COMBINED QUANTIZER AND LINEAR ERROR CONTROL CODE DESIGN FOR NOISY CHANNELS

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ABSTRACT

Most optimal quantizer design algorithms do not take into account the changes of the channel characteristics due to the inserted channel coder. In this paper, the overall channel characteristics including the channel coder is examined and an approximation model is proposed. Based on this model, a method for designing a quantizer (source coder) and the error control code together to achieve the best overall performance is proposed. Preliminary, simulation results reinforce the speculation that the error control codes would be useful only when the raw error rate is below a certain value.

1. INTRODUCTION

Shannon [1] showed that in the asymptotical cases (very large data blocks), source coding and channel coding problems can be treated separately without sacrificing the overall optimality. However, in Shannon's derivations there is no constraint on the delay and the complexity of the coders. In practical systems, we often cannot identify the source and the channel models perfectly; thus, the "separation principle" may not be an efficient approach for image transmission over wireless channels, where the communication channel is time-varying and noisy and the data bit rate is very low. Under the realistic constraint of limited complexity, the combined source and channel coding approach may be able to offer a superior performance than the separated coding approach.

The goal of this paper is to design a source coder (a scalar or vector quantizer) and the error control code (ECC) simultaneously to achieve the best overall performance. The concept behind our approach is as follows. In many applications such as image over wireless channel, we are often given a bit rate upper bound and a raw transmission bit error probability. Now considering simply a continuous memoryless source (such as image transform coefficients), we look for the optimal scalar quantizer and error control code that together would produce the least distortion at the destination. The total distortion at the decoder has three portions: source compression error, channel transmission error, and the cross effect due to the interaction between these two terms. If we use a strong channel code, the transmission error can be neglected and the total distortion is caused mainly by the source coding. However, a strong channel code implies a significant amount of redundancy being introduced. That is, fewer bits can be used by the source coder and thus the source coding error increases. On the other hand, if the source coder uses up all the channel capacity, the transmission error probability is high and thus may become the dominate factor in the total destination distortion. What we look for is the optimal operating point at which the best compromise is achieved.

2. SUPER CHANNEL MODEL

Most optimal quantizer design algorithms for a noisy channel do not consider the effects caused by the channel coder [2]; that is, the design of a quantizer depends only on the channel noise without including the changes of channel characteristics due to the inserted channel coder. Figure 1 shows the performance of various 4-level Gaussian signal quantizers operated under different crossover probabilities of binary symmetric channels (BSC). The solid lines are quantizers designed for specific crossover error probabilities 0.0001, 0.001, 0.01, 0.1, 0.25, and 0.5. The dash line, in Figure 1, is the lower bound of these solid lines; that is, it is the trajectory of all the minimum MSE 4-level quantizers each is designed and operated at single crossover error probability. These results show that the quantizer designed for a lower noise channel, say, does not perform well on a very noisy channel.

Now, let us consider the effect of channel coding. We examine the overall channel characteristics including the channel coder and propose an approximation

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model. As shown in Figure 2, the *super channel*, representing the channel I/0 characteristics with channel coding, is a function of the original channel bit error probability and the channel coder. We like to find the *super channel* characteristics and derive the relationship between the original noisy channel and the super channel.

Assume a t error-correcting (N, K) linear block code is in use as shown by Figure 3, and the original channel is a memoryless binary symmetric channel (BSC) with crossover probability p. It is conjectured and proved by simulations that the behavior of the super channel is approximately also a BSC with a different crossover probability p'. We have not been able to derive the exact expression of p', but its upper and lower bounds can be estimated. According to the results given by Mac-Williams and Sloane [3] and Cain and Simpson [4], for a (N, K) linear block code, we can obtain the bounds of p', $p_{min} \leq p' \leq p_{max}$, where

$$p_{max} = \sum_{i=t+1}^{N-t-1} \frac{(t+i)}{N} \binom{N}{i} p^i (1-p)^{N-i} + \sum_{i=N-t}^{N} \binom{N}{i} p^i (1-p)^{N-i}, \quad (1)$$

$$p_{min} = \frac{1}{K} P_m(E), \quad (2)$$

and $P_m(E)$ is upper bounded by

$$P_m(E) \leq \sum_{i=t+1}^{N} \binom{N}{i} p^i (1-p)^{N-i}. \quad (3)$$

The equality holds for perfect codes. Figures 4 and 5 show the simulation results of a BCH $(255, K)$ code over a BSC with crossover probabilities 0.1 and 0.01.

3. COMBINED QUANTIZER AND ECC DESIGN

For a BSC with crossover probability p and a given length N, the crossover probability p' of the super channel and its bounds p_{min} and p_{max} are functions of t, the error correction length; that is, $p_{min}(t) \leq p'(t) \leq p_{max}(t)$. Now, we take the worst case to design the optimal quantizer over a super channel. In other words, we assume that the crossover probability $p'(t)$ equals to the upper bound $p_{max}(t)$. Then we look for the optimum parameters: the number of quantization levels M, the quantizer decision thresholds x_i^*, the reconstruction levels y_j^*, and the error-correction length t. Based on above assumptions, the MSE of the entire system, $d(\cdot)$, can be expressed as

$$d(x_i, y_j, t, M) = \sum_{i=1}^{M} \sum_{j=1}^{M} P_{j/i}(t) \int_{x_i}^{x_{i+1}} (x - y_i)^2 p_x(x) dx, \quad (4)$$

where x_i $(i = 2, 3, \ldots, M)$ is the decision threshold, y_j $(j = 1, 2, \ldots, M)$ is the reconstruction level, t refers to the error-correction length, and M is the number of the quantizer levels. The transition probability $P_{j/i}(t)$ in (4) is defined by

$$P_{j/i}(t) = p'(t)^{d_H(y_i, y_j)} (1 - p'(t))^{K - d_H(y_i, y_j)}, \quad (5)$$

where $p'(t)$ is the super channel crossover probability, K is the message length in a BCH (N, K) code, and $d_H(y_i, y_j)$ is the Hamming distance between y_i and y_j. Now, the problem becomes a multi-dimension optimization problem. However, not all the variables in $d(\cdot)$ are floating numbers; for example, t and M only allow integer values. To simplify the problem, we first assume that M is fixed. Then for a fixed t, the function $d(x_i, y_j, t)$ becomes $d_t(x_i, y_j)$. Hence, the necessary conditions for minimizing $d_t(x_i, y_j)$ are

$$\begin{cases} \frac{\partial f_t(x_i, y_j)}{\partial x_i} = 0; & i = 2, 3, \ldots, K; \\ \frac{\partial f_t(x_i, y_j)}{\partial y_j} = 0; & j = 1, 2, \ldots, K. \end{cases} \quad (6)$$

The optimum $x_i{}^*$ and $y_j{}^*$ become

$$x_i^* = \frac{\sum_{j=1}^{K} [y_j^{*2} (P_{j/(i-1)}(t) - P_{j/i}(t))]}{2 \sum_{j=1}^{K} [y_j^* (P_{j/(i-1)}(t) - P_{j/i}(t))]}, \quad (7)$$

and

$$y_j^* = \frac{\sum_{i=1}^{K} P_{j/i}(t) \int_{x_i^*}^{x_{i+1}^*} x p_x(x) dx}{\sum_{i=1}^{K} P_{j/i}(t) \int_{x_i^*}^{x_{i+1}^*} p_x(x) dx}. \quad (8)$$

Thus far, the optimal x_i^* and y_j^* for a specific t have been derived. The performance of three 4-level quantizers is shown in Figure 6. The dashed-line is the quantizer optimized for a crossover probability (in Figure 1) and its outputs passing through a channel with the same crossover probability. The dotted line is the same quantizer outputs passing through a $BCH(255, 179)$ code ($t = 10$) before entering the same channel. The solid line is the quantizer designed for the super channel containing both the BCH codec and the original raw channel, the so-called combined-designed quantizer. We use the upper bound in modeling the super channel. When the raw rate > 0.04, this upper bound is greater than the raw rate. Hence, the optimal quantizer with BCH code has a higher MSE than the one without. The combined-designed quantizer takes the advantage of the changing characteristics of the channel when a BCH code is added. When the channel crossover probability ≤ 0.04, the combined quantizer with BCH code achieves a better performance than the quantizer designed without considering channel coder. However, when the crossover probability > 0.04, it is the opposite. This is due to the fact that when the average error rate is greater than 10 error bits (255×0.04) in one block, the channel coder can not decode the error bits correctly. It turns out the decoded error probability (upper bound) is larger than that of the uncoded one. Therefore, if the raw error rate is higher a certain threshold rate (related to t), the combined quantizer with ECC is not preferred.

The ultimate justification of a coder performance is its rate-distortion curve (R-D curve). Figure 7 indicates that for a certain portion of the channel crossover probability (error probability < 0.02) the combined method is the best (close to the ideal Gaussian source $R-D$ curve, the lower left solid line). This is the region where the BCH code is most effective. Note that the horizontal axis in Figure 7 is the transmission bit rate over channel (including BCH-insteated bits). The mismatched case (optimal quantizer with BCH code) is inferior because it has higher MSE and also a higher bit rate.

4. CONCLUSION

The goal of this paper is to design a scalar quantizer and the error control code simultaneously to obtain the best performance. In section 2, we propose a super channel model which represents the channel I/O characteristics when the channel coder is included. The super channel channel model is a function of the original channel bit error probability and the channel coder. Based on the super channel model, a combined quantizer and linear ECC design for noisy channels has been proposed. The preliminary simulation results show several interesting results. They reinforce the speculation that the error control codes are useful only when the raw error rate is smaller than a certain value. In addition, the combined design produces a better (smaller MSE) quantizer for small raw error rates.

5. REFERENCES

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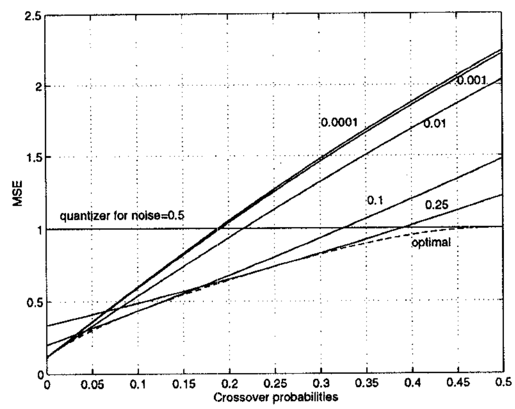

Figure 1: *The performance of an optimum quantizer over noisy channels.*

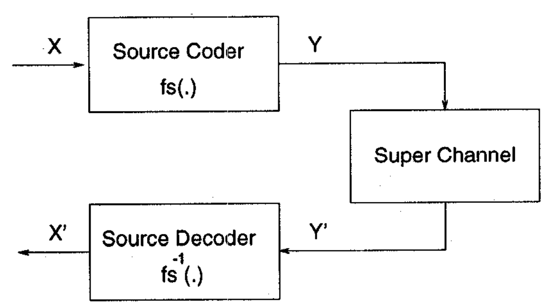

Figure 2: *A communication system block diagram.*

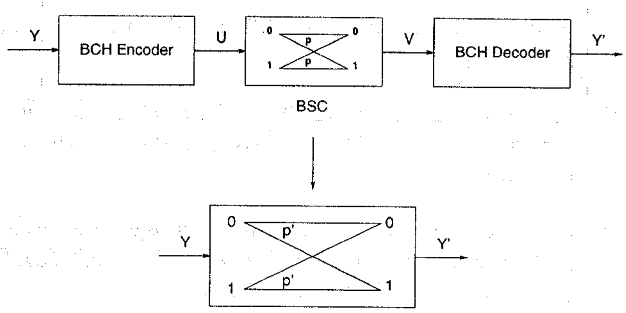

Figure 3: *Super channel.*

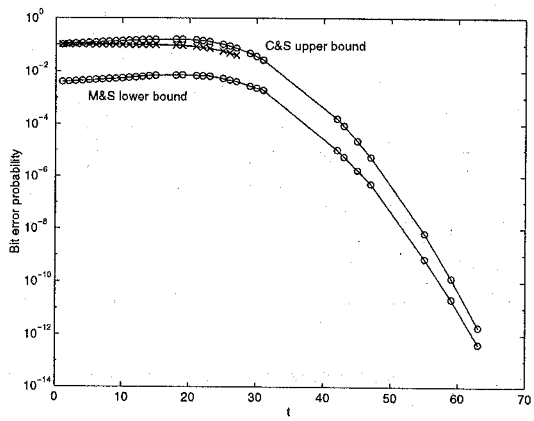

Figure 4: *The bounds of the crossover probability p' of the supper channel ($p = 0.1$ and BCH(255,K) codes).*

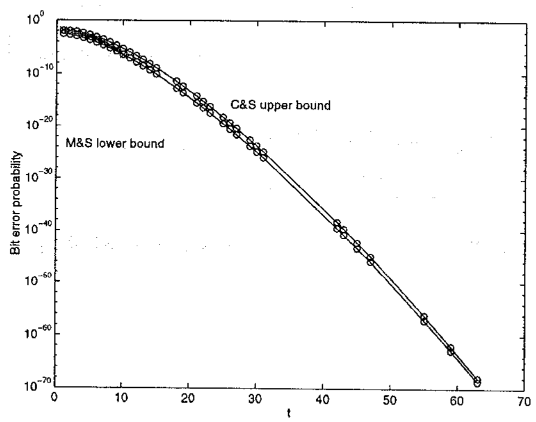

Figure 5: *The bounds of the crossover probability p' of the supper channel ($p = 0.01$ and BCH(255,K) codes).*

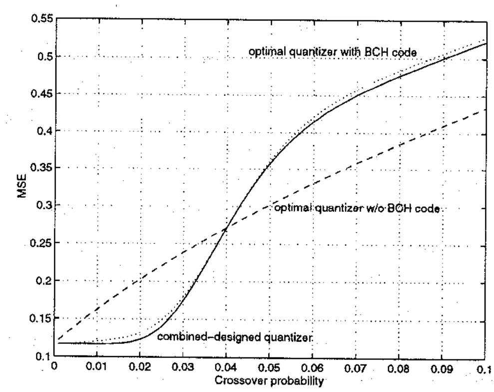

Figure 6: *The performance of three quantizer with and without combined method over noisy channels.(BCH(255,179) code is used.)*

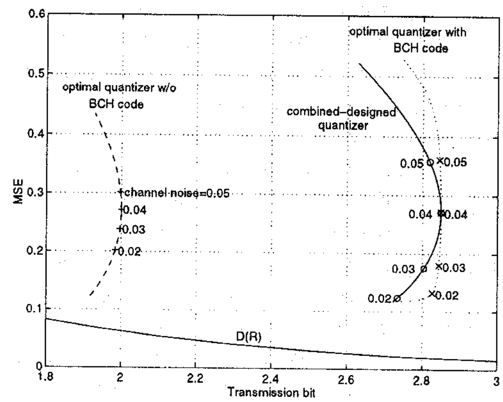

Figure 7: *MSE v.s. Transmission bit for a optimum quantizer with and without a combined method.*