function	Fourier Transform	Fourier series coeffs (if periodic)
$\Pi\left(\frac{t}{2T_0}\right)$	$\frac{2\sin(\Omega T_0)}{\Omega} = 2T_0 \mathrm{sinc}\left(\frac{\Omega T_0}{\pi}\right)$	—
$\frac{\sin(Wt)}{\pi t} = \frac{W}{\pi}\mathrm{sinc}\left(\frac{Wt}{\pi}\right)$	$\Pi\left(\frac{\Omega}{2W}\right)$	—
$e^{-at}u(t), \quad \mathcal{R}e\{a\} > 0$	$\frac{1}{a+j\Omega}$	—
$te^{-at}u(t), \quad \mathcal{R}e\{a\} > 0$	$\frac{1}{(a+j\Omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t), \quad \mathcal{R}e\{a\} > 0$	$\frac{1}{(a+j\Omega)^n}$	—
$\delta(t)$	1	—
$x(t) = 1$	$2\pi\delta(\Omega)$	$X[0] = 1, X[k] = 0, \text{ for } k \neq 0$
$\delta\left(t - t_0\right)$	$\exp\left(-j\Omega t_0\right)$	—
$x(t) = \exp\left(j\Omega_0 t\right)$	$2\pi\delta\left(\Omega - \Omega_0\right)$	$X[1] = 1, X[k] = 0, \text{ otherwise}$
$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$	—
$x(t) = \cos\left(\Omega_0 t\right)$	$\pi\left[\delta\left(\Omega - \Omega_0\right) + \delta\left(\Omega + \Omega_0\right)\right]$	$X[1] = X[-1] = \frac{1}{2}$ $X[k] = 0,$ otherwise
$x(t) = \sin\left(\Omega_0 t\right)$	$\frac{\pi}{j}\left[\delta\left(\Omega - \Omega_0\right) - \delta\left(\Omega + \Omega_0\right)\right]$	$X[1] = -X[-1] = \frac{1}{j2}$ $X[k] = 0,$ otherwise
$\sum_n \delta\left(t - nT_0\right)$	$\frac{2\pi}{T_0}\sum_k \delta\left(j\left(\Omega - \frac{2\pi}{T_0}k\right)\right)$	$X[k] = \frac{1}{T_0}, \forall k$

TABLE II

FOURIER TRANSFORM PROPERTIES

Property	Mathematical Description $x(t) \Longleftrightarrow X\left(j\Omega\right)$ $y(t) \Longleftrightarrow Y\left(j\Omega\right)$
Linearity	$ax(t)+by(t) \Longleftrightarrow aX\left(j\Omega\right)+bY\left(j\Omega\right)$, where a and b are constants
Time shifting	$x\left(t-t_0\right), \quad t_0 \in \mathbb{R} \Longleftrightarrow e^{-j\Omega t_0}X(j\Omega)$
Frequency shifting	$e^{j\Omega_0 t}x(t) \Longleftrightarrow X\left(j(\Omega-\Omega_0)\right)$
Conjugation	$x^*(t) \Longleftrightarrow X^*\left(-j\Omega\right)$
Time Reversal	$x(-t) \Longleftrightarrow X\left(-j\Omega\right)$
Time scaling	$x(at) \Longleftrightarrow \frac{1}{\lvert a\rvert}X\left(\frac{j\Omega}{a}\right)$, a is a constant
Convolution in the time domain	$x(t)*y(t) \Longleftrightarrow X\left(j\Omega\right)Y\left(j\Omega\right)$
Multiplication in the time domain	$x(t)y(t) \Longleftrightarrow \frac{1}{2\pi}\int_\theta X\left(j\theta\right)Y\left(j(\Omega-\theta\right)\ d\theta = \frac{1}{2\pi}X(j\Omega)*Y(j\Omega)$
Differentiation in the time domain	$\frac{d}{dt}x(t) \Longleftrightarrow j\Omega X(j\Omega)$
Integration in the time domain	$\int_{-\infty}^{t} x(\tau)\ d\tau \Longleftrightarrow \frac{1}{j\Omega}X(j\Omega)+\pi X(j0)\delta(\Omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real $\Longleftrightarrow \begin{cases} X(j\Omega)=X^*(-j\Omega) \\ \mathcal{R}e\{X(j\Omega)\}=\mathcal{R}e\{X(-j\Omega)\} \\ \mathcal{I}m\{X(j\Omega)\}=-\mathcal{I}m\{X(-j\Omega)\} \\ \lvert X(j\Omega)\rvert=\lvert X(-j\Omega)\rvert \\ \angle X(j\Omega)=-\angle X(-j\Omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even $\Longleftrightarrow X(j\Omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd $\Longleftrightarrow X(j\Omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \text{Even}\{x(t)\}$, $x(t)$ real $\Longleftrightarrow \mathcal{R}e\{X(j\Omega\}$ $x_o(t) = \text{Odd}\{x(t)\}$, $x(t)$ real $\Longleftrightarrow j\mathcal{I}m\{X(j\Omega\}$
Parseval's Theorem	$\int_t \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_\Omega \lvert X(j\Omega)\rvert^2 d\Omega$
Duality	If $x(t) \Longleftrightarrow X(j\Omega)$ then $X(jt) \Longleftrightarrow 2\pi x(-j\Omega)$

TABLE I

DISCRETE-TIME FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_k 2\pi\delta(\omega+2\pi k)$
$a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_k \pi\delta(\omega+2\pi k)$
$(n+1)a^n u[n] \quad (\|a\| < 1)$	$\frac{1}{\left(1-ae^{-j\omega}\right)^2}$
$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\|r\| < 1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2 e^{-j2\omega}}$
$\frac{\sin\omega_c n}{\pi n}$	$X\left(e^{j\omega}\right) = \begin{cases} 1, & \|\omega\| < \omega_c, \\ 0, & \omega_c < \|\omega\| \le \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_k 2\pi\delta(\omega-\omega_0+2\pi k)$
$\cos(\omega_0 n+\phi)$	$\sum_k \left[\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k) + \pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)\right]$

TABLE II

DISCRETE-TIME FOURIER TRANSFORM THEOREMS

Sequence $x[n]$ $y[n]$	Fourier Transform $X\left(e^{j\omega}\right)$ $Y\left(e^{j\omega}\right)$
$ax[n]+by[n]$	$aX\left(e^{j\omega}\right)+bY\left(e^{j\omega}\right)$
$x[n-n_d] \quad (n_d$ an integer$)$	$e^{-j\omega n_d} X\left(e^{j\omega}\right)$
$e^{j\omega_0 n}$	$X\left(e^{j(\omega-\omega_0)}\right)$
$x[-n]$	$X\left(e^{-j\omega}\right)$
$nx[n]$	$j\frac{\partial X\left(e^{j\omega}\right)}{\partial\omega}$
$x[n]*y[n]$	$X\left(e^{j\omega}\right)Y\left(e^{j\omega}\right)$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\theta}\right)Y\left(e^{j(\omega-\theta)}\right)\,d\theta$ (periodic convolution)

Parseval's theorem:

$\sum_n \|x[n]\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\|X\left(e^{j\omega}\right)\right\|^2\,d\omega$

$\sum_n x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)Y^*\left(e^{j\omega}\right)\,d\omega$

TABLE III

DISCRETE FOURIER SERIES PROPERTIES

Periodic Sequence (Period N)	DFS Coefficients (Period N)
$\widetilde{x}[n]$	$\widetilde{X}[k]$ periodic with period N
$\widetilde{x}_1[n]$, $\widetilde{x}_2[n]$	$\widetilde{X}_1[k]$, $\widetilde{X}_2[k]$ periodic with period N
$a\widetilde{x}_1[n] + b\widetilde{x}_2[n]$	$a\widetilde{X}_1[k] + b\widetilde{X}_2[k]$
$\widetilde{X}[n]$	$N\widetilde{x}[-k]$ (Duality)
$\widetilde{x}[n-m]$	$e^{-j\frac{2\pi}{N}km}\widetilde{X}[k]$
$e^{j\frac{2\pi}{N}\ell n}\widetilde{x}[n]$	$\widetilde{X}[k-\ell]$
$\sum_{m=0}^{N-1}\widetilde{x}_1[m]\widetilde{x}_2[n-m]$ (periodic convolution)	$\widetilde{X}_1[k]\widetilde{X}_2[k]$
$\widetilde{x}_1[n]\widetilde{x}_2[n]$	$\frac{1}{N}\sum_{\ell=0}^{N-1}\widetilde{X}_1[\ell]\widetilde{X}_2[k-\ell]$ (periodic convolution)
$\widetilde{x}^*[n]$	$\widetilde{X}^*[-k]$
$\widetilde{x}^*[-n]$	$\widetilde{X}^*[k]$
$Re\{\widetilde{x}[n]\}$	$\widetilde{X}_e[k] = \frac{1}{2}\left(\widetilde{X}[k] + \widetilde{X}^*[-k]\right)$
$jIm\{\widetilde{x}[n]\}$	$\widetilde{X}_o[k] = \frac{1}{2}\left(\widetilde{X}[k] - \widetilde{X}^*[-k]\right)$
$\widetilde{x}_e[n] = \frac{1}{2}\left(\widetilde{x}[n] + \widetilde{x}^*[-n]\right)$	$Re\{\widetilde{X}[k]\}$
$\widetilde{x}_o[n] = \frac{1}{2}\left(\widetilde{x}[n] - \widetilde{x}^*[-n]\right)$	$jIm\{\widetilde{X}[k]\}$
The following properties apply only when $x[n]$ is real	
Symmetry properties	$\begin{cases} \widetilde{X}[k] = \widetilde{X}^*[-k] \\ Re\{\widetilde{X}[k]\} = Re\{\widetilde{X}[-k]\} \\ Im\{\widetilde{X}[k]\} = -Im\{\widetilde{X}[-k]\} \\ \lvert\widetilde{X}[k]\rvert = \lvert\widetilde{X}[-k]\rvert \\ \measuredangle\widetilde{X}[k] = -\measuredangle\widetilde{X}[-k] \end{cases}$
$\widetilde{x}_e[n] = \frac{1}{2}\left(\widetilde{x}[n] + \widetilde{x}[-n]\right)$	$Re\{\widetilde{X}[k]\}$
$\widetilde{x}_o[n] = \frac{1}{2}\left(\widetilde{x}[n] - \widetilde{x}[-n]\right)$	$jIm\{\widetilde{X}[k]\}$

TABLE I

LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Time Function	Transform	ROC
$\delta(t)$	1	All s
$u(t)$	$\frac{1}{s}$	$\mathcal{R}e\{s\} > 0$
$-u(-t)$	$\frac{1}{s}$	$\mathcal{R}e\{s\} < 0$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\mathcal{R}e\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\mathcal{R}e\{s\} < 0$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} > -\alpha$
$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} < -\alpha$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{R}e\{s\} > -\alpha$
$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{R}e\{s\} < -\alpha$
$\delta(t-T_0)$	e^{-sT_0}	All s
$[\cos(\Omega_0 t)]\, u(t)$	$\frac{s}{s^2+\Omega_0^2}$	$\mathcal{R}e\{s\} > 0$
$[\sin(\Omega_0 t)]\, u(t)$	$\frac{\Omega_0}{s^2+\Omega_0^2}$	$\mathcal{R}e\{s\} > 0$
$\left[e^{-\alpha t}\cos(\Omega_0 t)\right] u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\Omega_0^2}$	$\mathcal{R}e\{s\} > -\alpha$
$\left[e^{-\alpha t}\sin(\Omega_0 t)\right] u(t)$	$\frac{\Omega_0}{(s+\alpha)^2+\Omega_0^2}$	$\mathcal{R}e\{s\} > -\alpha$
$u_n(t) \triangleq \frac{d^n\delta(t)}{dt^n}$	s^n	All s
$u_{-n}(t) \triangleq \underbrace{u(t) * \ldots * u(t)}_{n \text{ times}, * \text{ denotes convolution}}$	$\frac{1}{s^n}$	$\mathcal{R}e\{s\} > 0$

TABLE II

PROPERTIES OF LAPLACE TRANSFORM

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1 \bigcap R_2$
Time shifting	$x\left(t-t_0\right)$	$e^{-st_0}X(s)$	R
Shifting in s-domain	$e^{s_0 t}x(t)$	$X(s-s_0)$	Shifted version of R (i.e. s in the ROC if $s-s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e. s is in the ROC if $\frac{s}{a}$ is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t)*x_2(t)$	$X_1(s)Y(s)$	At least $R_1 \bigcap R_2$
Differentiation in the time domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the time domain	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\frac{1}{s}X(s)$	At least $R\bigcap\{\mathcal{R}e\{s\}>0\}$

Initial- and Final-Value Theorems

If $x(t)=0$ for $t<0$ and $x(t)$ contains no impulses or higher-order singularities at $t=0$, then

$$x(0^+)=\lim_{s\to\infty} sX(s).$$

If $x(t)=0$ for $t<0$ and $x(t)$ has a finite limit as $t\to\infty$, then

$$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s).$$