“Imperfect” Equalization and Blind Channel Estimation using HOS

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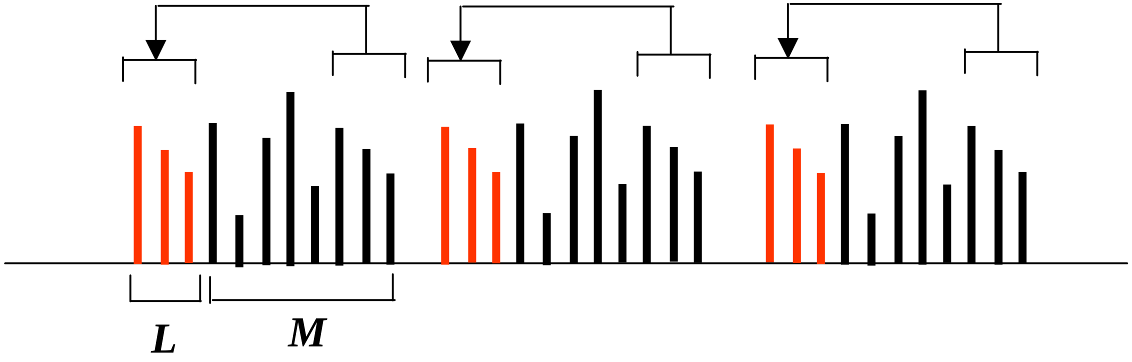
Part I: “Imperfect” Equalization and Channel Shortening

“Imperfect” Equalization and Channel Shortening

- Used in DMT-based systems such as xDSL
  - Imperfect because ISI is allowed to corrupt cyclic prefix (CP)
- Motivation
  - Simple equalization
  - But require “long” CP if channel IR is long
- TEQ
  - Shorten channel to maximize bit rate
  - BUT bit error rate should be taken into account too

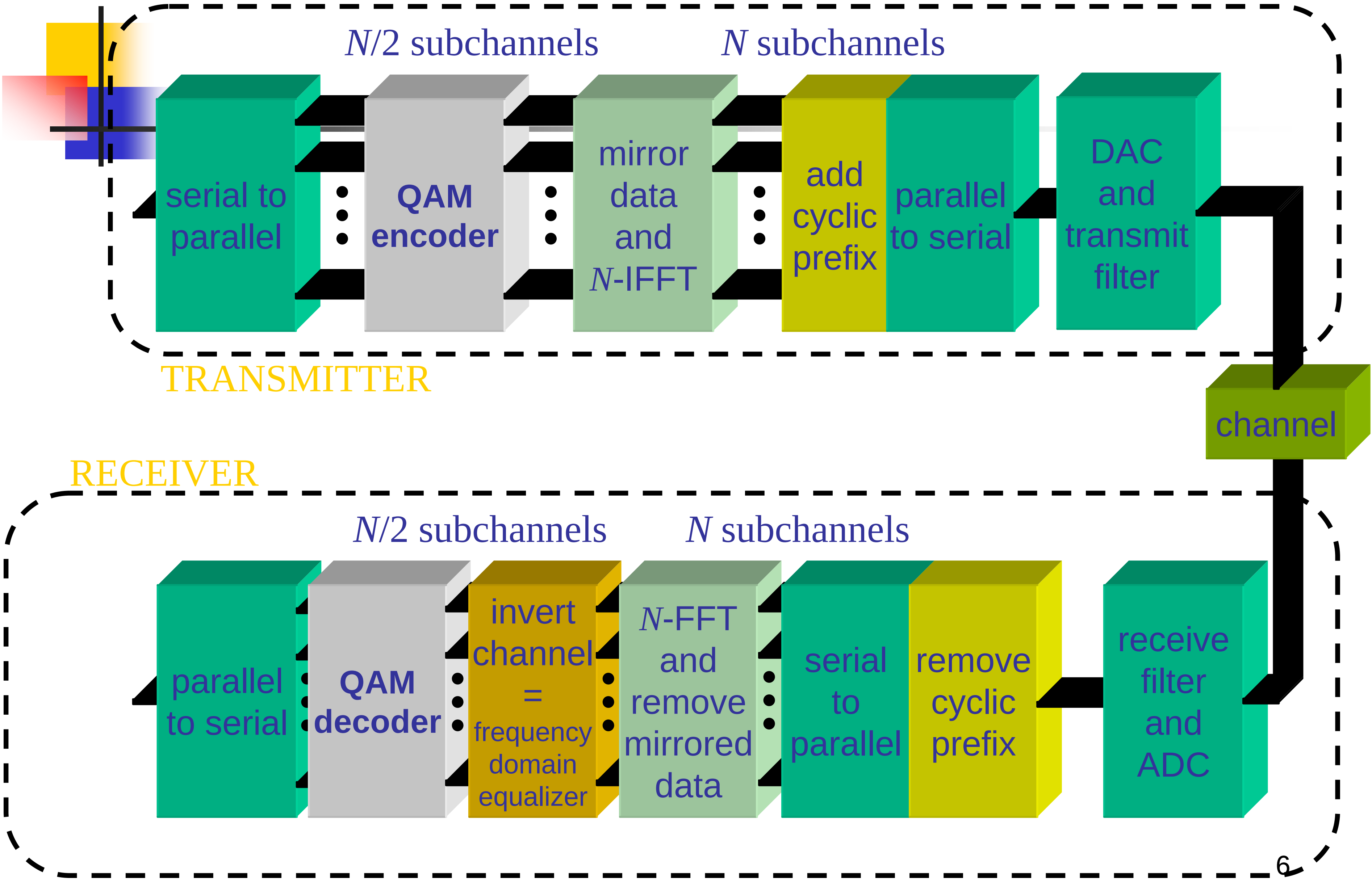
Cyclic Prefix (1)

- Cyclic prefix (CP) is used to minimize ISI and ICI
- Divide input stream into blocks, say of length *M*
- *L* symbols at the end of each block is copied to form the CP – periodic sequence
- ISI only affects the L samples in each block



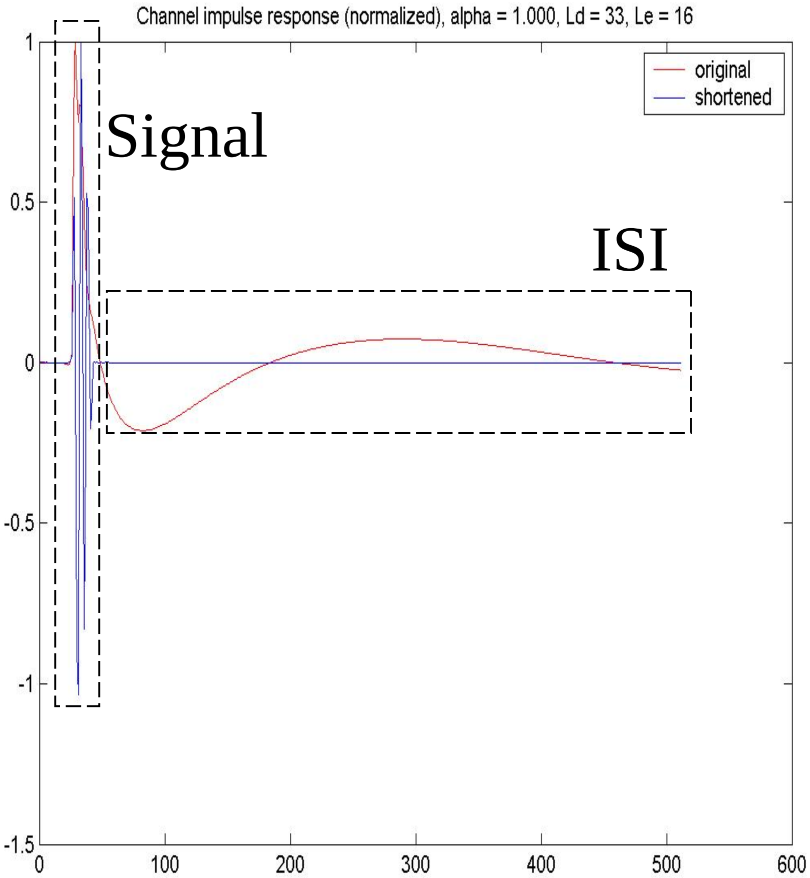
Cyclic Prefix (2)

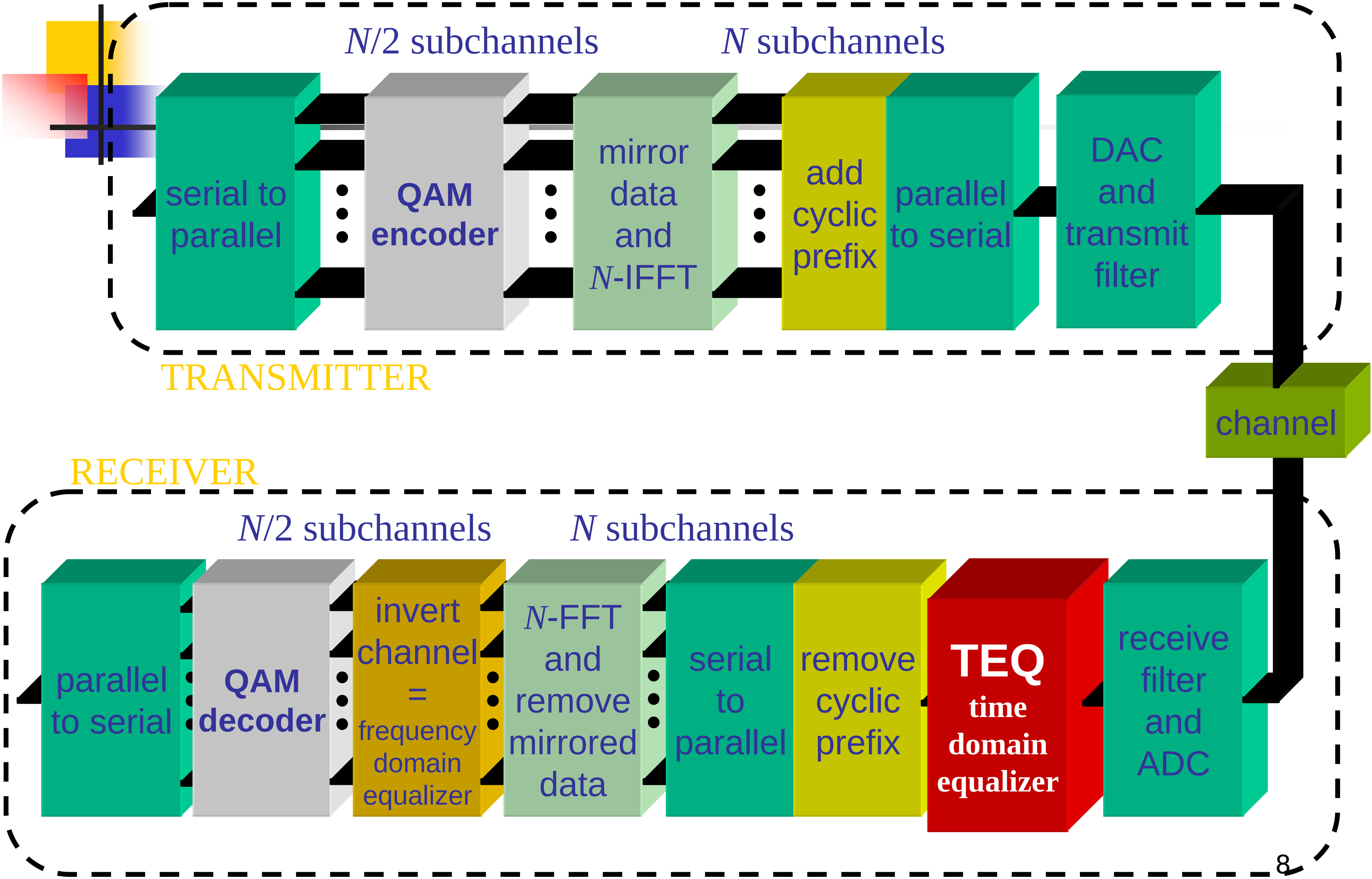
- Advantages:
  - **C** does not have to be minimum phase
  - robust toward channel noise amplification
  - Easy to implement
  - Only need a simple frequency-domain equalizer to cancel magnitude and phase distortion in the remaining M samples
- Disadvantage:
  - Decrease transmission efficiency by M/(M+L)


N/2 subchannels
N subchannels
serial to parallel
QAM encoder
mirror data and N-IFFT
add cyclic prefix
parallel to serial
DAC and transmit filter
TRANSMITTER
channel
RECEIVER
N/2 subchannels
N subchannels
parallel to serial
QAM decoder
invert channel = frequency domain equalizer
N-FFT and remove mirrored data
serial to parallel
remove cyclic prefix
receive filter and ADC

What is Time-Domain Equalizer?

- CP length ≥ L_c +1
- L_c shortened → CP length shortened → higher throughput
- Shorten channel with TEQ
  - window out the undesired portion of the channel
- Some design objectives
  - Minimize ISI power
  - Maximize bit rate
  - Minimize BER

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parallel to serial
QAM decoder
invert channel = frequency domain equalizer
N-FFT and remove mirrored data
serial to parallel
remove cyclic prefix
TEQ time domain equalizer
receive filter and ADC

Past Work

- various TEQ design approach
  - MMSE *[Falconer & Magee, 1973]*
  - Maximize shortening SNR (MSSNR)
    - *[Melsa & et. al., 1996]*
    - *[Wang & et. al., 1999]*
  - Maximize Bit Rate
    - *MGSNR [Al-Dhahir & Cioffi, 1967, Farhang-Boroujeny & Ding, 2001]*
    - *MBR [Arslan, Evans and et. al.,2001]*
  - ISI power/AWGN power
    - *ISI only [Schur & Speidel, 2001]*
    - *ISI + AWGN [Tkacenko & Vaidyanathan, 2002]*

Maximize Bit Rate

- Bit rate expression:

$$b_{DMT} = \sum_{i=1}^{N/2} \log_2\left(1 + \frac{SNR_i}{\Gamma_i}\right) \qquad \text{bits/symbol}$$

i : subchannel index

SNR_i : SNR in the i^{th} subchannel

Γ : $\Gamma(P_e, C)$ - SNR gap for achieving Shannon channel capacity

C - line code, function of basis function (modulation) and signal constellation

MBR (1)

- Works with original b_{DMT} expression with

$$SNR_i = \frac{\text{signal power}}{\text{additive noise power} + \text{ISI power}}$$

  - SNR_i expression that includes ISI power
  - assume $\Gamma_i = \Gamma$ only
  - achieve near optimal solution for achievable bit rate

$$b_{DMT} = \sum_{i=1}^{N/2} \log_2\left(1 + \frac{1}{\Gamma}\frac{\mathbf{h}^T\mathbf{A_i h}}{\mathbf{h}^T\mathbf{B}_i\mathbf{h}}\right)$$

$\mathbf{A}$: signal power inside window

$\mathbf{B}$: signal power outside window + additive noise power

MBR (2)

- Disadvantages:
  - High BER (compared to our design)
  - Requires nonlinear optimization

Eigenfilter TEQ (1)

- Trade-off between additive noise and ISI power to allow more design freedom
- Able to get a global optimal using Rayleigh quotient

$$J = \frac{\alpha(\text{ISI power}) + (1-\alpha)(\text{additive noise power})}{\text{signal power}}$$

$$= \frac{\alpha\sigma_{x_{res}}^2 + (1-\alpha)\sigma_q^2}{\sigma_{x_{des}}^2}$$

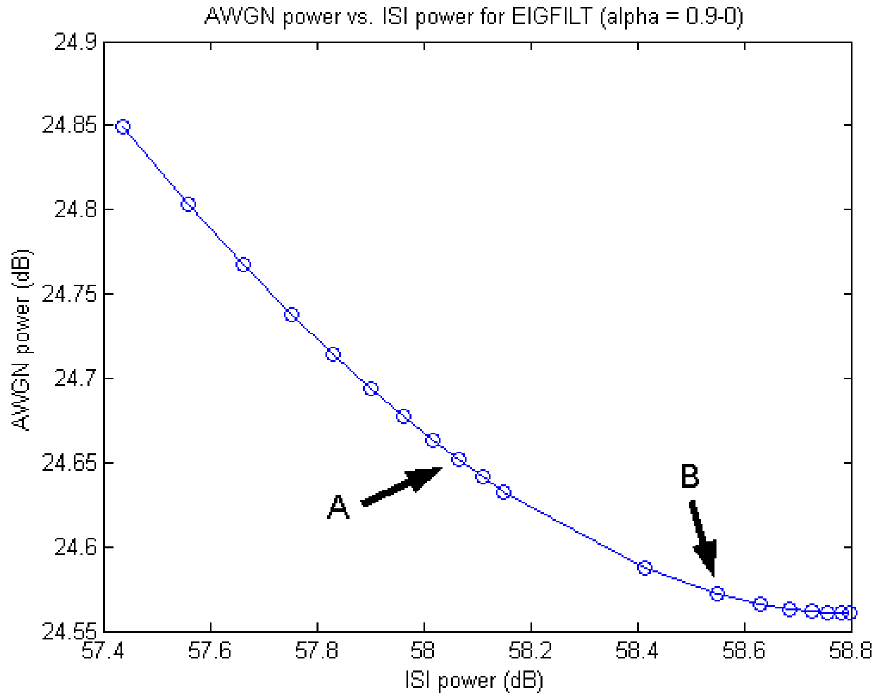
$$\Rightarrow J = \min_{\mathbf{v}} \frac{\mathbf{v}^H \mathbf{T} \mathbf{v}}{\mathbf{v}^H \mathbf{v}}$$

EIGFILT (2)

- Disadvantage:
  - high BER (compared to our design)
  - does not account for the bit rate

Can we do better on BER?

- ISI taken care of by the CP
- Minimize the channel noise → minimize the BER
  - fixed ISI noise power as a constraint



BER bound

- Exact computation of BER not available analytically
  - Minimize a tight bound instead → Chernoff bound
- Chernoff bound of Q-function

$$P_e \equiv \frac{2}{N}\sum_{i=1}^{N/2} Q\left(\sqrt{k_m SNIR_i}\right)$$

$$Q\left(\sqrt{k_m SNIR_i}\right) \leq \exp\left(-\frac{k_m SNIR_i}{2}\right)$$

Notations

$$\mathbf{h} \equiv [h(0)\ \ h(1)\ \ \ldots\ \ h(L_e - 1)]$$

$$\mathbf{c} \equiv [c(0)\ \ c(1)\ \ \ldots\ \ c(L_c - 1)]$$

$$\mathbf{C} \equiv \begin{bmatrix} c(0) & c(1) & \ldots & c(L_c-1) & 0 & \ldots & 0 \\ 0 & c(0) & c(1) & \ldots & c(L_c-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & c(0) & c(1) & \ldots & c(L_c-1) \end{bmatrix}$$

$$\mathbf{W}_\Delta \equiv \begin{bmatrix} \mathbf{0}_\Delta & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{L_d} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0}_{L_c+L_e-L_d-1-\Delta} \end{bmatrix}$$

Δ : delay

$\mathrm{L_e}$: Equalizer length

$\mathrm{L_c}$: Channel length

$\mathrm{L_d}$: Desired effective/shortened channel length

Chernoff TEQ design

$$SNIR_i = \frac{\sigma_x^2 \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^H \mathbf{h}^H}{\sigma_x^2 \mathbf{h}\mathbf{C}\overline{\mathbf{W}}_\Delta \mathbf{C}^H \mathbf{h}^H + \mathbf{h}\mathbf{R}_\eta \mathbf{h}^H}$$

$$J = \min_{\mathbf{h}} \exp\left(-\frac{\sigma_x^2 \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^H \mathbf{h}^H}{2\left(\sigma_x^2 \mathbf{h}\mathbf{C}\overline{\mathbf{W}}_\Delta \mathbf{C}^H \mathbf{h}^H + \mathbf{h}\mathbf{R}_\eta \mathbf{h}^H\right)} \right)$$

$$\text{s.t. } \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^H \mathbf{h}^H = \mu \mathbf{c}\mathbf{c}^H = \mu E_c$$

$$\text{since } \overline{\mathbf{W}}_\Delta = \mathbf{I} - \mathbf{W}_\Delta$$

$$J = \min_{\mathbf{h}} \ \mathbf{h}\left(\mathbf{C}\mathbf{C}^H + \sigma_x^{-2}\mathbf{R}\right)\mathbf{h}^+$$

$$s.t. \ \ \mathbf{h}\mathbf{C}\mathbf{W}_\Delta \mathbf{C}^+ \mathbf{h}^+ = \mu E_c$$

$$\Rightarrow$$

$$J = \min_{\mathbf{h}} \mathbf{h}\mathbf{P}\mathbf{h}^H$$

$$\text{s.t. } \mathbf{h}\mathbf{Q}\mathbf{h}^H = \mu E_c$$

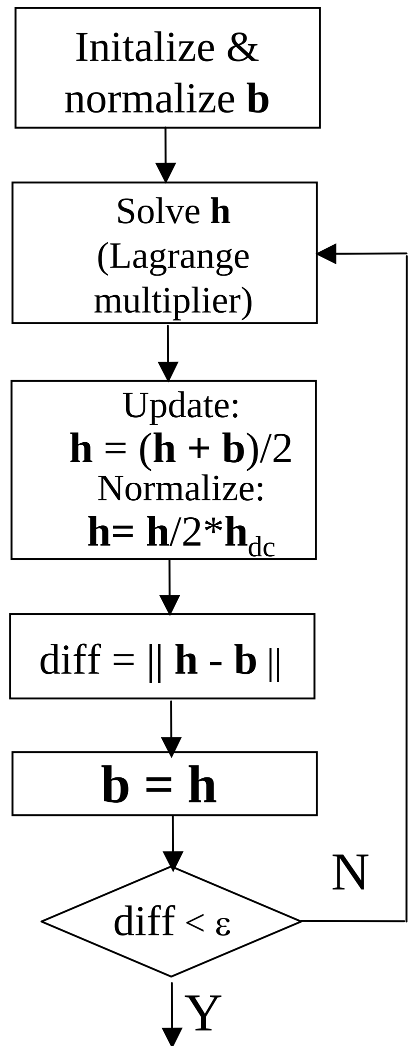
Solutions

- Nonlinear Optimization (*fmincon* in MATLAB)
- Iterative technique
  - linearize the constraint $\mathbf{hQh}^H$
  - $\mathbf{hQh}^H$ ==> $\mathbf{bQh}^H$ ==> $\mathbf{Kh}^H$ ==> constraint is now linear
  - $\mathbf{b} = \mathbf{h}^{(k-1)}$, *k* is the current iteration

$$L(h,\lambda) = \mathbf{hPh}^H + \lambda(\mathbf{Kh} - \mu E_c)$$

$$\begin{bmatrix} 2\mathbf{P} & \mathbf{K}^H \\ \mathbf{K} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{h}^H \\ \lambda^H \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mu E_c \end{bmatrix}$$

Iterative Algorithm



Comparison

- Chernoff (Fung & Kok, 2003)
  - minimize BER
- EIGFILT (Tkacenko & Vaidyanathan, 2002)
  - minimize ISI power/additive noise power
- MBR (Arslans, Evans & et. al., 2001)
  - maximize bit rate

Design Objective Comparison

TEQ	Bit rate	ISI power	AWGN power
Chernoff	--	fixed ISI power	minimize AWGN power
EIGFILT	--	tradeoff AWGN power	tradeoff ISI power
MBR	optimal	--	--

TEQ Design Parameters

	Chernoff	EIGFILT	MBR
Input signal power, σ_x^2 (dBm)	14	14	23
AWGN power, σ_η^2 (dBm)	-110	-110	-140
Equalizer length, L_e (samples)	16	16	16
Desired EIR length, L_d (samples)	33	33	33
Delay, Δ (samples)	10	10	24
Carrier Service Loop (CSA), L_c = 512	1	1	1
DFT size	512	512	512
Sampling frequency, fs (MHz)	2.208	2.208	2.208
μ	0.1-0.95	--	--
α	--	0.1-1.0	--

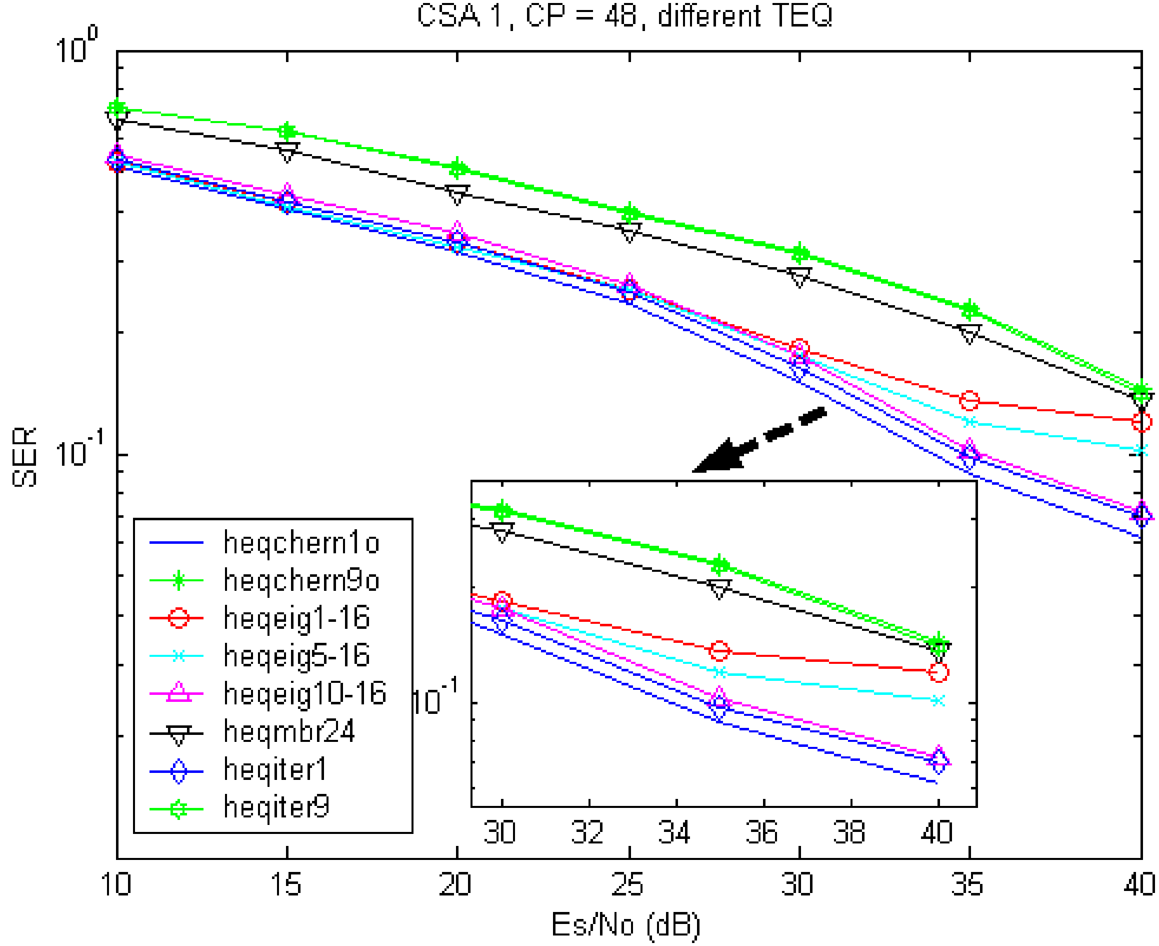
Simulation Parameters

- CP = 48
- 16 QAM modulation
- 1 tap FEQ/subcarrier (zero-forcing)
- CSA 1, 2, 6 (DMT Toolbox)

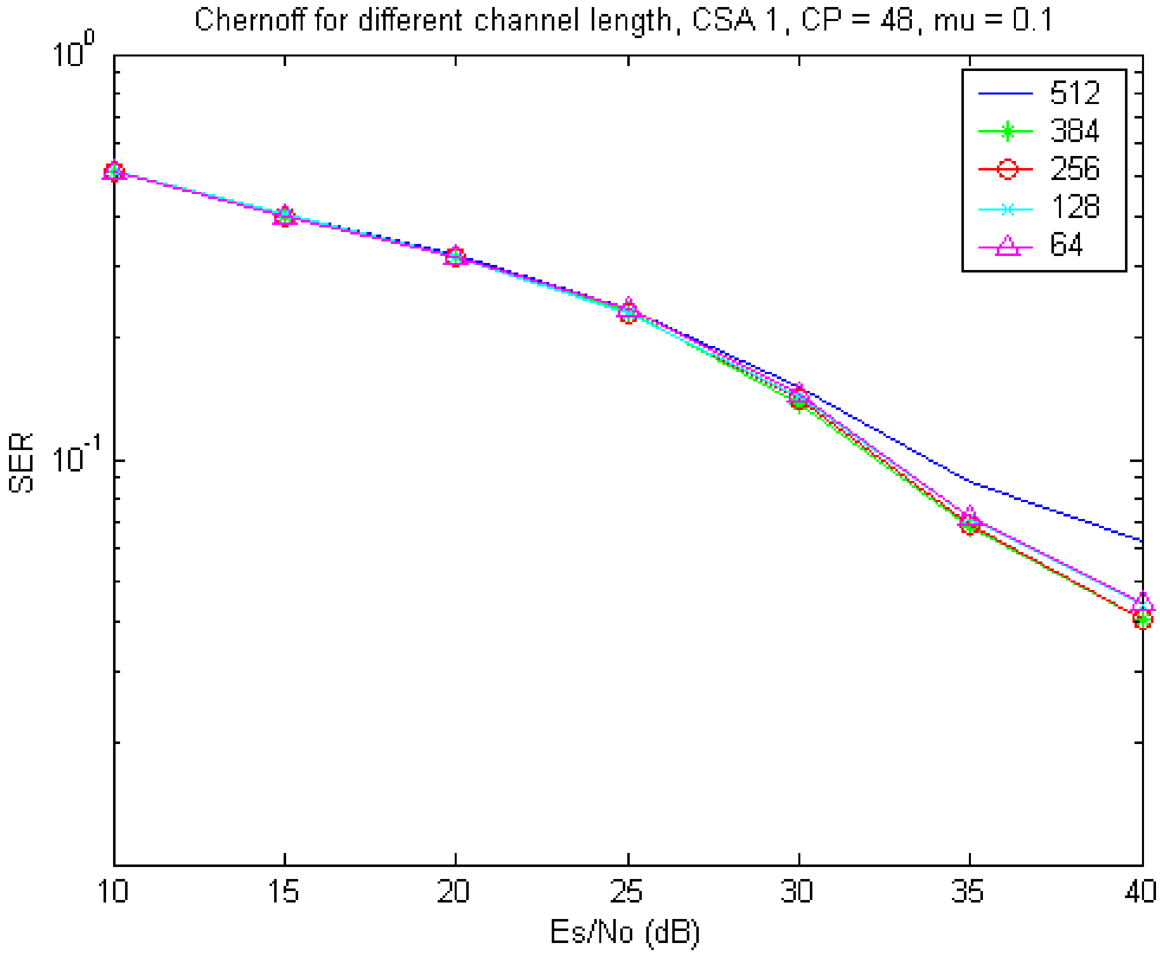
TEQ Naming Convention

Name	Type	μ	α
heqchern1o	Chernoff (nonlinear opt)	0.1	--
heqchern9o	Chernoff (nonlinear opt)	0.9	--
heqiter1	Chernoff (iterative)	0.1	--
heqiter9	Chernoff (iterative)	0.9	--
heqeig1-16	EIGFILT	--	0.1
heqeig5-16	EIGFILT	--	0.9
heqeig10-16	EIGFILT	--	1.0
heqmbr24	MBR	--	--

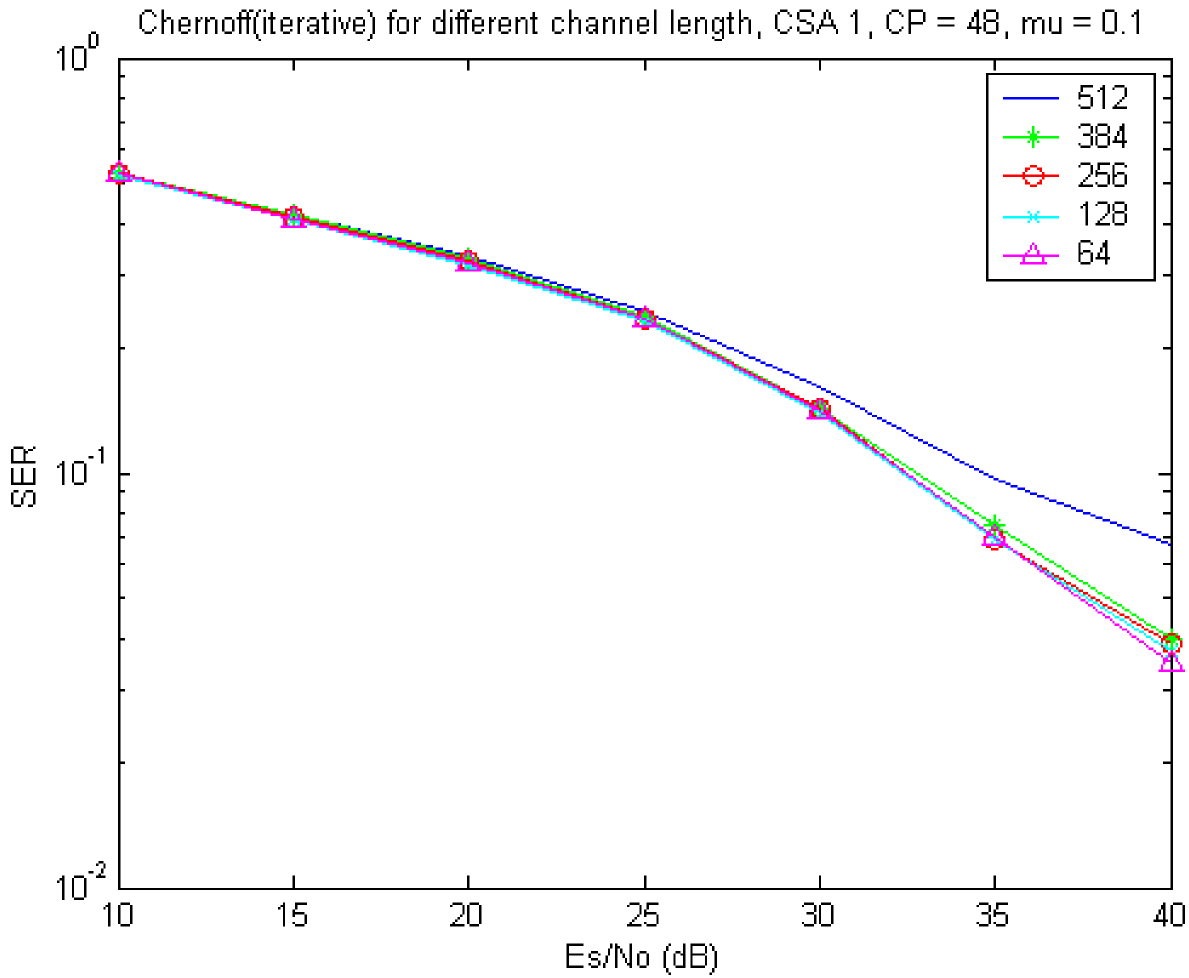
Results: different TEQs – high SNR



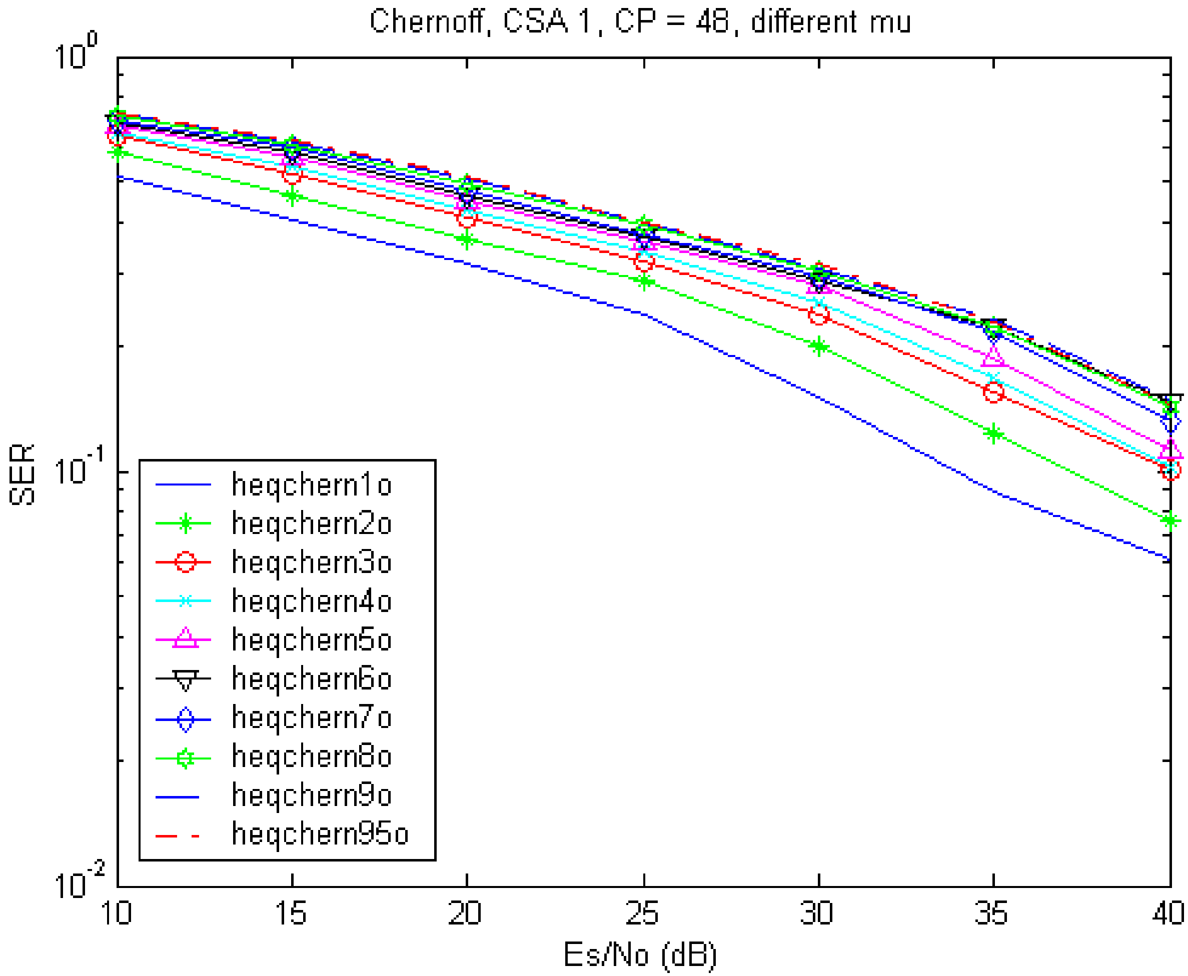
Results: Robustness against different channel lengths – Chernoff (nonlinear opt)



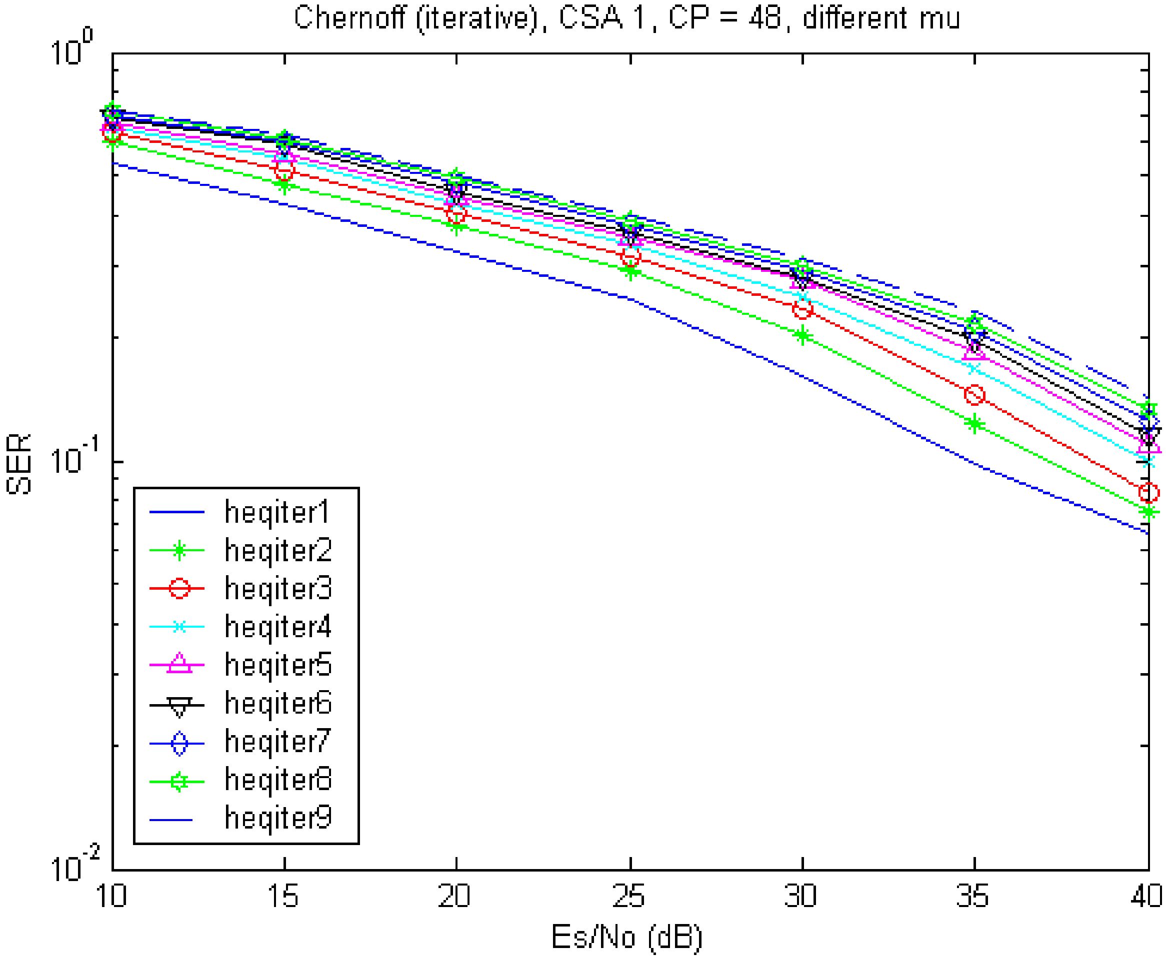
Results: Robustness against different channel lengths – Chernoff (iterative)



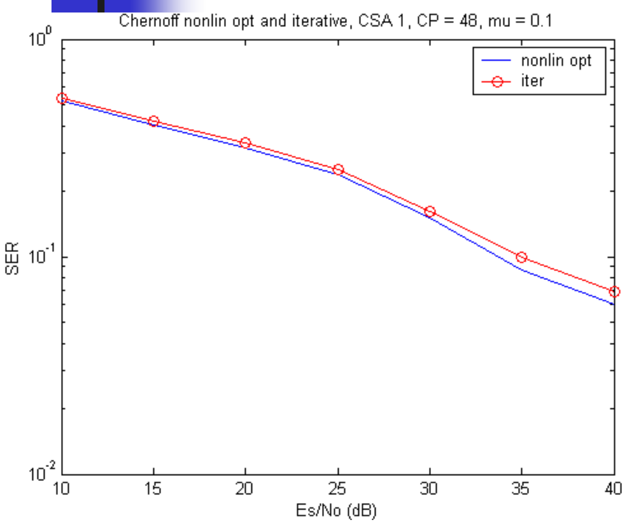
Results: Different μ (nonlinear opt)

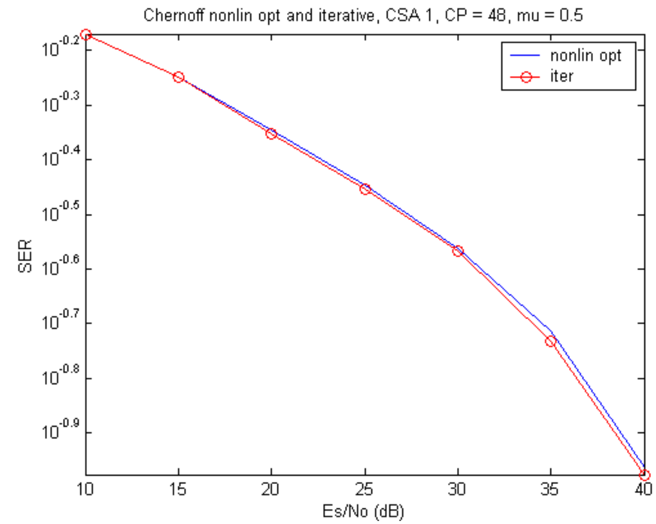


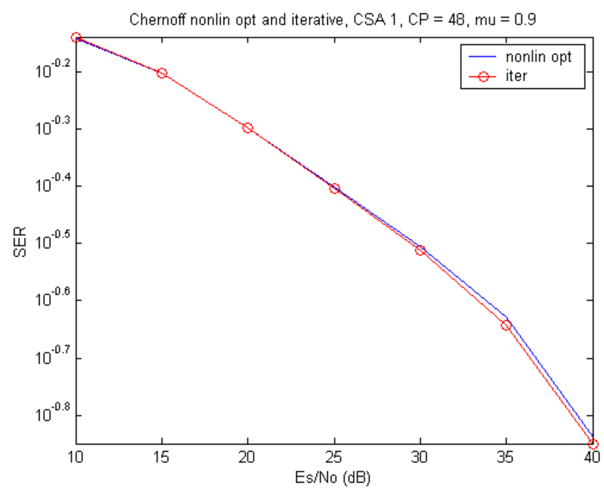
Results: Different μ (iterative)



Results: nonlinear opt *vs.* iterative







Conclusion

- BER optimized TEQ design
  - 2 solutions
- Better BER than EIGENFILT and MBR
- Robustness:
  - different channel length
- Future work
  - Effects of delay and window size

Publications

- C.C. Fung and C.-W. Kok, “Bit Error Rate Optimized Time-Domain Equalizers for DMT Systems”, *Proc. of the 14th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications,* Sept. 2003.
- C.C. Fung and C.-W. Kok, “Bit Error Rate Optimized Time-Domain Equalizers for DMT Systems”, *submitted to the IEEE Trans. on Communications,* Nov. 2003.

Part II: Blind Estimation Using HOS

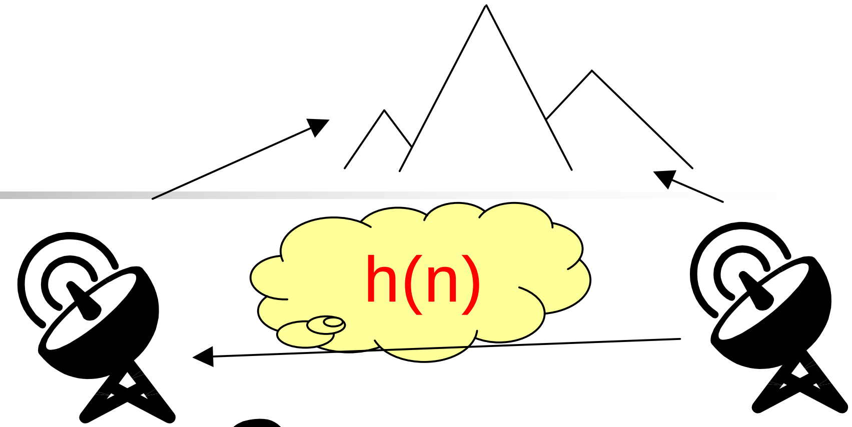
(joint work with Prof. Zhi Ding at UC Davis)

Problem and Objective

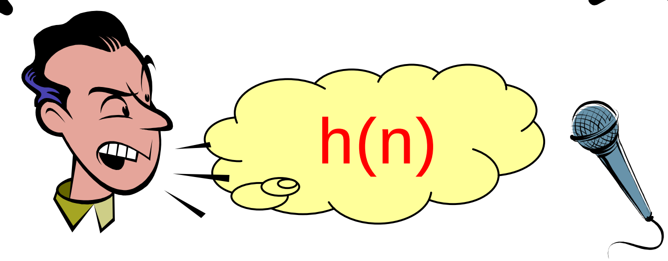
- Knowns
  - Output signal
  - Statistics of input signal
- Unknowns
  - Input signal
  - System/channel response
  - Additive noise
- Goals:
  - Estimate unknown system/channel coefficients
  - Estimate channel length
  - Equalize effect of channel and additive noise

Applications

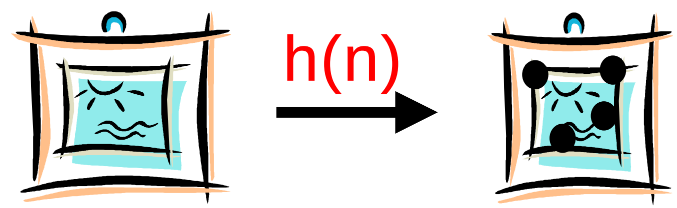
- Communications



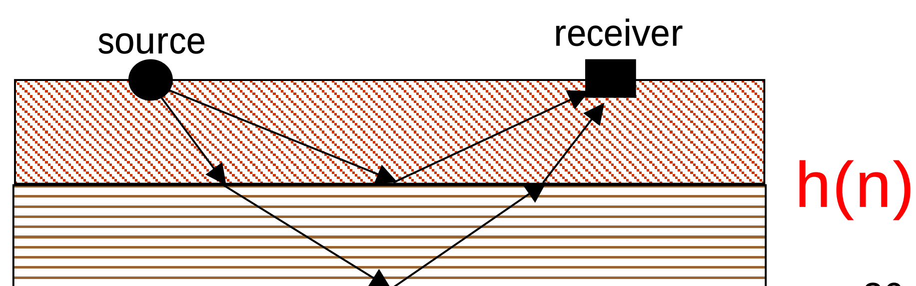
- Speech recognition and reverberation cancellation



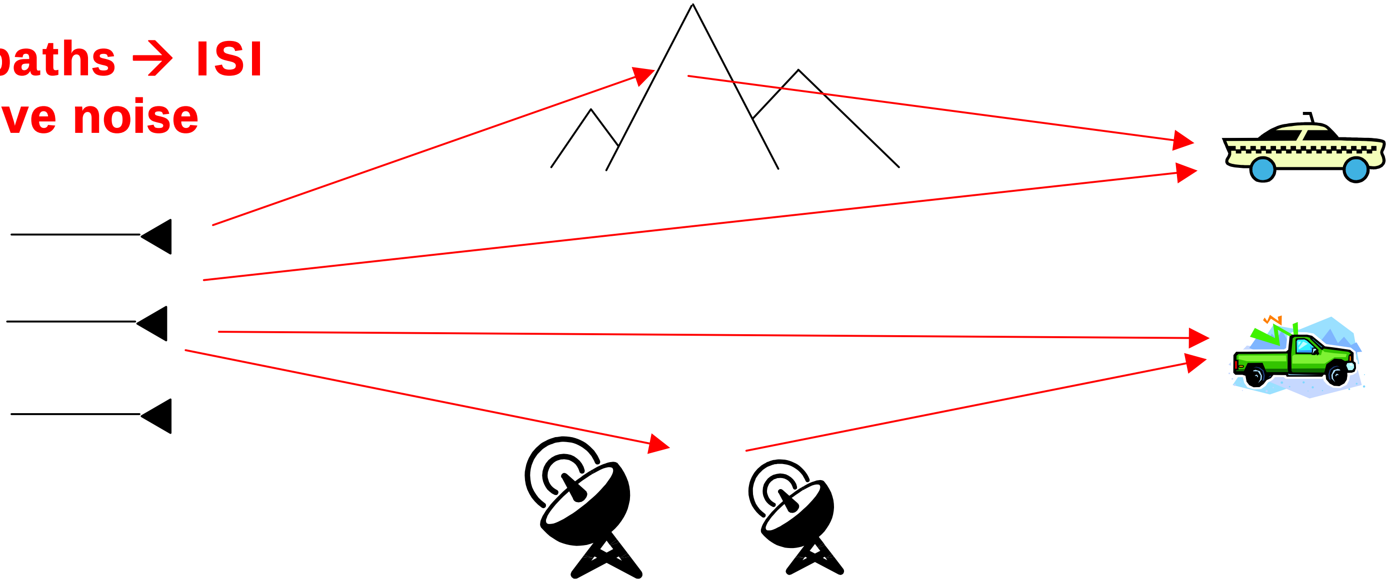
- Image restoration

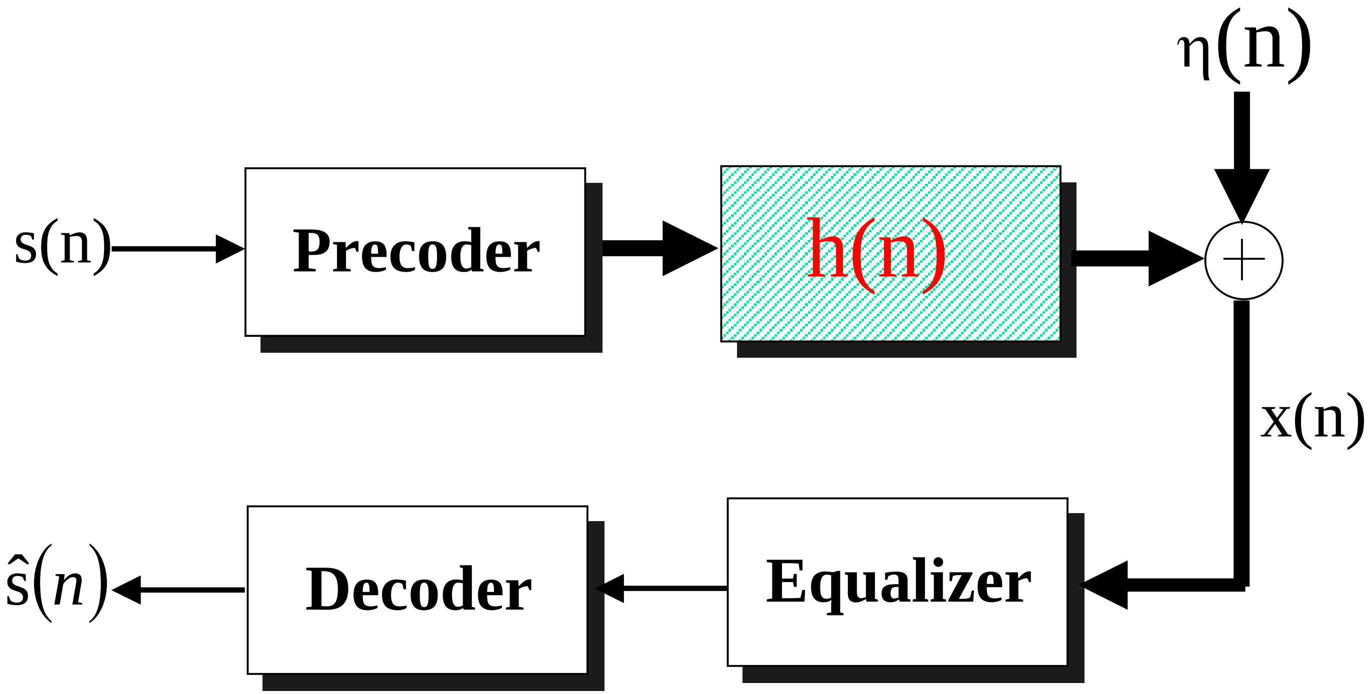


- Seismology



- Multipaths → ISI
- Additive noise





Some Blind Estimation Techniques

- Maximum Likelihood
- Adaptive
  - Constant-Modulus Algorithm
- Second-order statistics (SOS)
  - Cyclostationary signal (e.g. Tong, Gardner)
- Higher-order statistics (HOS)
  - Fourth-order cumulant (e.g. GM method, Tugnait, Ding)
- Linear prediction method

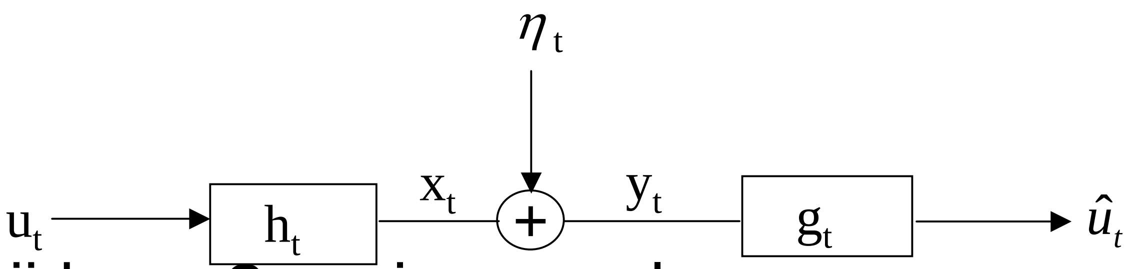
Advantages & Disadvantages of using 4th order Cumulant

- Advantages:
  - Good low SNR estimator when noise is Gaussian distributed
  - Can identify “singular” channels (common zeros)
- Disadvantages:
  - Requires more data to obtain good estimates compared to SOS based techniques

Our Formulations

- Formulate the blind channel estimation problem as a singular matrix pencil problem using cumulant slice
- Will show:
  - Eigenvalues contain the magnitude of the channel coefficients
  - Eigenvectors contain the zero-forcing equalizer (ZFE) coefficients

SISO system



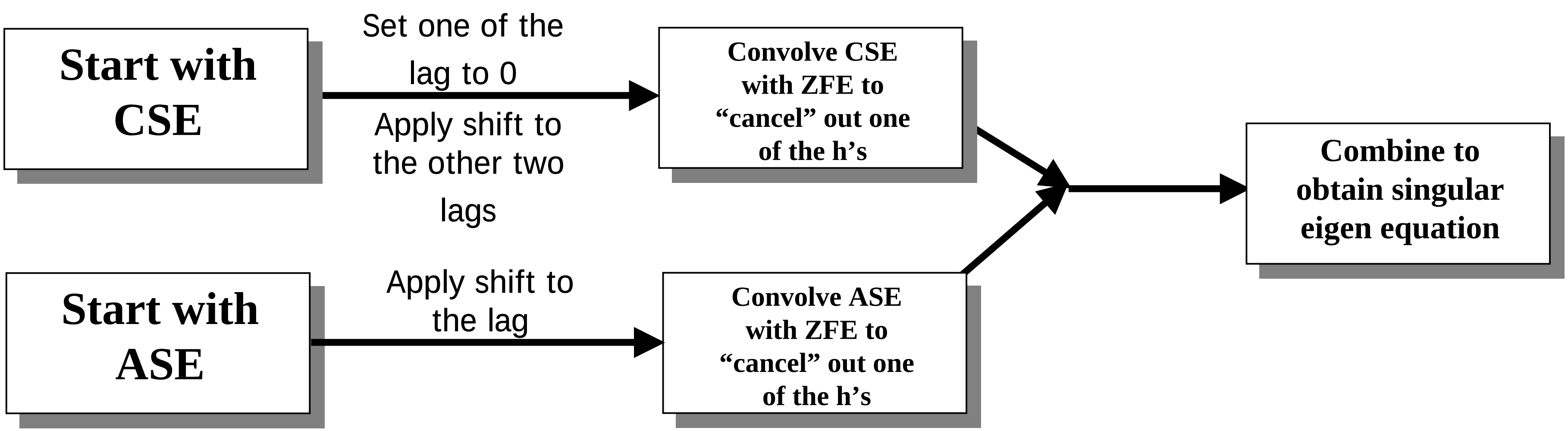
- u_t iid non-Gaussian complex r.v.
- η_t additive Gaussian noise
- h_0 and h_q are not equal 0
- q is assumed to be known
- $g_t * h_t = \delta(t-D)$, i.e. g_t is a zero-forcing equalizer
  - *delay D ignored*

Basic Idea for Cumulant Slice

Autocorrelation System Equation (ASE) : $r(m) = \gamma_{1,1} \sum_{k=0}^{q} h_k h_{k+m}^{*}$ and $s(m) E\left[u_t^2\right] \sum_{k=0}^{q} h_k h_{k+m}$

Cumulant System Equation (CSE) : $c(l, m, n) = \mu_{2,2} \sum_{t=0}^{q} h_t h_{t+l}^{*} h_{t+m}^{*} h_{t+n}$

Definition : $c(l, m, n) = d(l, m, n) - r(l) r(m-n) - r(m) r(l-n) - s(n) s^{*}(m-l)$



Conclusion

- Blind channel estimation problem formulated as an generalized eigen problem
  - eigenvalue = magnitude of channel coefficients
  - eigenvector = zero-forcing equalizer
- Channel coefficients can be estimated by using the eigenvector
- Channel order may be estimated by noting number of non-zero eigenvalues
- Future Work
  - MIMO
  - Frequency Estimation
  - DOA

References – TEQ (1)

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References – TEQ (2)

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References – Cumulant

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