Improved Detection Performance for Channel Shortening

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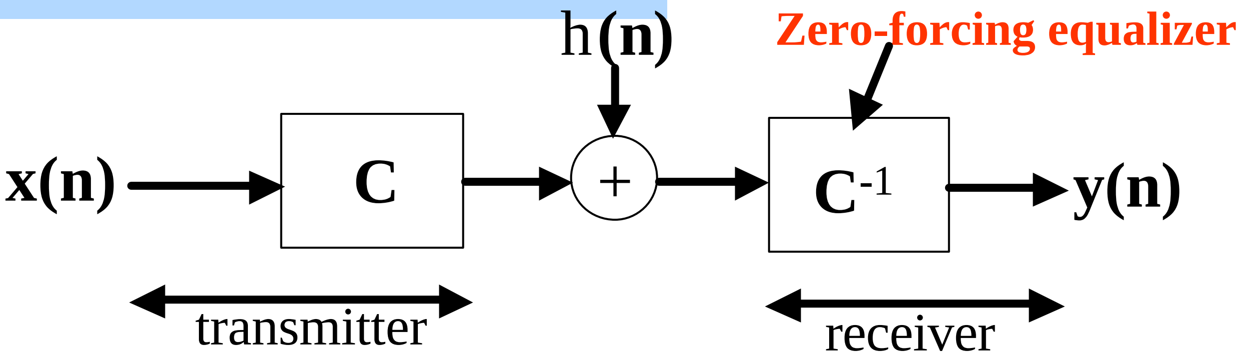
Channel Shortening and DMT

- Extensively used in Discrete Multitone (DMT) System to minimize ISI and ICI
  - *x*DSL
  - OFDM
- DMT
  - divide overall channel into multiple subchannels
  - IFFT/FFT used for transmit and receive, respectively
  - no ISI if each subchannel has constant gain and perfect sampling (ideal case)

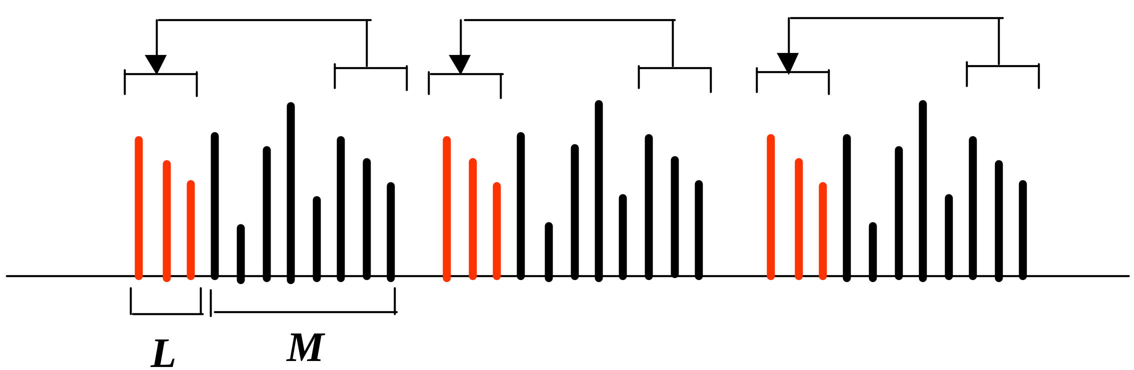
Zero-forcing Equalizer



- Assume FIR model for **C**
- Problems:
  - **C** has to be minimum phase
  - zeros close to the unit circle in **C**
    - poles close to unit circle in $\mathbf{C}^{-1}$ → large noise gain

"Imperfect" Equalization

- Cyclic prefix (CP) is used to minimize ISI and ICI
- Divide input stream into blocks, say of length *M*
- *L* symbols at the end of each block is copied to form the CP – periodic sequence
- ISI only affects the L samples in each block



Cyclic Prefix

- Advantages:
  - **C** does not have to be minimum phase
  - robust toward channel noise amplification
  - Easy to implement
  - Only need a simple frequency-domain equalizer to cancel magnitude and phase distortion in the remaining M samples
- Disadvantage:
  - Decrease transmission efficiency by M/(M+L)

DMT System Analysis (1)

- If L < M, system can be modeled as:

$$\mathbf{y}(m) = \mathbf{C}\mathbf{x}(m)$$

$$\mathbf{x}(m) = [x(mM) \quad x(mM+1) \quad \cdots \quad x(mM+M-1)]^T$$

$$\mathbf{y}(m) = [y(J_m) \quad y(J_m+1) \quad \cdots \quad y(J_m+M-1)]^T$$

$$J_m = m(L+M) + L$$

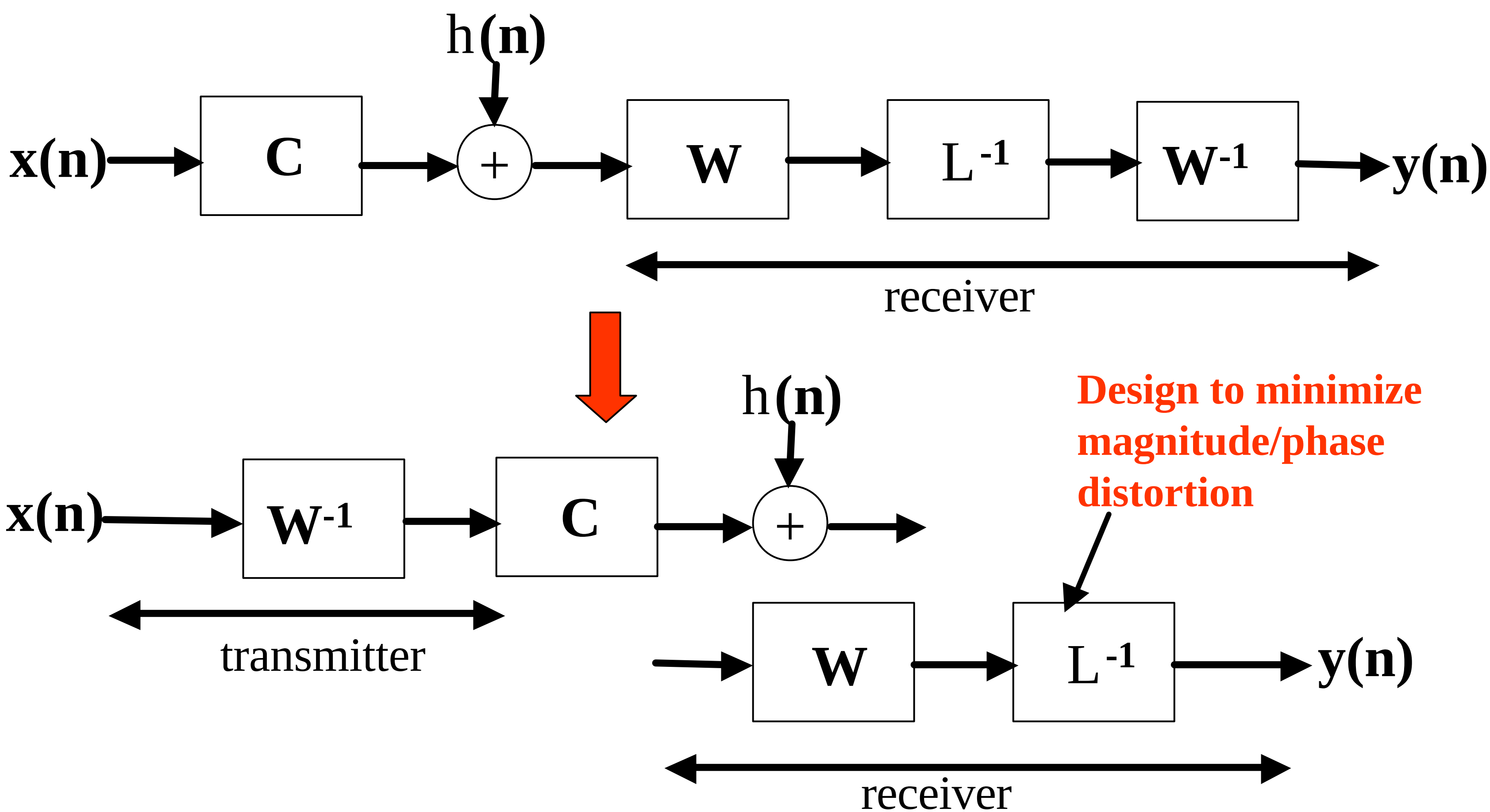
- C is a circulant matrix of the form (L=3, M=6)

$$\mathbf{C} = \begin{bmatrix} c(0) & 0 & 0 & c(3) & c(2) & c(1) \\ c(1) & c(0) & 0 & 0 & c(3) & c(2) \\ c(2) & c(1) & c(0) & 0 & 0 & c(3) \\ c(3) & c(2) & c(1) & c(0) & 0 & 0 \\ 0 & c(3) & c(2) & c(1) & c(0) & 0 \\ 0 & c(4) & c(3) & c(2) & c(1) & c(0) \end{bmatrix}$$

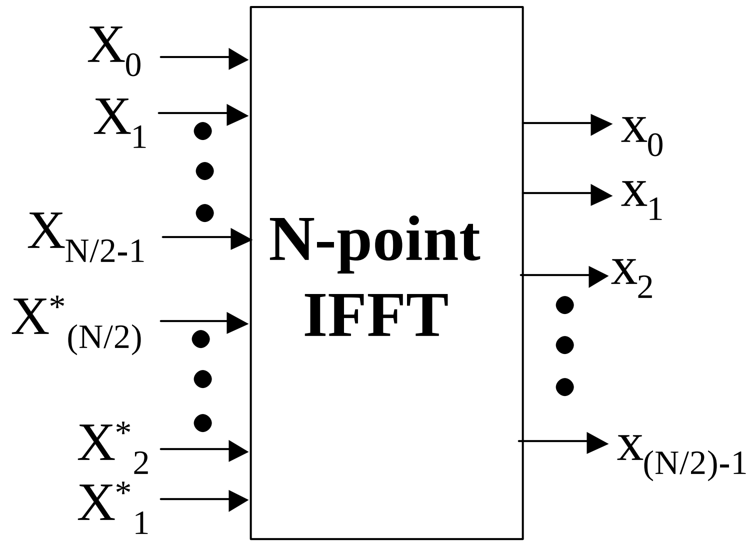
DMT System Analysis (2)

- Can diagonalize the circulant matrix using DFT matrix: $\mathbf{C} = \mathbf{W}^{-1}\mathrm{L}\mathbf{W}$
- Zero-forcing equalizer requires having $\mathbf{C}^{-1}$ at the receiving: $\mathbf{C}^{-1} = \mathbf{W}^{-1}\mathrm{L}^{-1}\mathbf{W}$
- $\mathbf{W}^{-1} = (\mathrm{L}^{-1}\mathbf{W}\mathbf{C})^{-1}$

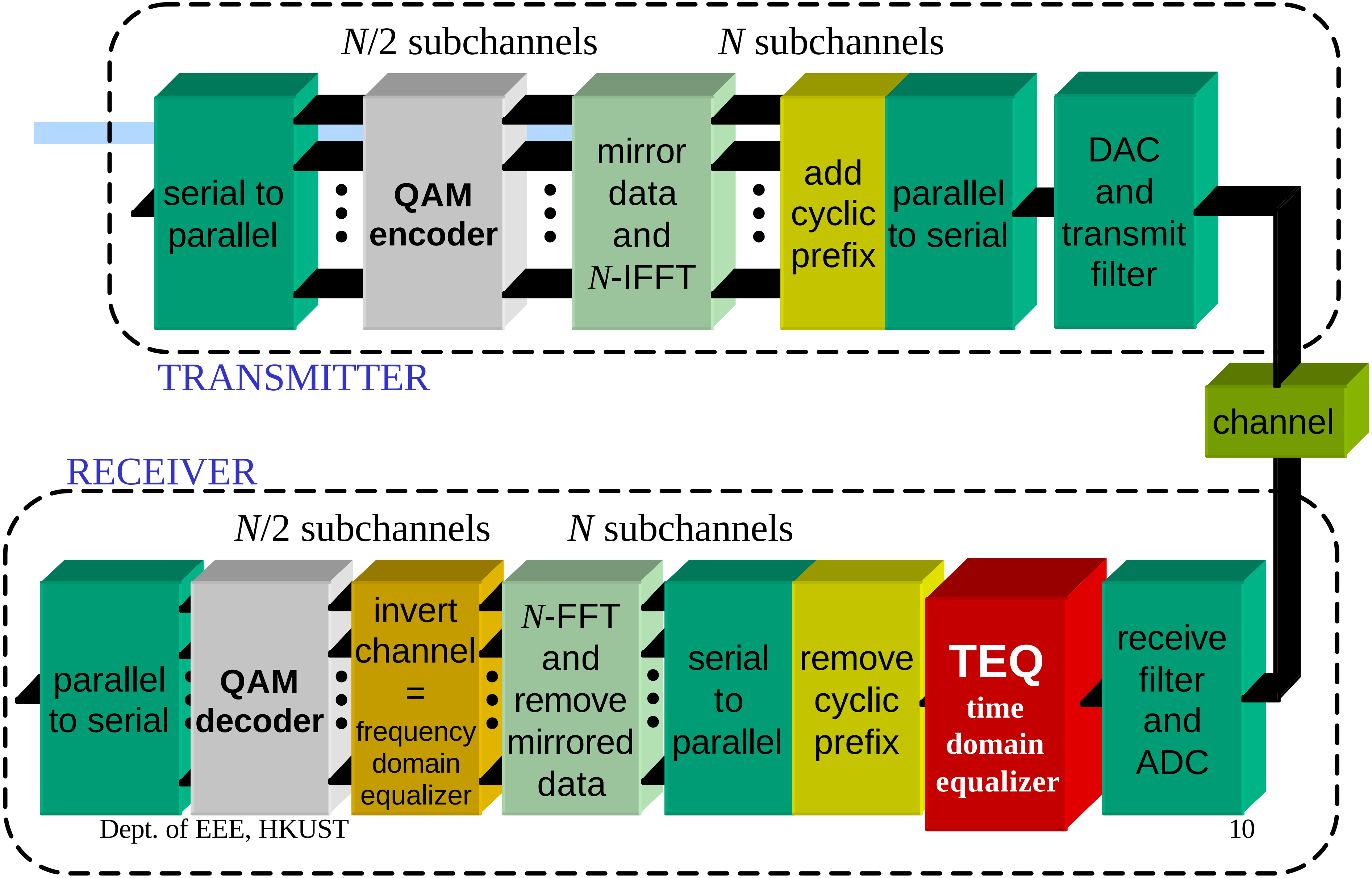
DMT System Analysis (3)



IFFT Transmitter



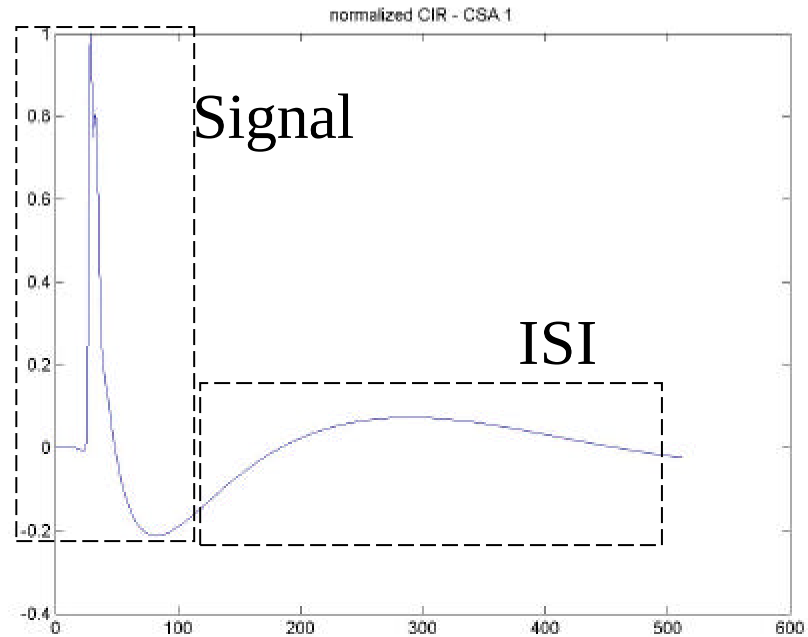
- *N* DMT symbols in → N/2 symbols out
- Ensures real-valued time domain output



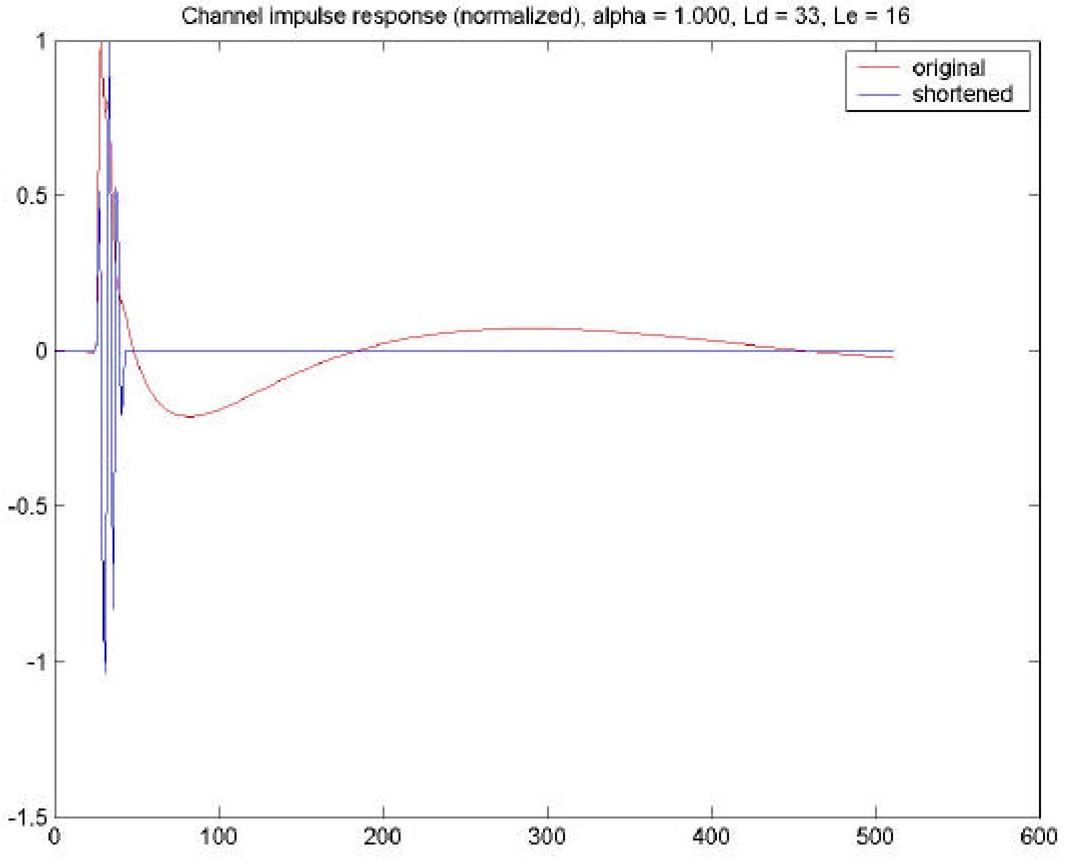
N/2 subchannels
N subchannels
serial to parallel
QAM encoder
mirror data and N-IFFT
add cyclic prefix
parallel to serial
DAC and transmit filter
TRANSMITTER
channel
RECEIVER
N/2 subchannels
N subchannels
parallel to serial
QAM decoder
invert channel = frequency domain equalizer
N-FFT and remove mirrored data
serial to parallel
remove cyclic prefix
TEQ time domain equalizer
receive filter and ADC

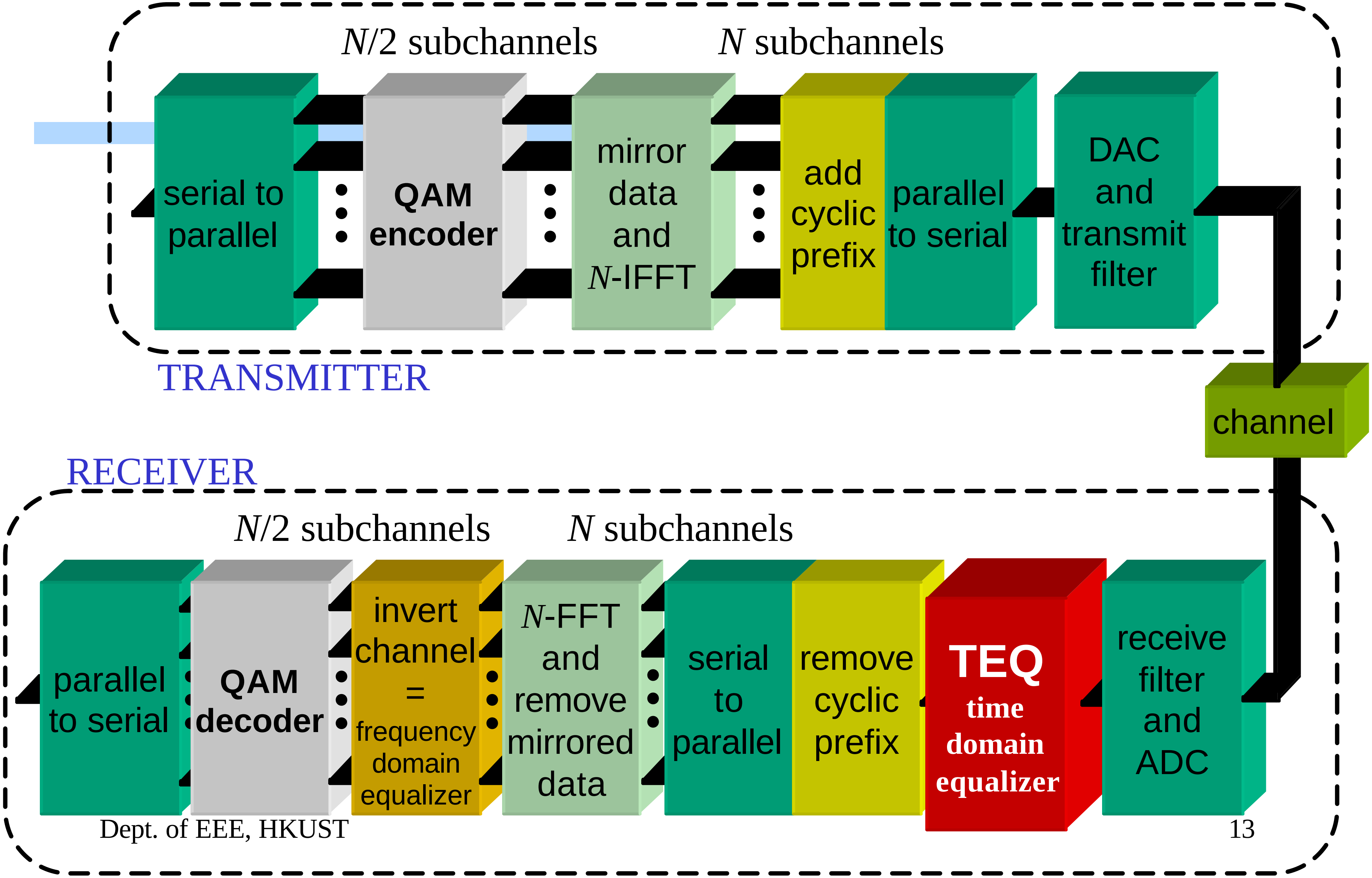
What is Time-Domain Equalizer?

- CP length ≥ L_c +1
- L_c shortened → CP length shortened → higher throughput
- Shorten channel with TEQ
  - window out the undesired portion of the channel
- Some design objectives
  - Minimize ISI power
  - Maximize bit rate
  - Minimize BER



Effect of TEQ

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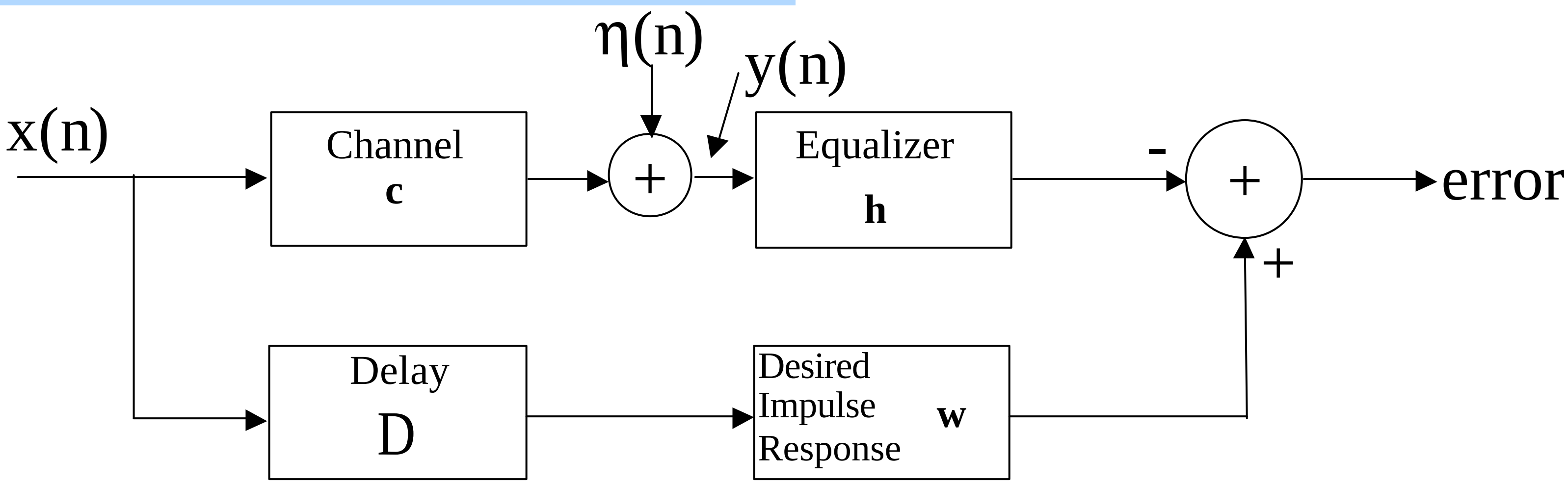
Past Work

- various TEQ design approach
  - MMSE *[Falconer & Magee, 1973]*
  - Maximize shortening SNR (MSSNR)
    - *[Melsa & et. al., 1996]*
    - *[Wang & et. al., 1999]*
  - Maximize Bit Rate
    - *MGSNR [Al-Dhahir & Cioffi, 1967, Farhang-Boroujeny & Ding, 2001]*
    - *MBR [Arslan, Evans and et. al.,2001]*
  - ISI power/AWGN
    - *ISI only [Schur & Speidel, 2001]*
    - *ISI + AWGN [Tkacenko & Vaidyanathan, 2002]*

MMSE



- MMSE achieved when: $\mathbf{w}^T\mathbf{R}_{xy} = \mathbf{h}^T\mathbf{R}_{yy}$
- Disadvantages:
  - don't consider BER
  - don't consider bit rate

MSSNR (1)

- Deal with shortening channel directly
- Idea:
  - ISI lies outside the shortened CIR
  - Maximize/minimize SNR inside/outside TEQ window

$$
\begin{aligned}
\max_{\mathbf{h}}(\text{SSNR in dB}) &= \max_{\mathbf{h}} 10\log_{10}\left(\frac{\text{energy inside window after TEQ}}{\text{energy outside window after TEQ}}\right) \\
&= \max_{\mathbf{h}} 10\log_{10}\frac{\mathbf{c}_{in}^{T}\mathbf{c}_{in}^{T}}{\mathbf{c}_{out}^{T}\mathbf{c}_{out}^{T}} \\
&= \max_{\mathbf{h}} 10\log_{10}\frac{\mathbf{h}^{T}\mathbf{C}_{in}^{T}\mathbf{C}_{in}\mathbf{h}}{\mathbf{h}^{T}\mathbf{C}_{out}^{T}\mathbf{C}_{out}\mathbf{h}} \\
&= \max_{\mathbf{h}} 10\log_{10}\frac{\mathbf{h}^{T}\mathbf{B}\mathbf{h}}{\mathbf{h}^{T}\mathbf{A}\mathbf{h}} \quad \text{s.t. } \mathbf{h}^{T}\mathbf{B}\mathbf{h} = 1
\end{aligned}
$$

MSSNR (2)

$$\mathbf{h}_{opt} = \left(\sqrt{\mathbf{B}^T}\right)^{-1}\mathbf{q}_{\min} \qquad \mathbf{q}_{\min} : \text{eigenvector of min eigenvalue of } \mathbf{C}$$

$$\mathbf{C} = \left(\sqrt{\mathbf{B}}\right)^{-1}\mathbf{A}\left(\sqrt{\mathbf{B}^T}\right)^{-1}$$

- Disadvantages:
  - leakage effect of FFT subbands not taken care of
  - doesn't account for additive noise

Maximize Bit Rate

- Bit rate expression:

$$b_{DMT} = \sum_{i=1}^{N/2} \log_2\left(1 + \frac{SNR_i}{\Gamma_i}\right) \quad \text{bits/symbol}$$

i : subchannel index

SNR_i : SNR in the ith subchannel

Γ : $\Gamma(P_e, C)$ - SNR gap for achieving Shannon channel capacity

C - line code, function of basis function (modulation) and signal constellation

MGSNR (1)

- Maximize geometric SNR (MGSNR)
  - replace SNR with GSNR

$$GSNR \equiv \Gamma\left[\left(\prod_{i=1}^{\frac{N}{2}} 1+\frac{SNR_i}{\Gamma}\right)^{\frac{2}{N}}-1\right]$$

$$\approx \left[\prod_{i=1}^{\frac{N}{2}} SNR_i\right]^{\frac{2}{N}}$$

- Assume:

$\Gamma_i = \Gamma$ (assume same P_e for all subchannels)

$SNR_i = \frac{S_x|H_i|^2}{R_{n,i}}$ (assume flat input energy across subchannels)

$$\Rightarrow b_{DMT} = \frac{N}{2}\log_2\left(1+\frac{GSNR}{\Gamma}\right)$$

MGSNR (2)

- Maximize bit rate becomes maximizing the GSNR
- Disadvantage: cannot achieve optimal solution (as shown in MBR)
  - too much assumptions and approximations in GSNR expression
  - does not include ISI power

MBR (1)

- Works with original b_{DMT} expression with

$$SNR_i = \frac{\text{signal power}}{\text{additive noise power} + \text{ISI power}}$$

  – SNR_i expression that includes ISI power
  – assume $\Gamma_i = \Gamma$ only
  – achieve near optimal solution for achievable bit rate

$$b_{DMT} = \sum_{i=1}^{N/2} \log_2\left(1 + \frac{1}{\Gamma}\frac{\mathbf{h}^T \mathbf{A_i} \mathbf{h}}{\mathbf{h}^T \mathbf{B}_i \mathbf{h}}\right)$$

$\mathbf{A}$: signal power inside window

$\mathbf{B}$: signal power outside window + additive noise power

MBR (2)

- Disadvantages:
  - High BER (compared to our design)
  - Requires nonlinear optimization

Eigenfilter TEQ (1)

- Trade-off between additive noise and ISI power to allow more design freedom
- Able to get a global optimal using Rayleigh quotient

$$J = \frac{a\left(\text{ISI power}\right) + \left(1 - a\right)\left(\text{additive noise power}\right)}{\text{signal power}}$$

$$= \frac{a s_{x_{res}}^{2} + \left(1 - a\right) s_{q}^{2}}{s_{x_{des}}^{2}}$$

$$\Rightarrow J = \min_{\mathbf{v}} \frac{\mathbf{v}^{H}\mathbf{T}\mathbf{v}}{\mathbf{v}^{H}\mathbf{v}}$$

EIGFILT (2)

- Disadvantage:
  - high BER (compared to our design)
  - does not account for the bit rate

Can we do better on BER?

- ISI taken care of by the CP
- Minimize the channel noise → minimize the BER
  - fixed ISI noise as a constraint
- Exact computation of BER not available analytically
  - Minimize a tight bound instead → Chernoff bound
- Chernoff bound of *Q*-function

$$P_e \equiv \frac{2}{N}\sum_{i=1}^{N/2} Q\left(\sqrt{k_m SNIR_i}\right)$$

$$Q\left(\sqrt{k_m SNIR_i}\right) \leq \exp\left(-\frac{k_m SNIR_i}{2}\right)$$

Chernoff TEQ design

$$SNIR_i = \frac{s_x^2 \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^H \mathbf{h}^H}{s_x^2 \mathbf{h}\mathbf{C}\overline{\mathbf{W}}_\Delta \mathbf{C}^H \mathbf{h}^H + \mathbf{h}\mathbf{R}_h \mathbf{h}^H}$$

$$J = \min_{\mathbf{h}} \exp\left(- \frac{s_x^2 \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^H \mathbf{h}^H}{2\left(s_x^2 \mathbf{h}\mathbf{C}\overline{\mathbf{W}}_\Delta \mathbf{C}^H \mathbf{h}^H + \mathbf{h}\mathbf{R}_h \mathbf{h}^H\right)} \right)$$

$$\text{s.t. } \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^H \mathbf{h}^H = m\mathbf{c}\mathbf{c}^H = mE_c$$

$$\text{since } \overline{\mathbf{W}}_\Delta = \mathbf{I} - \mathbf{W}_\Delta$$

$$J = \min_{\mathbf{h}} \ \mathbf{h}\left(\mathbf{C}\mathbf{C}^H + s_x^{-2}\mathbf{R}\right)\mathbf{h}^+$$

$$s.t. \ \ \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^+ \mathbf{h}^+ = mE_c$$

$\Rightarrow$

$$J = \min_{\mathbf{h}} \mathbf{h}\mathbf{P}\mathbf{h}^H$$

$$\text{s.t. } \mathbf{h}\mathbf{Q}\mathbf{h}^H = mE_c$$

Notations

$$\mathbf{h} \equiv [h(0)\ \ h(1)\ \ \ldots\ \ h(L_e - 1)]$$

$$\mathbf{c} \equiv [c(0)\ \ c(1)\ \ \ldots\ \ c(L_c - 1)]$$

$$\mathbf{C} \equiv \begin{bmatrix} c(0) & c(1) & \ldots & c(L_c-1) & 0 & \ldots & 0 \\ 0 & c(0) & c(1) & \ldots & c(L_c-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & c(0) & c(1) & \ldots & c(L_c-1) \end{bmatrix}$$

$$\mathbf{W}_\Delta \equiv \begin{bmatrix} \mathbf{0}_\Delta & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{L_d} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0}_{L_c+L_e-L_d-1-\Delta} \end{bmatrix}$$

Δ : delay

$\mathrm{L_e}$: Equalizer length

$\mathrm{L_c}$: Channel length

$\mathrm{L_d}$: Desired effective/shortened channel length

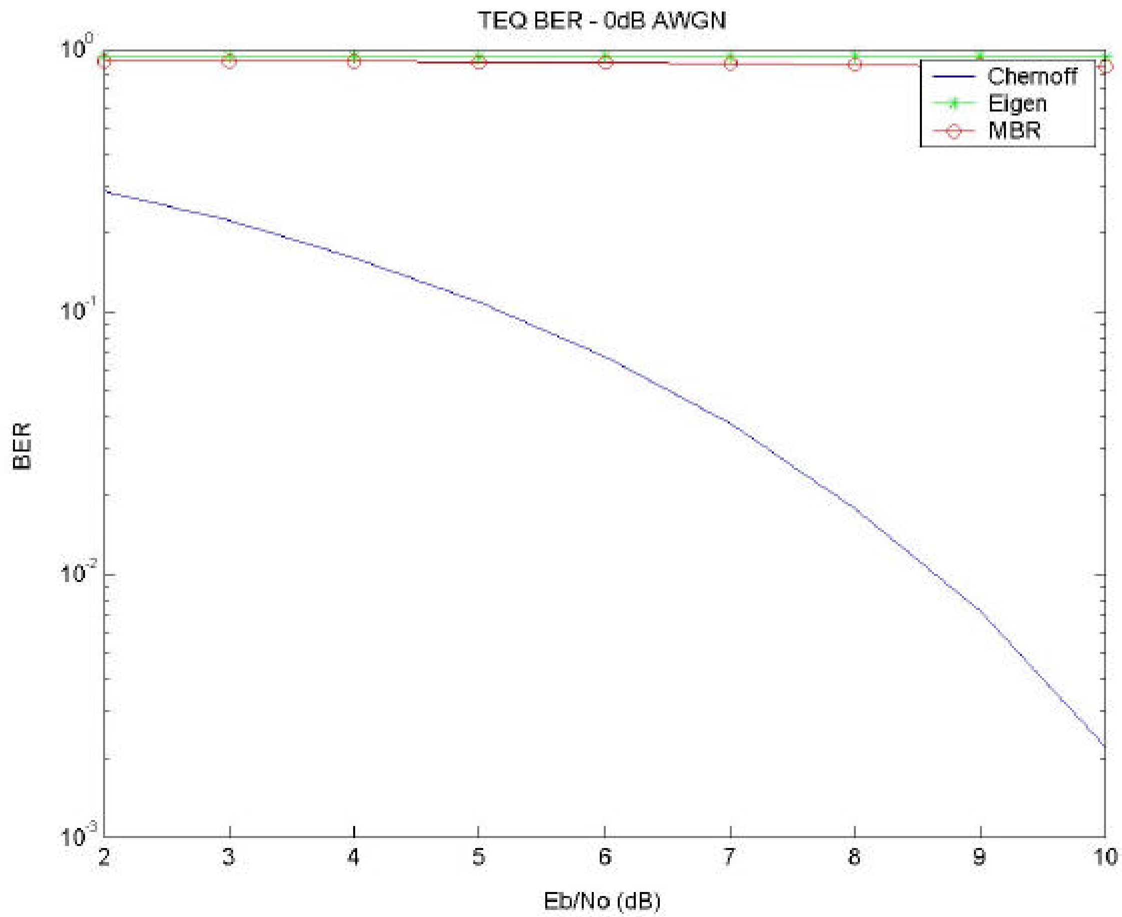
Comparison

- Chernoff (Fung & Kok, 2003)
  - minimize BER
- EIGFILT (Tkacenko & Vaidyanathan, 2002)
  - minimize ISI power/additive noise power
- MBR (Arslans, Evans & et. al., 2001)
  - maximize bit rate

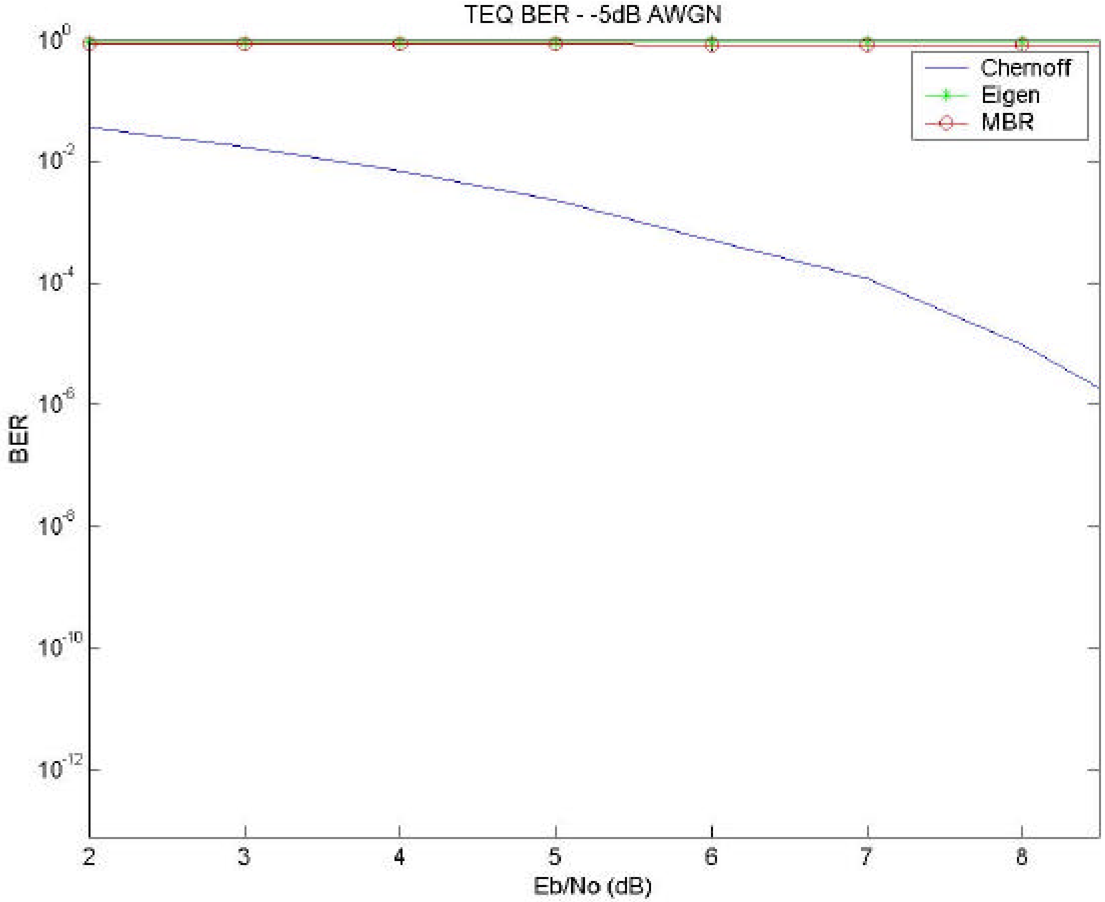
Design Parameters

Input signal power	14 dBm
AWGN power	-110 dBm
Length of equalizer	16 – 45 taps
Desired length of effective channel	33/48
Delay of effective channel	10 (Chernoff and EIGFILT)
Channel	CSA loop 1, 2, 6
Channel length	512
Size of DFT	512
Sampling frequency	2.208 MHz
μ	0.5 – 0.95

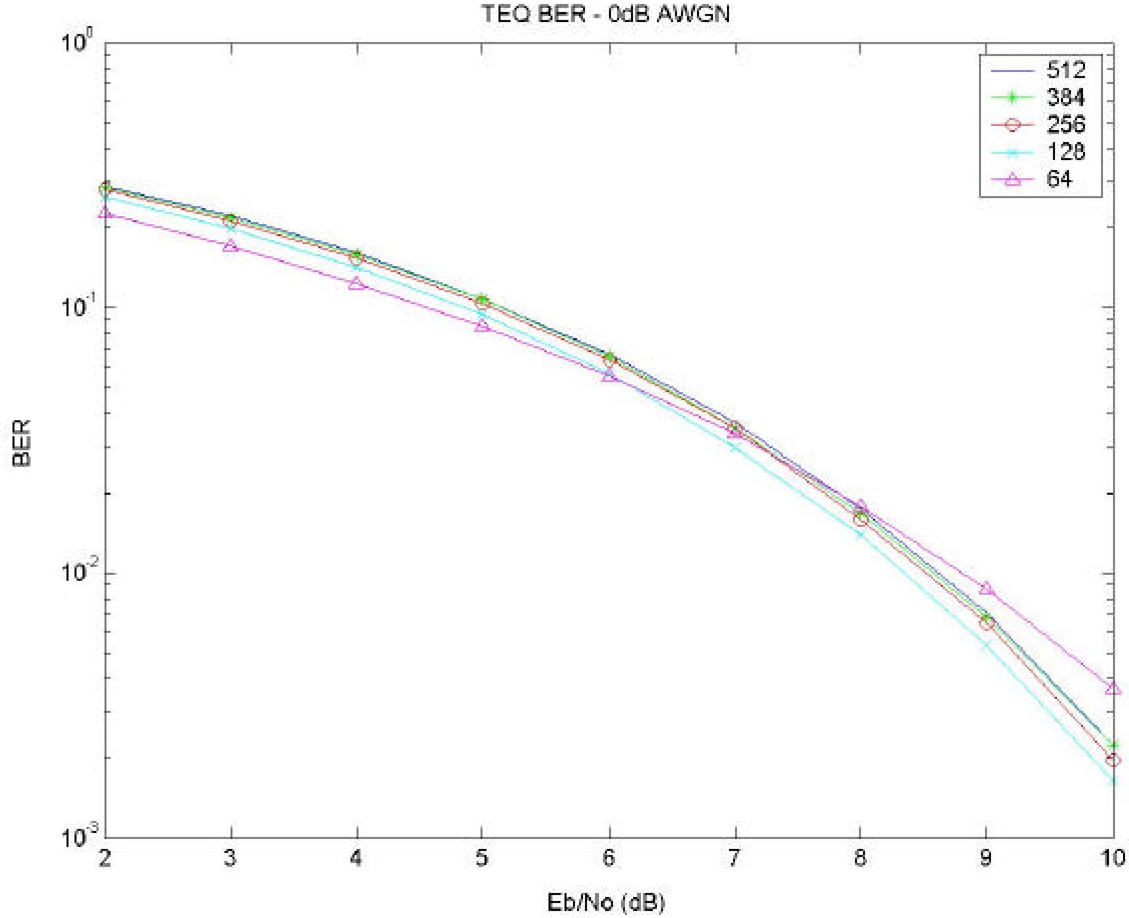
Results - different TEQs (1)



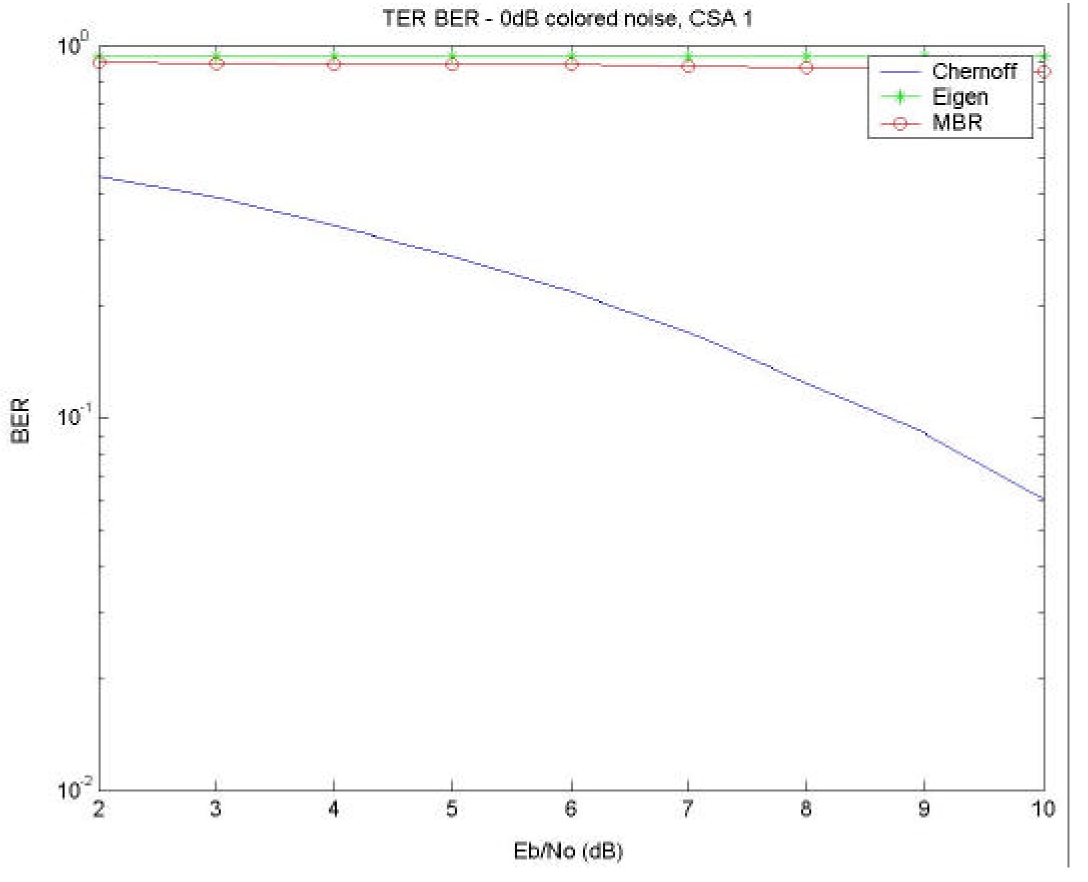
Results - different TEQs (2)



Robustness - different channel lengths



Robustness - colored noise



Conclusion

- BER optimized TEQ design
- Better BER than EIGENFILT and MBR
- Robustness:
  - different channel length
  - colored noise
- Closed form expression?
  - Iterative solution possible
- Further investigation on the effect of μ
- Can we incorporate both objectives?
  - Maximize bit rate
  - Minimize BER

References (1)

- A. Peled and A. Ruiz, “Frequency Domain Data Transmission using Reduced Computational Complexity Algorithms”, *IEEE ICASSP*, vol. 1, p. 964-967, Apr. 1980.
- P. P. Vaidyanathan and B. Vrcelj, “On Power Allocation for Generalized Cyclic-Prefix based Channel-Equalizers”, *IEEE ISCAS*, vol. 1, p. I-1 – I-4, 2002.
- D. D. Falconer and F. R. Magee, Jr., “Adaptive Channel Memory Truncation for Maximum Likelihood Sequence Estimation”, *The Bell System Technical Journal*, vol. 52, p. 1551-1562, Nov. 1973.
- P. J. W. Melsa, C. Younce, and C. E. Rohrs, “Impulse Response Shortening for Discrete Multitone Transceivers”, *IEEE Trans. on Communications*, vol. 44(12), p. 1662-1672, Dec. 1996.
- B. Wang, T. Adali, Q. Liu, and M. Vlajnic, “Generalized Channel Impulse Response Shortening for Discrete Multitone Transceivers”, *Proc. Of the 33rd Asilomar Conference on Signals, Systems, and Computers*, vol. 1, p. 276-280, Oct. 1999.

References (2)

- N. Al-Dhahir and J. M. Cioffi, “Optimum Finite-Length Equalization for Multicarrier Transceivers”, *IEEE Trans. on Communications*, vol. 44(1), p. 56-64, Jan. 1996.
- N. Al-Dhahir and J. M. Cioffi, “A Bandwidth-Optimized Reduced-Complexity Equalized Multicarrier Transceiver”, *IEEE Trans. on Communications*, vol. 45(8), p. 948-956, Aug. 1997.
- B. Farhang-Boroujeny and M. Ding, “Design Methods for Time-Domain Equalizers in DMT Transceivers”, *IEEE Trans. on Communications*, vol. 49(3), p. 554-562, Mar. 2001.
- G. Arslan, B. L. Evans, S. Kiaei, “Equalization for Discrete Multitone Transceivers to Maximize Bit Rate”, *IEEE Trans. on Signal Processing*, vol. 49(12), p. 3123-3135, Dec. 2001.
- A. Tkacenko and P. P. Vaidyanathan, “A New Eigenfilter Based Method for Optimal Design Channel Shortening Equalizers”, *IEEE ISCAS*, vol. 2, p. 504-507, 2002.

References (3)

- A. P. Iserte, A. I. Perez-Neira, D. P. Palomar, M. A. Lagunas, “Power Allocation Techniques for Joint Beamforming in OFDM-MIMO Channels”, *11th European Signal Processing Conference*, Sept. 2002.
- J. Zhang and W. Ser, “Joint Impulse Response Shortening for DMT Transceivers”, *Electronics Letters*, vol. 38(24), p. 1603-1604, Nov. 21, 2002.
- G. Arslan, B. Lu, and B. L. Evans, *Matlab DMTTEQ Toolbox*, http://signal.ece.utexas.edu/~arslan/dmtteq/dmtteq.html